Solving quadratic equations: $ax^2+bx+c=0$

Method :

- If necessary, re-arrange the equation so one side of the equation is "0"
- Factorise the quadratic expression (bracket_1)(bracket_2) = 0
- Solve bracket_1 = 0 and bracket_2 = 0 to find the two solutions

The equation is already factorised

- Solve $(x+2)(x-4)=0$

  $x+2=0 \;\; or \;\; x-4=0$

  $x=-2 \;\; or \;\; x=4$

- Solve $(3x-2)(x+3)=0$

  $3x-2=0 \;\; or \;\; x+3=0$

  $x=\dfrac{2}{3} \;\; or \;\; x=-3$

Factorising x^2+bx+c and solving $x^2+bx+c=0$

If you want to factorise the expression x^2+bx+c

you need to find two numbers, p and q so that $p\times q=c$ *and* $p+q=b$

then $x^2+bx+c=(x+p)(x+q)$

- Solve $x^2+8x+15=0$ $\qquad 15=1\times15=3\times5$ *and* $3+5=8$

  $(x+3)(x+5)=0$

  $x+3=0 \;\; or \;\; x+5=0$

  $x=-3 \;\; or \;\; x=-5$

- Solve $x^2-3x-10=0$ $\qquad -10=-1\times10=1\times-10=2\times-5=-2\times5$

  $(x-5)(x+2)=0$ $\qquad$ and $2+-5=-3$

  $x-5=0 \;\; or \;\; x+2=0$

  $x=5 \;\; or \;\; x=-2$

Re-arrange, factorise and solve

- Solve $x^2+10x=-24$ $\quad$ ($+24$ *to both sides*)

  $x^2+10x+24=0$

  $(x+6)(x+4)=0$

  $x+6=0 \;\; or \;\; x+4=0$

  $x=-6 \;\; or \;\; x=-4$

- Solve $(x+1)(x-2)=4$ $\quad$ (expand *the brackets*)

  $x^2+x-2x-2=4$

  $x^2-x-2=4$

  $x^2-x-6=0 \Leftrightarrow (x-3)(x+2)=0$

  $x-3=0 \;\; or \;\; x+2=0 \Leftrightarrow x=3 \;\; or \;\; x=-2$

The quadratic formula

The purpose of this formula is to solve quadratic equations

in the form $ax^2 + bx + c = 0$

Step 1: *state the value of* a, b *and* c

(*including the sign before the number*)

$a = \ldots\ldots\ ,\ b = \ldots\ldots\ ,\ c = \ldots\ldots$

Step 2: Work out the discriminant : $\Delta = b^2 - 4ac = (b)^2 - 4 \times a \times c$

(*use brackets around* b)

Step 3: Work out the solutions

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{\Delta}}{2a}$$

(*use the fraction button of your calculator* $\frac{\square}{\square}$)

- Solve the equation $2x^2 + x - 8 = 0$, giving your answers to 2 decimal places.

  ⊗ $a = 2\ ,\ b = 1\ ,\ c = -8$

  ⊗ The discriminant $b^2 - 4ac = (1)^2 - 4 \times 2 \times -8 = 65$

  ⊗ The solutions are : $x_1 = \frac{-1 + \sqrt{65}}{4}$ *and* $x_2 = \frac{-1 - \sqrt{65}}{4}$

  $x_1 = 1.77 \qquad x_2 = -2.27$

The completed square form

- The expression $x^2 + bx$ can be written $(x + p)^2 - p^2$ where $p = \frac{b}{2}$

Examples: $x^2 + 4x = (x + 2)^2 - 4 \qquad x^2 - 6x = (x - 3)^2 - 9$

This fact can be used to transform any expression of the form $x^2 + bx + c$

Examples: $\underbrace{x^2 + 4x}_{(x+2)^2 - 4} + 3 = (x + 2)^2 - 4 + 3 = (x + 2)^2 - 1$

$\underbrace{x^2 - 6x}_{(x-3)^2 - 9} - 7 = (x - 3)^2 - 9 - 7 = (x - 3)^2 - 16$

Solving equations using the completed square form

- $\underbrace{x^2 + 6x}_{(x+3)^2 - 9} - 16 = 0$

  $(x + 3)^2 - 9 - 16 = 0$

  $(x + 3)^2 - 25 = 0$ *Now make* x *the subject* :

  $(x + 3)^2 = 25$

  $x + 3 = 5$ *or* $x + 3 = -5$

  $x = 2$ *or* $x = -8$

- $\underbrace{x^2 + 14x}_{(x+7)^2 - 49} - 5 = 0$

  $(x + 7)^2 - 49 - 5 = 0$

  $(x + 7)^2 - 54 = 0$

  $(x + 7)^2 = 54$

  $x + 7 = \sqrt{54}$ *or* $x + 7 = -\sqrt{54}$

  $x = \sqrt{54} - 7$ *or* $x = -\sqrt{54} - 7$