Dimensional analysis – exam questions

Question 1: Jun 2006 – Q1

The time T taken for a simple pendulum to make a single small oscillation is thought to depend only on its length l, its mass m and the acceleration due to gravity g.

By using dimensional analysis:

(a) show that T does **not** depend on m; *(3 marks)*

(b) express T in terms of l, g and k, where k is a dimensionless constant. *(4 marks)*

Question 2: June 2007 – Q1

The magnitude of the gravitational force, F, between two planets of masses m_1 and m_2 with centres at a distance x apart is given by

$$F = \frac{Gm_1m_2}{x^2}$$

where G is a constant.

(a) By using dimensional analysis, find the dimensions of G. *(3 marks)*

(b) The lifetime, t, of a planet is thought to depend on its mass, m, its initial radius, R, the constant G and a dimensionless constant, k, so that

$$t = km^{\alpha}R^{\beta}G^{\gamma}$$

where α, β and γ are constants.

Find the values of α, β and γ. *(5 marks)*

Question 3: June 2008 – Q1

The speed, $v \text{ m s}^{-1}$, of a wave travelling along the surface of a sea is believed to depend on

the depth of the sea, d m,
the density of the water, $\rho \text{ kg m}^{-3}$,
the acceleration due to gravity, g, and
a dimensionless constant, k

so that

$$v = kd^{\alpha}\rho^{\beta}g^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, show that $\beta = 0$ and find the values of α and γ. *(6 marks)*

Question 4: June 2009 – Q1

A ball of mass m is travelling vertically downwards with speed u when it hits a horizontal floor. The ball bounces vertically upwards to a height h.

It is thought that h depends on m, u, the acceleration due to gravity g, and a dimensionless constant k, such that

$$h = km^{\alpha}u^{\beta}g^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, find the values of α, β and γ. *(5 marks)*

Question 5: June 2010 –Q1

A tank containing a liquid has a small hole in the bottom through which the liquid escapes. The speed, $u \text{ m s}^{-1}$, at which the liquid escapes is given by

$$u = CV\rho g$$

where $V \text{ m}^3$ is the volume of the liquid in the tank, $\rho \text{ kg m}^{-3}$ is the density of the liquid, g is the acceleration due to gravity and C is a constant.

By using dimensional analysis, find the dimensions of C. *(5 marks)*

Question 6: June 2011 – Q2

The time, t, for a single vibration of a piece of taut string is believed to depend on

the length of the taut string, l,
the tension in the string, F,
the mass per unit length of the string, q, and
a dimensionless constant, k,

such that

$$t = kl^{\alpha}F^{\beta}q^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, find the values of α, β and γ. *(5 marks)*

Question 7: June 2012 – Q2

A pile driver of mass m_1 falls from a height h onto a pile of mass m_2, driving the pile a distance s into the ground. The pile driver remains in contact with the pile after the impact. A resistance force R opposes the motion of the pile into the ground.

Elizabeth finds an expression for R as

$$R = \frac{g}{s}\left[s(m_1 + m_2) + \frac{h(m_1)^2}{m_1 + m_2}\right]$$

where g is the acceleration due to gravity.

Determine whether the expression is dimensionally consistent. *(4 marks)*

Dimensional analysis – exam questions

Question 1: Jun 2006 – Q1

(i)	$T^1 = L^a \times M^b \times (LT^{-2})^c$	M1A1	
	There is no M on the left, so $b = 0$	E1	3
ii)	$T^1 = L^{a+c} \times M^0 \times T^{-2}$	M1	
	$\begin{cases} -2c = 1 \\ a + c = 0 \end{cases}$	m1	
	$a = \frac{1}{2},\ c = -\frac{1}{2}$	m1	
	$\therefore$ Period $= kl^{\frac{1}{2}} g^{-\frac{1}{2}}$	A1F	4
	Total		**7**

Question 2: June 2007 – Q1

(a)	$MLT^{-2} = \frac{[G]MM}{L^2}$	M1 A1	
	$[G] = L^3M^{-1}T^{-2}$	A1F	3
(b)	$t = km^{\alpha}R^{\beta}G^{\gamma}$		
	$T = M^{\alpha}L^{\beta}M^{-\gamma}L^{3\gamma}T^{-2\gamma}$	M1 A1F	
	$-2\gamma = 1 \quad \Rightarrow \quad \gamma = -\frac{1}{2}$		
	$\alpha - \gamma = 0 \quad \Rightarrow \quad \alpha = -\frac{1}{2}$	m1 m1	
	$\beta + 3\gamma = 0 \quad \Rightarrow \quad \beta = \frac{3}{2}$	A1F	5
	Total		**8**

Question 3: June 2008 – Q1

1	$LT^{-1} = L^{\alpha} \times (ML^3)^{\beta}(LT^{-2})^{\gamma}$	M1	
	There is no M on the left hand side, so $\beta = 0$.	E1	
	$LT^{-1} = L^{\alpha+\gamma}T^{-2\gamma}$	m1	
	$\alpha + \gamma = 1$ $-2\gamma = -1$	m1	
	$\gamma = \frac{1}{2}$	A1	
	$\alpha = \frac{1}{2}$	A1	6
	Total		**6**

Question 4: June 2009 – Q1

1	$L = M^{\alpha}(LT^{-1})^{\beta}(LT^{-2})^{\gamma}$	M1A1	
	$\beta + \gamma = 1$ $-\beta - 2\gamma = 0$ $\alpha = 0$	m1	
	$\gamma = -1$	m1	
	$\beta = 2$	A1F	5
	Total		**5**

Question 5: June 2010 –Q1

1	LT^{-1}	B1	
	$LT^{-1} = M^{\alpha}L^{\beta}T^{\gamma} \times L^3 \times ML^{-3} \times LT^{-2}$	M1 A1	
	$1 = \beta + 1$ $-1 = \gamma - 2$		
	$0 = \alpha + 1$	m1	
	$\beta = 0,\ \ \alpha = -1,\ \ \gamma = 1$		
	The dimensions of C are $M^{-1}T$	A1F	5
	Alternative :		
	LT^{-1}	(B1)	
	$LT^{-1} = C \times L^3 \times ML^{-3} \times LT^{-2}$	(M1A1)	
	$LT^{-1} = C \times L\ MT^{-2}$	(m1)	
	The dimensions of C are $M^{-1}T$	(A1F)	5
	Total		**5**

Question 6: June 2011 – Q2

$T^1 = L^{\alpha}(MLT^{-2})^{\beta}(ML^{-1})^{\gamma}$	M1 A1	
$\alpha + \beta - \gamma = 0$ $\beta + \gamma = 0$	m1	
$-2\beta = 1$ $\beta = -\frac{1}{2}$		
$\gamma = \frac{1}{2}$	m1 A1F	5
$\alpha = 1$		
		5

Question 7: June 2012 – Q2

Dimension of g is LT^{-2} Dimension of s is L Dimension of h is L Dimension of m_1 and m_2 is M	B1	
Dimension of $\frac{g}{s}[s(m_1 + m_2) + \frac{hm_1^2}{m_1 + m_2}]$ is		
$\frac{LT^{-2}}{L}[LM + \frac{LM^2}{M}] \cong MLT^{-2} + MLT^{-2}$	M1	
$\cong MLT^{-2}$	A1	
which is a force	B1	4
Total		**4**