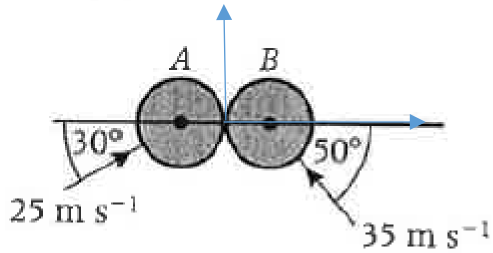
TYPICAL 2D collision exercise

When solving problems involving oblique collisions between smooth spheres:

- note that the components of velocity perpendicular to the line of centres will be unchanged,
- apply conservation of momentum along the line of centres,
- use the coefficient of restitution with the components of velocity along the line of centres.

The diagram shows two identical smooth spheres colliding obliquely. The coefficient of restitution for collision between the spheres is 0.7. Find the speed of sphere A immediately after the collision and the angle between its direction of motion and the line of centres.

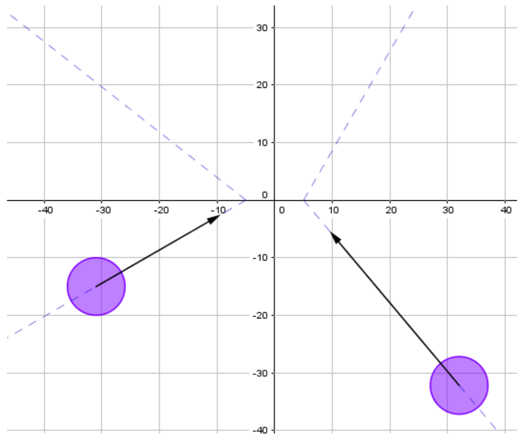


Initial velocity vectors

$$u_A = \begin{pmatrix} 25Cos(30) \\ 25Sin(30) \end{pmatrix} \qquad u_B = \begin{pmatrix} -35Cos(50) \\ 35Sin(50) \end{pmatrix}$$

Velocity vectors after collision

$$v_A = \begin{pmatrix} v_{Ax} \\ 25Sin(30) \end{pmatrix} \qquad v_B = \begin{pmatrix} v_{Bx} \\ 35Sin(50) \end{pmatrix}$$



• *The y – components are unchanged*

•Conservation of momentum along the line of centres:

$$m_A u_{Ax} + m_B u_{Bx} = m_A v_{Ax} + m_B v_{Bx}$$

$$m \times 25Cos(30) + m \times -35Cos(50) = mv_{Ax} + mv_{Bx}$$

$$-0.847 = v_{Ax} + v_{Bx}$$

• Law of restitution:

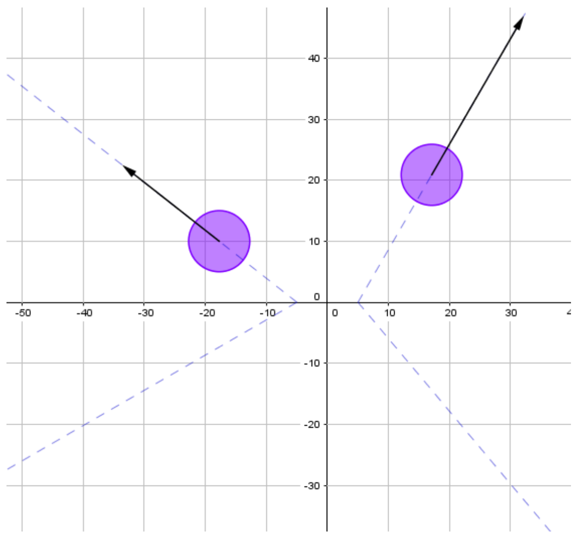
$$v_{Ax} - v_{Bx} = -e(u_{Ax} - u_{Bx})$$

$$v_{Ax} - v_{Bx} = -0.7(25Cos(30) - (-35Cos(50)))$$

$$v_{Ax} - v_{Bx} = -30.904$$

Solve simultaneously :

$$\begin{cases} v_{Ax} + v_{Bx} = -0.847 \\ v_{Ax} - v_{Bx} = -30.904 \end{cases} \qquad \begin{cases} v_{Ax} = -15.9\ m/s \\ v_{Bx} = 15.1 m/s \end{cases}$$



Conclusion :

$$v_A = \begin{pmatrix} -15.9 \\ 12.5 \end{pmatrix} \qquad v_B = \begin{pmatrix} 15.1 \\ 26.8 \end{pmatrix}$$

The speed of sphere A: $\sqrt{(-15.9)^2 + 12.5^2} = 20.2m/s$

The speed of sphere B: $\sqrt{15.1^2 + 26.8^2} = 30.8m/s$

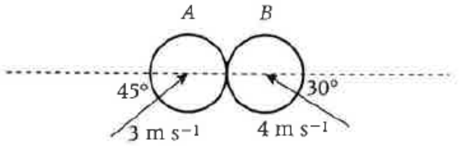
"Direction of motion"=angle made with the line of centres

Sphere A: $Tan^{-1}\left(\frac{12.5}{15.9}\right) = 38.2^o$

Sphere B: $Tan^{-1}\left(\frac{26.8}{15.1}\right) = 60.6^o$

Have a go:

Two smooth spheres, A and B, are travelling on a smooth horizontal surface when they collide as shown in the diagram. The coefficient of restitution between the two spheres is 0.5. The mass of A is 2 kg and the mass of B is 4 kg.



Find the speeds of the spheres after the collision and the directions in which they move.

Initial velocity vectors

$u_A = \begin{pmatrix} \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \end{pmatrix} \qquad u_B = \begin{pmatrix} \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \end{pmatrix}$

Velocity vectors after collision

$v_A = \begin{pmatrix} v_{Ax} \\ \ldots\ldots\ldots\ldots \end{pmatrix} \qquad v_B = \begin{pmatrix} v_{Bx} \\ \ldots\ldots\ldots\ldots \end{pmatrix}$

• *The y – components are unchanged*

• Conservation of momentum along the line of centres:

$$m_A u_{Ax} + m_B u_{Bx} = m_A v_{Ax} + m_B v_{Bx}$$

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• Law of restitution:

$$v_{Ax} - v_{Bx} = -e(u_{Ax} - u_{Bx})$$

$v_{Ax} - v_{Bx} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$v_{Ax} - v_{Bx} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Solve simultaneously:

$\begin{cases} \ldots\ldots v_{Ax} + \ldots\ldots v_{Bx} = \ldots\ldots \\ v_{Ax} - v_{Bx} = \ldots\ldots\ldots \end{cases} \qquad \begin{cases} v_{Ax} = \ldots\ldots\ m/s \\ v_{Bx} = \ldots\ldots\ m/s \end{cases}$

Conclusion:

$v_A = \begin{pmatrix} \ldots\ldots\ldots \\ \ldots\ldots\ldots \end{pmatrix} \qquad v_B = \begin{pmatrix} \ldots\ldots\ldots \\ \ldots\ldots\ldots \end{pmatrix}$

The speed of sphere A: $\sqrt{(\ldots\ldots)^2 + (\ldots\ldots)^2} = \ldots\ldots m/s$

The speed of sphere B: $\sqrt{(\ldots\ldots)^2 + (\ldots\ldots)^2} = \ldots\ldots m/s$

"Direction of motion"=angle made with the line of centres

Sphere A: $Tan^{-1}\left(\dfrac{\ldots\ldots}{\ldots\ldots}\right) = \ldots\ldots^o$

Sphere B: $Tan^{-1}\left(\dfrac{\ldots\ldots}{\ldots\ldots}\right) = \ldots\ldots^o$

$v_A = 4.06 m/s \ \ at \ \ 31.5^o$

$v_B = 2.11 m/s \ \ at \ \ 108.6^o$