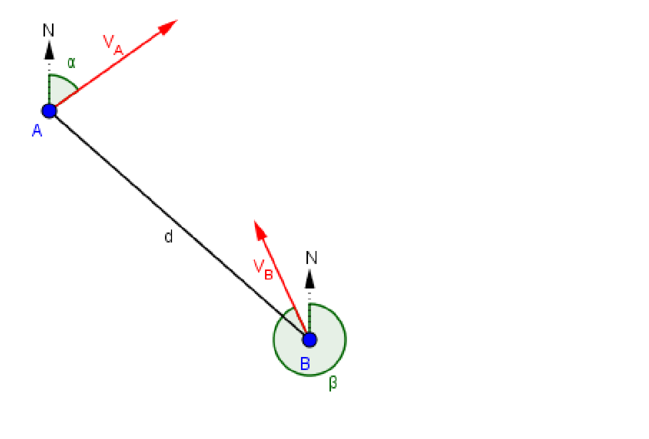
Relative motion –Speeds and bearings

<u>*The typical setting*</u>:

Object A: V_A *at a bearing of* α

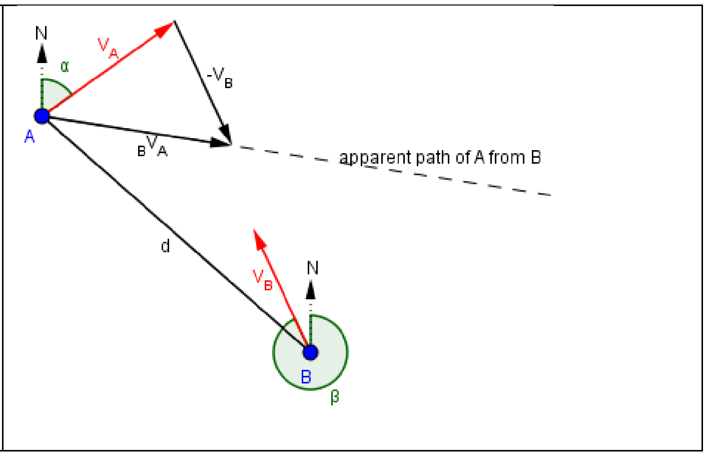
Object B: V_B *at a bearing of* β

The object B is at a distance d *from A at a bearing of* γ



Let's work out /construct the vector ${}_BV_A = V_A - V_B$

Velocity of A relative to B

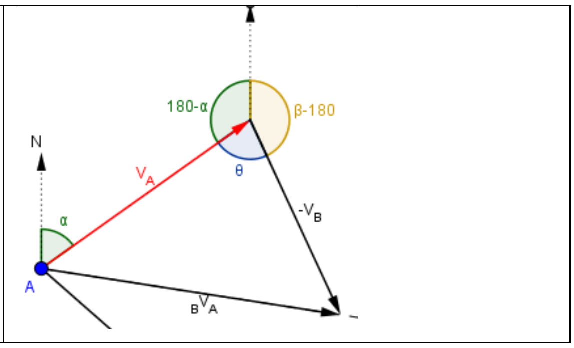


To work out ${}_BV_A$, use trigonometry in the triangle formed by the three vectors

The angle θ can be worked out using angle properties and the "Cos rule" gives

$${}_BV_A^{\,2} = V_A^{\,2} + V_B^{\,2} - 2\times V_A \times V_B \times Cos\theta$$

The bearing of ${}_BV_A$ can be found using the "Sin rule"



Example:

A cyclist, *A*, is travelling east at 20 km h^{-1}. A second cyclist,
B, is travelling on a bearing of 120° at a speed of 15 km h^{-1}.
Find the magnitude and bearing of the velocity of *A* relative
to *B*.

$-V_B$ *has a bearing of* 300^0, *angle* $GEA = 90 - 60 = 30^0$

• ${}_BV_A^{\,2} = 20^2 + 15^2 - 2\times 20\times 15\times Cos(30) = 105.38$

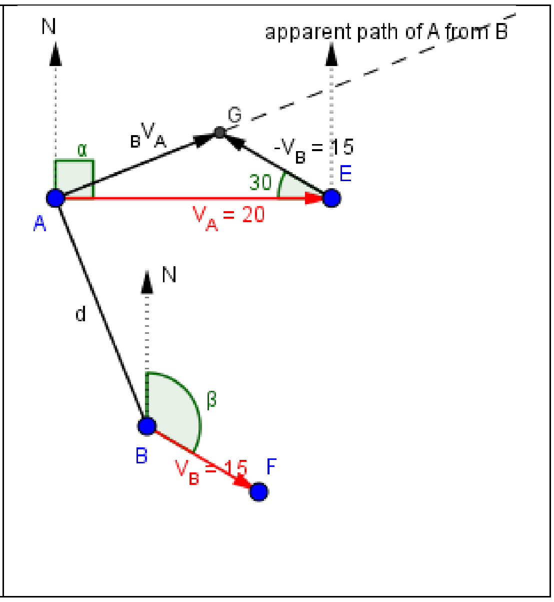
${}_BV_A = 10.3\,km.h^{-1}$

• *The angle GAE*:

$$\frac{Sin\,GAE}{15} = \frac{Sin30}{10.3} \qquad Angle\ GAE = 46.73$$

The bearing of the velocity of A relative B is

$90 - 46.73 = 043.2^o$



Interception

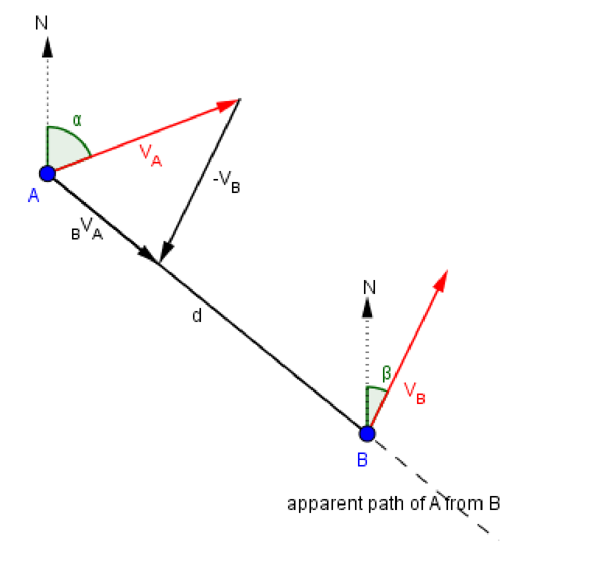
Let's called A_0 *and* B_0 *the initial position of A and B*

The object A and B will INTERCEPT if/when

$_BV_A$ *is parallel to the line* A_0B_0

i.e: the apparent path of A (view from B) will go through B

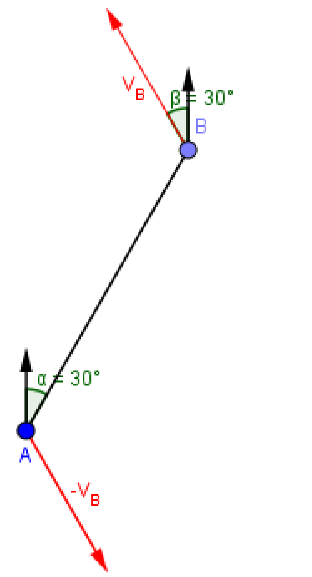
The exercises will often ask you to find the bearing of one of the velocity so that the interception is possible. (Make a sketch to scale using a ruler and compasses)



Example:

A ship, A, is to intercept a second ship, B, which is 100 km away on a bearing if 030°. B is sailing on a course of 330° at 20 km h^{-1}. If the speed of A is 40 km h^{-1}, find the course that A must steer, to the nearest degree, and also the time taken for A to reach B.

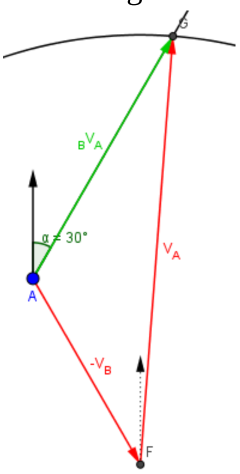
Step 1:



Step 2:

We need to construct V_A *so that* $_BV_A = V_A - V_B$ *or* $-V_B + V_A$ *is parallel to* A_0B_0

- Using your compasses, draw a circle (radius 40km/h to scale) at the end of $-V_B$. The circle crosses AB. We can complete the "triangle of velocities".



The angle GAF=120^o $(300^o - 180^o)$

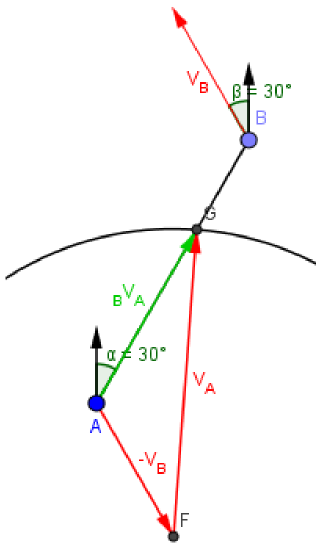
Using the Sin rule : $\dfrac{Sin\ AGF}{20} = \dfrac{Sin120}{40}$ *so* $AGF = 25.66^o$

and therefore *Angle* AFG $= 34.34^o$

The bearing of V_A *is* $34.34 - 30 = 004^o$ (*Draw the vector North from F*)

- Using the cos rule: $_BV_A = 26.1 km/h$
- *Time taken* $T = \dfrac{D}{S} = \dfrac{100}{26.1} = 3.84h = 3h50\min$

Step 2:



The interception will not occur: CLOSEST approach

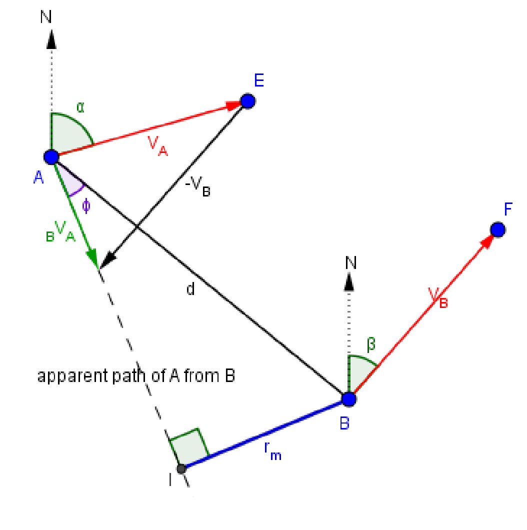
The object A and B will not intercept, but we would like to know the minimum distance between A and B and when it will happen.

Construct ${}_BV_A$ and the apparent path then draw the perpendicular to the path going through B. BI is the closest A and B will be.

Find the angle labelled ϕ, then

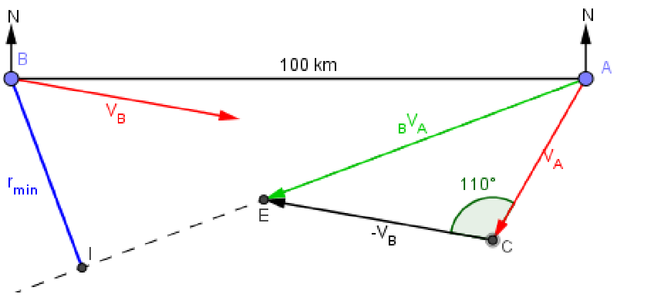
$r_{min} = d \times Sin\phi$

$$t = \frac{d \times Cos\phi}{{}_BV_A}$$



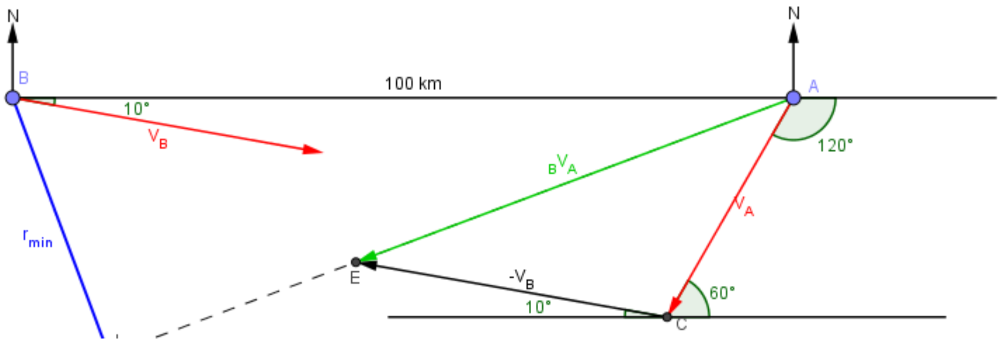
Example:

Two ships, A and B, have velocities of 20 km h^{-1} and 25 km h^{-1} on bearings of 210° and 100° respectively. At noon A is 100 km east of B. Find the closest distance between the ships in the ensuing motion and the time that they are closest, assuming that their velocities remain constant.



WHY 110°?

To work it out I drew the line parallel to AB through C and used the properties of angles (alternate corresponding etc..)



Now solving the problem:

- $_BV_A = \sqrt{20^2 + 25^2 - 2 \times 20 \times 25 \times Cos110} = 36.97 km\,h^{-1}$
- $\frac{Sin(EAC)}{25} = \frac{Sin110}{36.97}$ so Angle $EAC = 39.45^o$
- The angle $BAE = 180 - 120 - 39.45 = 20.55^o$
- $r_{min} = 100 \times Sin20.55 = 35.1 km$
- The length $AI = 100 \times Cos20.55 = 93.64\,km$

The time taken to travel AI is $t = \frac{93.64}{36.97} = 2.53h = 2h32\min$

They are the closest to each other at 14:32

Course to the closest approach

In this section, the interception of A and B is not possible. In most exercises, you will be asked to find the bearing of the velocity V_A so that A approach B as close as possible.

Construction:

It is a similar construction to the one seen in the "interception" section.

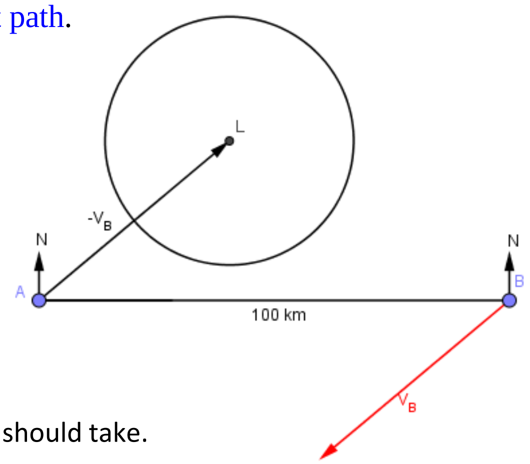
The difference: the circle will not cross the line A_0B_0.

The apparent path (view from B) that A should take to approach B as close as possible will be the TANGENT to the circle drawn from A.

Consequence: the velocity V_A is perpendicular to the apparent path.

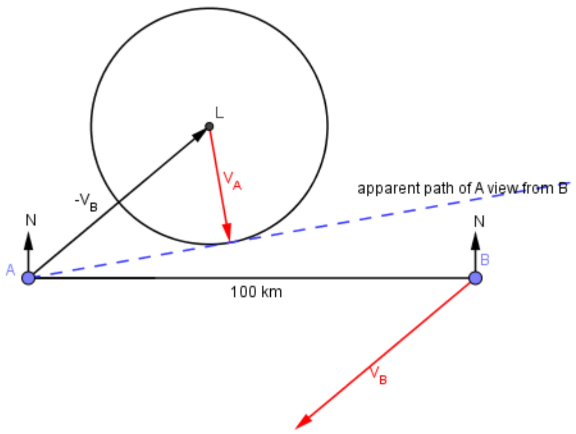
Example

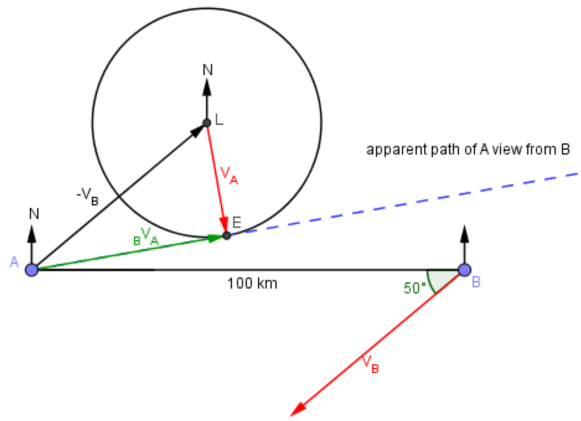
Two ships, A and B, are 100 km apart, with A due west of B. Ship B is travelling at 30 km h^{-1} on a bearing of 220°. The maximum speed of A is 15 km h^{-1}. Find the course that A should take if it is to approach B as closely as possible, and find the shortest distance between the ships.



Step1: The circle has it centre at the end of $-V_B$ and has radius V_A (to scale)

Step 2: Draw the tangent to the circle through A. This is the path A should take. V_A is the radius perpendicular to the tangent





Step 3: Draw the north vector on L and work out the bearing of V_A.

(Use SOHCAHTOA and Pythagoras' theorem as the "velocity triangle" is right-angled)

Solving the problem:

- $Angle\ ALE = Cos^{-1}\left(\frac{15}{30}\right) = 60^o$ so the bearing of V_A is $220 - 60 = 160^o$
- $Angle\ LAE = 180 - 90 - 60 = 30^o$ $so\ angle\ EAB = 50 - 30 = 20^o$
- $r_{\min} = 100 \times Sin20 = 34.2\,km$