AQA – M3 – June 2017

Question 1:

$[C] = MLT^{-1} \times L^{-2} = ML^{-1}T^{-2}$

$[P] = ML^{-1}T^{-2}$ (because same units as C)

$[\rho gh] = ML^{-3} \times LT^{-2} \times L = ML^{-1}T^{-2}$

$[\frac{1}{2}\rho v^2] = ML^{-3} \times M^2T^{-2} = ML^{-1}T^{-2}$

All the terms have the same dimension
therefore the formula is consistent.

Question 2:

$$\vec{a}\begin{pmatrix}0\\-g\end{pmatrix} \quad \vec{v}\begin{pmatrix}70Cos\theta\\-gt+70Sin\theta\end{pmatrix} \quad \vec{s}\begin{pmatrix}70tCos\theta\\-\frac{1}{2}gt^2+70tSin\theta\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$$

$70tCos\theta = x$ gives $t = \dfrac{x}{70Cos\theta}$

$$y = -\frac{1}{2}gt^2 + 70tSin\theta = -\frac{1}{2}g\left(\frac{x}{70Cos\theta}\right)^2 + 70\left(\frac{x}{70Cos\theta}\right)Sin\theta$$

$$y = -\frac{1}{2} \times \frac{g}{70^2} \times \frac{x^2}{Cos^2\theta} + xTan\theta \qquad \text{Using } Cos^2\theta = 1 + Tan^2\theta$$

$$y = xTan\theta - \frac{x^2}{1000}(1+Tan^2\theta)$$

$b)i)$ When $x = 500$, $y = -25$

$$-25 = 500Tan\theta - \frac{500^2}{1000}(1+Tan^2\theta) \quad \text{which simplifies to}$$

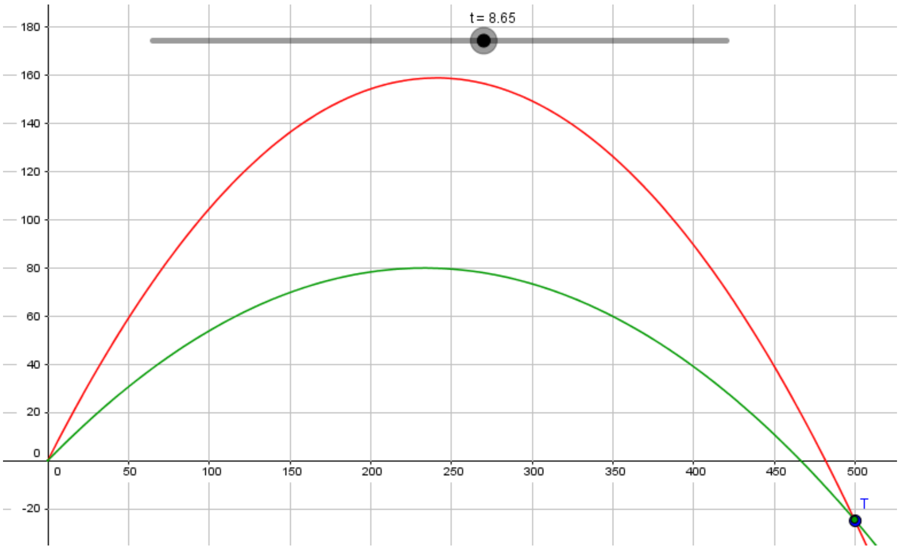
$$10Tan^2\theta - 20Tan\theta + 9 = 0$$

$$Tan\theta = 0.684 \text{ or } Tan\theta = 1.3162$$

$$\theta = 34.4^o \text{ or } \theta = 52.8^o$$

$$ii)\ t = \frac{x}{70Cos\theta} = \frac{500}{70Cos34.4} \text{ or } t = \frac{500}{70Cos52.8^o}$$

$t = 8.7$ sec. or t = 11.8 sec.



Question 3:

$a)\, I = \int_0^4 F(t)dt = \int_0^4 (10-2t)\,dt = \left[10t - t^2\right]_0^4 = 40 - 16 = 24Ns$

$b)\, I = mv - mu$

$24 = 2v - 2\times 3 \qquad V = 15\ ms^{-1}$

$c)\int_0^T 10 - 2t\,dt = 10T - T^2 = 2\times 11 - 2\times 3$

$T^2 - 10T + 16 = 0$

$(T-8)(T-2) = 0$

$T = 8s \quad or \quad T = 2s$

Question 4:

$\overrightarrow{u_A}\begin{pmatrix} uCos30 \\ -uSin30 \end{pmatrix} \; and \; \overrightarrow{u_B}\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Conservation of the component

perpendicular to the line of centres

$\overrightarrow{v_A}\begin{pmatrix} V_{Ax} \\ -uSin30 \end{pmatrix} \; and \; \overrightarrow{v_B}\begin{pmatrix} V_{Bx} \\ 0 \end{pmatrix}$

• Conservation of the momentum along the line of centres

$3mu_{Ax} + 2mu_{Bx} = 3mV_{Ax} + 2mV_{Bx}$

$3uCos30 = 3V_{Ax} + 2V_{Bx} \Leftrightarrow \qquad 3V_{Ax} + 2V_{Bx} = \frac{3\sqrt{3}}{2}u$

• $N.L.R$

$V_{Ax} - V_{Bx} = -\frac{2}{3}\left(u_{Ax} - u_{Bx}\right)$

$V_{Ax} - V_{Bx} = -\frac{2}{3}\left(uCos30 - 0\right) \Leftrightarrow \quad V_{Ax} - V_{Bx} = -\frac{\sqrt{3}}{3}u$

Solving the equation simultaneously:

$V_{Ax} = \frac{\sqrt{3}}{6}u \; and \; V_{Bx} = \frac{\sqrt{3}}{2}u$

$Conclusion:$

$\overrightarrow{v_A}\begin{pmatrix} \frac{\sqrt{3}}{6}u \\ -\frac{1}{2}u \end{pmatrix} and \; \overrightarrow{v_B}\begin{pmatrix} \frac{\sqrt{3}}{2}u \\ 0 \end{pmatrix}$

The angle made with horizontal

$Tan^{-1}\left(\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{6}}\right) = Tan^{-1}\left(-\sqrt{3}\right) = -60^o$

$\alpha = -60^o$

$I_{A\to B} = 2mV_{Bx} - 2mu_{Bx} = 2m\times\frac{\sqrt{3}}{2}u = 0$

$I_{A\to B} = \sqrt{3}\,mu$

Question 5:

$$\vec{a}\begin{pmatrix}-g\,Sin\alpha\\-g\,Cos\alpha\end{pmatrix}\quad \vec{v}\begin{pmatrix}-gtSin\alpha+uCos\theta\\-gtCos\alpha+uSin\theta\end{pmatrix}\quad \vec{s}\begin{pmatrix}-\frac{1}{2}gt^2Sin\alpha+utCos\theta\\-\frac{1}{2}gt^2Cos\alpha+utSin\theta\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$$

$$a)\ At\ \ A,\ v_x=0\ \ \Leftrightarrow -gtSin\alpha+uCos\theta=0\ \ so\ \ t=\frac{u\,Cos\theta}{g\,Sin\alpha}$$

$$and\ \ y=0\ \ so\ \ -\frac{1}{2}gt^2Cos\alpha+utSin\theta=0$$

$$-\frac{1}{2}g\left(\frac{u\,Cos\theta}{g\,Sin\alpha}\right)^2Cos\alpha+u\left(\frac{u\,Cos\theta}{g\,Sin\alpha}\right)Sin\theta=0$$

$$-\frac{u^2Cos^2\theta\,Cos\alpha}{2gSin^2\alpha}+\frac{u^2\,Cos\theta\,Sin\theta}{g\,Sin\alpha}=0$$

$$\frac{u^2Cos\theta}{g\,Sin\alpha}\left(-\frac{Cos\theta\,Cos\alpha}{2Sin\alpha}+Sin\theta\right)=0$$

$$-\frac{Cos\theta\,Cos\alpha}{2Sin\alpha}+Sin\theta=0$$

$$Cos\theta Cos\alpha=2Sin\theta Sin\alpha$$

$$\frac{Cos\alpha}{2Sin\alpha}=\frac{Sin\theta}{Cos\theta}\qquad\qquad Tan\theta=\frac{1}{2Tan\alpha}=\frac{3}{4}$$

$$b)\ At\ \ A,\ v_x=0\ \Leftrightarrow -gtSin\alpha+uCos\theta=0\ \ so\ \ u=\frac{gtSin\alpha}{Cos\theta}$$

$$x=20\ \Leftrightarrow -\frac{1}{2}gt^2Sin\alpha+utCos\theta=20$$

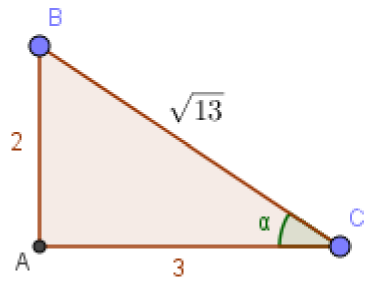
$$-\frac{1}{2}gt^2Sin\alpha+\left(\frac{gtSin\alpha}{Cos\theta}\right)tCos\theta=20$$

$$-\frac{1}{2}gt^2Sin\alpha+gt^2Sin\alpha=20$$

$$\frac{1}{2}gt^2Sin\alpha=20\ \ \Leftrightarrow t=\sqrt{\frac{40}{gSin\alpha}}$$

$$Sin\alpha=\frac{2}{\sqrt{2^2+3^2}}=\frac{2}{\sqrt{13}}\qquad\qquad t=\sqrt{\frac{40\sqrt{13}}{2\times 9.8}}=2.71\sec$$

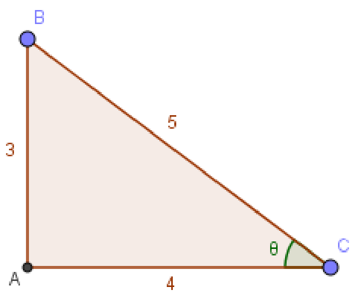
$$c)\ u=\frac{gtSin\alpha}{Cos\theta}=\frac{9.8\times 2.71\times\frac{2}{\sqrt{13}}}{\frac{4}{5}}=18.4\,m/s$$



$$Tan(\alpha)=\frac{2}{3}$$

$$Cos(\alpha)=\frac{3}{\sqrt{13}}$$

$$Sin(\alpha)=\frac{2}{\sqrt{13}}$$



$$Tan(\alpha)=\frac{3}{4}$$

$$Cos(\alpha)=\frac{4}{5}$$

$$Sin(\alpha)=\frac{3}{5}$$

Question 6:

$C.L.M: 5um + 4um = mV_A + 2mV_B$

$V_A + 2V_B = 9u \quad (1)$

$N.L.R: V_A - V_B = -\frac{2}{3}(5u - 2u)$

$V_A - V_B = -2u \quad (2)$

Solving simultaneously:

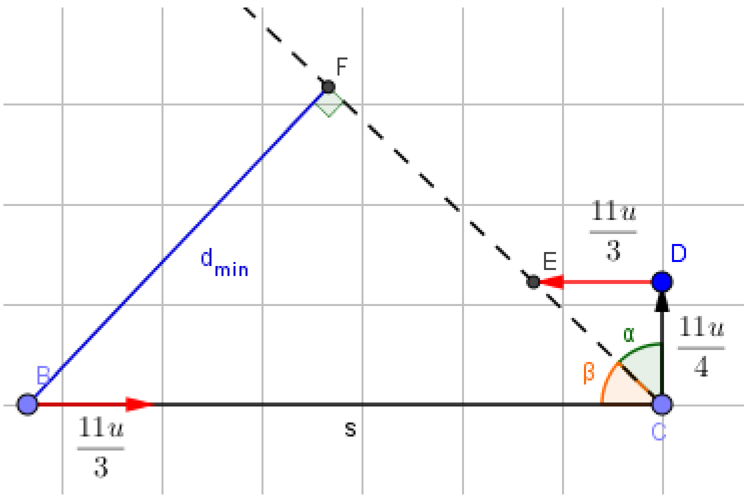
$(1)-(2) \ gives \ 3V_B = 11u \ \ so \ V_B = \frac{11u}{3}$

$V_A = -2u + V_B = \frac{5u}{3}$

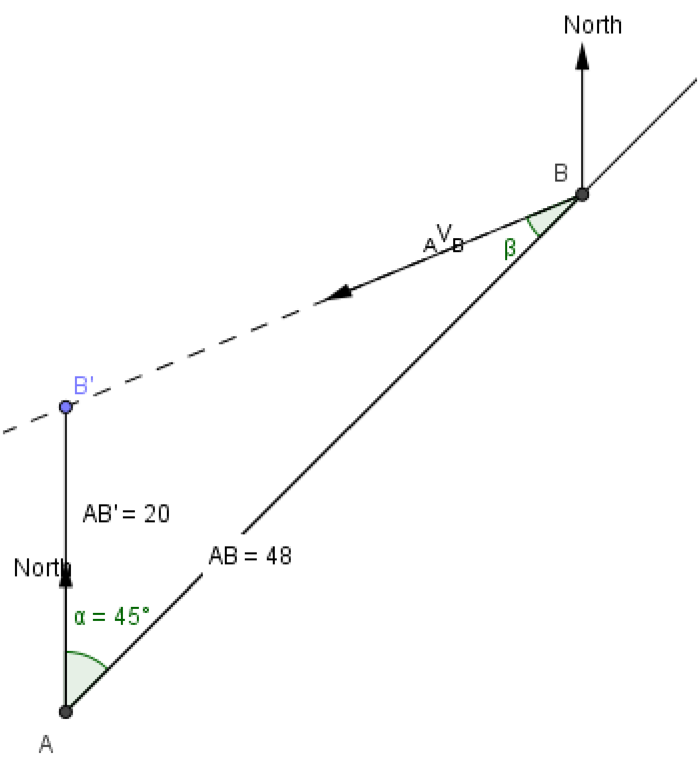
$b)\ \alpha = Tan^{-1}\left(\frac{11u/3}{11u/4}\right) = Tan^{-1}(\frac{4}{3}) = 53.13^o$

$\beta = 90 - 53.13 = 36.87^o$

$d_{min} = s \times Sin 36.87 = 0.6s$



Question 7:



Using the "Cos rule",

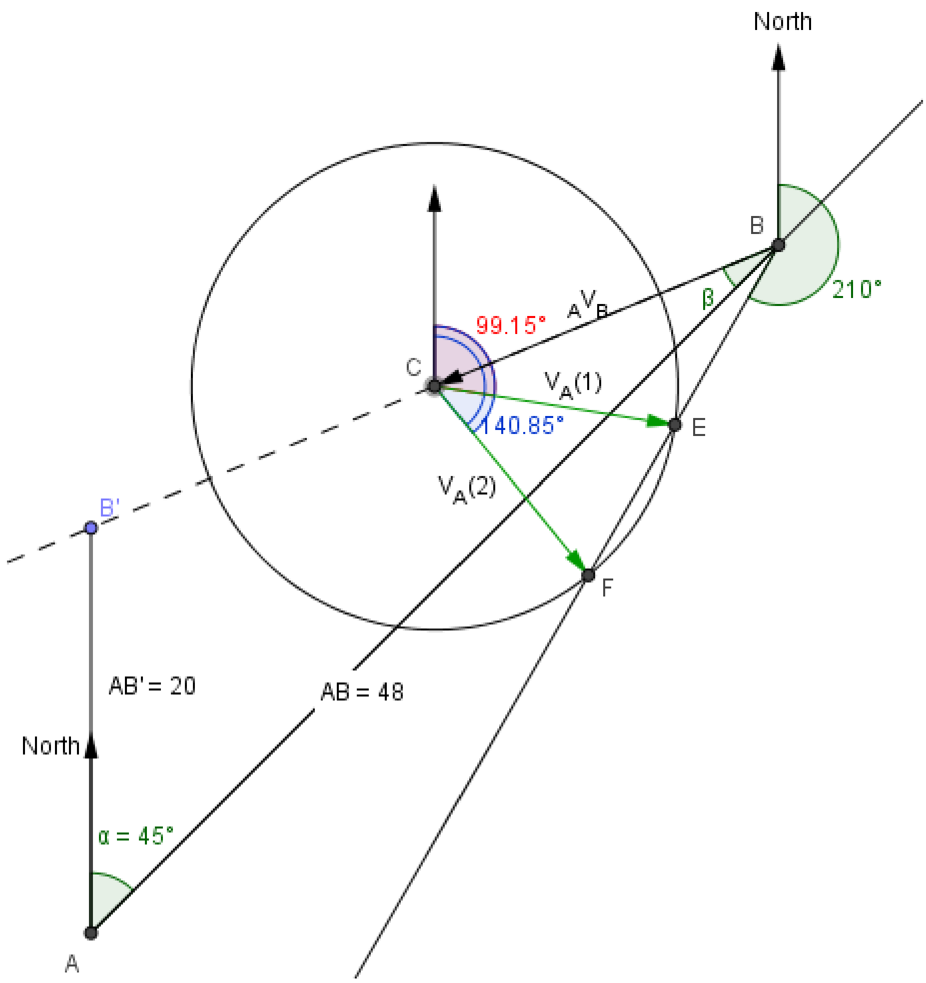
$BB' = \sqrt{20^2 + 48^2 - 2 \times 20 \times 48 \times Cos45} = 36.69\,km$

$_AV_B = \frac{36.7km}{2h} = 18.34\,km/h$

Using the "Sin rule"

$\frac{Sin\beta}{20} = \frac{Sin45}{36.7} \quad so \ \ Sin\beta = 0.3853 \ \ and \ \ \beta = 22.7^o$

$The \ \ bearing \ \ is \ \ \ 45 + 180 + 22.7 = 247.7^o$



We want to construct $_A\vec{V}_B + \vec{V}_A = \vec{V}_B$ so that the bearing of $\vec{V}_B = 210^o$

To do so, we draw a circle radius 12 at the extremity of $_AV_B$ *AND* a line from B at a bearing of 210^o

The circle and the line cross at two points, which gives the two possible vectors $\vec{V}_A$

We are going to work out the angles BEC and BFC susing the Sin rule

$$\frac{Sin\,\gamma}{18.3} = \frac{Sin(247.7 - 210)}{12} \quad so \quad Sin\,\gamma = 0.9318$$

Therefore the possible values for γ *are* 68.7^o *or* $180 - 68.7^o = 111.2^o$

The bearing of $V_A(1) = 67.7 + (180 - 37.7 - 68.7) = 141^o$

The bearing of $V_A(2) = 67.7 + (180 - 37.7 - 11.2) = 99^o$

ii) In the triangle BCF,

$$\frac{V_B}{Sin(141 - 67.7)} = \frac{12}{Sin\,37.7} \quad so \quad V_B = 18.8\,km/h$$

Some Angles exp*lained* :