AQA

A-level MATHEMATICS

Unit Mechanics 3

Wednesday 8 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 At a firing range, a man holds a gun and fires a bullet horizontally. The bullet is fired with a horizontal velocity of $400\,\mathrm{m\,s^{-1}}$. The mass of the gun is $1.5\,\mathrm{kg}$ and the mass of the bullet is 30 grams.

(a) Find the speed of recoil of the gun.

[2 marks]

(b) Find the magnitude of the impulse exerted by the man on the gun in bringing the gun to rest after the bullet is fired.

[2 marks]

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2 A lunar mapping satellite of mass m_1 measured in kg is in an elliptic orbit around the moon, which has mass m_2 measured in kg. The effective potential, E, of the satellite is given by

$$E = \frac{K^2}{2m_1r^2} - \frac{Gm_1m_2}{r}$$

where r measured in metres is the distance of the satellite from the moon, $G\ \mathrm{Nm^2kg^{-2}}$ is the universal gravitational constant, and K is the angular momentum of the satellite.

By using dimensional analysis, find the dimensions of:

(a) E,

[3 marks]

(b) K.

[3 marks]

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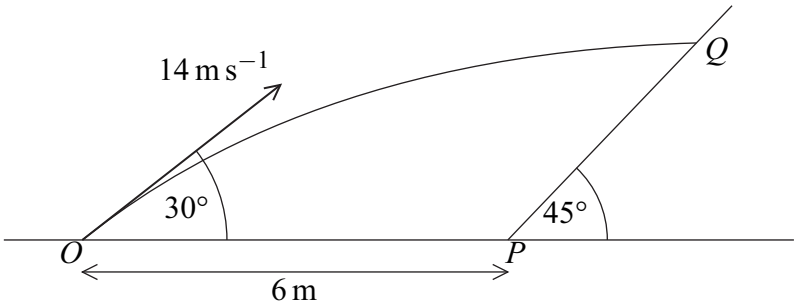
3 A ball is projected from a point O on horizontal ground with speed $14\,\mathrm{m\,s^{-1}}$ at an angle of elevation $30°$ above the horizontal. The ball travels in a vertical plane through the point O and hits a point Q on a plane which is inclined at $45°$ to the horizontal. The point O is 6 metres from P, the foot of the inclined plane, as shown in the diagram. The points O, P and Q lie in the same vertical plane. The line PQ is a line of greatest slope of the inclined plane.



(a) During its flight, the horizontal and upward vertical distances of the ball from O are x metres and y metres respectively.

Show that x and y satisfy the equation

$$y = x\frac{\sqrt{3}}{3} - \frac{x^2}{30}$$

Use $\cos 30° = \frac{\sqrt{3}}{2}$ and $\tan 30° = \frac{\sqrt{3}}{3}$.

[5 marks]

(b) Find the distance PQ.

[7 marks]

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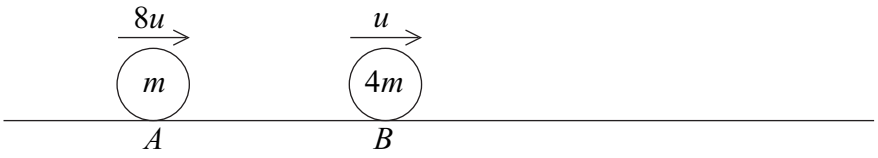
4 A smooth uniform sphere A, of mass m, is moving with velocity $8u$ in a straight line on a smooth horizontal table. A smooth uniform sphere B, of mass $4m$, has the same radius as A and is moving on the table with velocity u.



The sphere A collides directly with the sphere B.

The coefficient of restitution between A and B is e.

(a) (i) Find, in terms of u and e, the velocities of A and B immediately after the collision.

[6 marks]

(ii) The direction of motion of A is reversed by the collision. Show that $e > a$, where a is a constant to be determined.

[2 marks]

(b) Subsequently, B collides with a fixed smooth vertical wall which is at right angles to the direction of motion of A and B. The coefficient of restitution between B and the wall is $\frac{2}{3}$.

The sphere B collides with A again after rebounding from the wall.

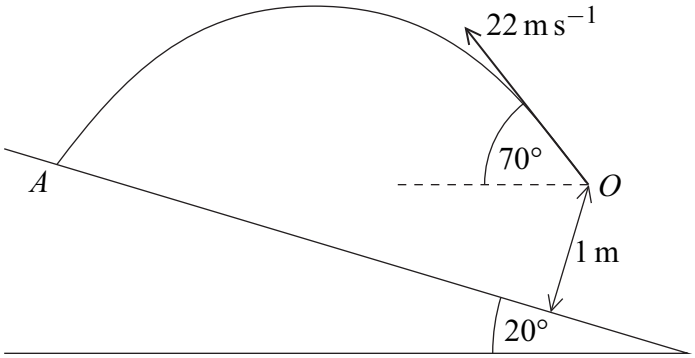
Show that $e < b$, where b is a constant to be determined.

[3 marks]

(c) Given that $e = \frac{4}{7}$, find, in terms of m and u, the magnitude of the impulse exerted on B by the wall.

[3 marks]

5 A ball is projected from a point O above a smooth plane which is inclined at an angle of $20°$ to the horizontal. The point O is at a perpendicular distance of $1\,\text{m}$ from the inclined plane. The ball is projected with velocity $22\,\text{m s}^{-1}$ at an angle of $70°$ above the **horizontal**. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane for the first time at a point A.



(a) (i) Find the time taken by the ball to travel from O to A.

[4 marks]

(ii) Find the components of the velocity of the ball, parallel and perpendicular to the inclined plane, as it strikes the plane at A.

[4 marks]

(b) After striking A, the ball rebounds and strikes the plane for a second time at a point further up than A.

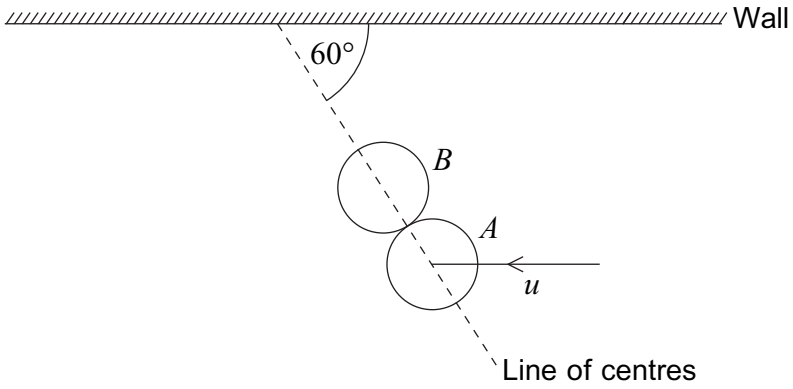
The coefficient of restitution between the ball and the inclined plane is e.

Show that $e < k$, where k is a constant to be determined.

[4 marks]

6 In this question use $\cos 30° = \sin 60° = \frac{\sqrt{3}}{2}$.

A smooth spherical ball, A, is moving with speed u in a straight line on a smooth horizontal table when it hits an identical ball, B, which is at rest on the table. Just before the collision, the direction of motion of A is parallel to a fixed smooth vertical wall. At the instant of collision, the line of centres of A and B makes an angle of $60°$ with the wall, as shown in the diagram.



The coefficient of restitution between A and B is e.

(a) Show that the speed of B immediately after the collision is $\frac{1}{4}u(1+e)$ and find, in terms of u and e, the components of the velocity of A, parallel and perpendicular to the line of centres, immediately after the collision.

[7 marks]

(b) Subsequently, B collides with the wall. After colliding with the wall, the direction of motion of B is parallel to the direction of motion of A after its collision with B.

Show that the coefficient of restitution between B and the wall is $\frac{1+e}{7-e}$.

[7 marks]

7 A quad-bike, a truck and a car are moving on a large, open, horizontal surface in a desert plain. Relative to the quad-bike, which is travelling due west at its maximum speed of $10\,\mathrm{m\,s^{-1}}$, the truck is moving on a bearing of $340°$. Relative to the car, which is travelling due east at a speed of $15\,\mathrm{m\,s^{-1}}$, the truck is moving on a bearing of $300°$.

(a) Show that the speed of the truck is approximately $24.7\,\mathrm{m\,s^{-1}}$ and that it is moving on a bearing of $318°$, correct to the nearest degree.

[8 marks]

(b) At the instant when the truck is at a distance of 400 metres from the quad-bike, the bearing of the truck from the quad-bike is $060°$. The truck continues to move with the same velocity as in part **(a)**. The quad-bike continues to move at a speed of $10\,\mathrm{m\,s^{-1}}$.

Find the bearing, to the nearest degree, on which the quad-bike should travel in order to approach the truck as closely as possible.

[5 marks]

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

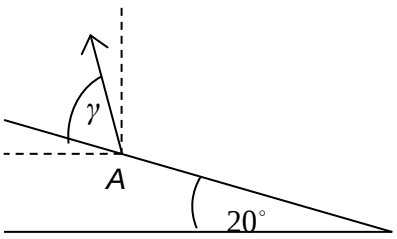
Q	Solution	Mark	Total	Comment
1 (a)	CLM:			
	$1.5(0)+0.03(0)=1.5(-v)+0.03(400)$	M1		M1: Correct terms, allow sign errors. OE, Condone the omission of the zero terms
			2	
	$v=8$ (ms^{-1}) OE	A1		A1: CAO
(b)				
	$I=1.5(8)-1.5(0)$	M1		M1: Seeing their *v* multiplied by 1.5 and condone the omission of the zero term
	$I=12$ (Ns)	A1	2	A1: CAO, must be positive.
	Total		**4**	

Q	Solution	Mark	Total	Comment
2 (a)	$[E]=\left[\frac{Gm_1m_2}{r}\right]$	M1		M1: Working with the 2nd term in the expression
	$=\text{MLT}^{-2}\text{L}^2\text{M}^{-2}\text{MML}^{-1}$	dM1		dM1: Correct unsimplified expression
	$=\text{ML}^2\text{T}^{-2}$	A1	3	A1: CAO
(b)	$\left[\frac{K^2}{2m_1r^2}\right]=\text{ML}^2\text{T}^{-2}$	M1		M1: Working with their answer to (a) and the first term of the expression
	$[K^2]=\text{ML}^2\text{ML}^2\text{T}^{-2}$	A1		A1: Correct dimensions for K^2
	$=\text{M}^2\text{L}^4\text{T}^{-2}$			
	$[K]=\text{ML}^2\text{T}^{-1}$	A1	3	A1:CAO
	Total		**6**	

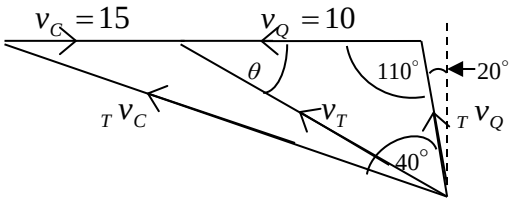
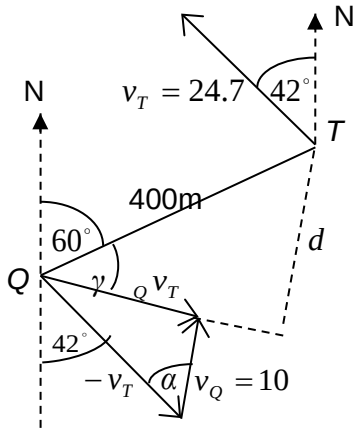
Q	Solution	Mark	Total	Comment
3(a)	$x = 14\cos 30^\circ t$	B1		B1: Correct horizontal eqn.
	$y = 14\sin 30^\circ t - \frac{1}{2}gt^2$	B1		B1: Correct vertical eqn.
	$t = \frac{x}{14\cos 30^\circ}$	M1		M1: Making t the subject of x
	$y = 14\sin 30^\circ \times \frac{x}{14\cos 30^\circ} - \frac{1}{2}(9.8)\left(\frac{x}{14\cos 30^\circ}\right)^2$	dM1		dM1:Elimination of t from their y
	$y = x\tan 30^\circ - \frac{x^2}{40\cos^2 30^\circ}$			
	$y = \frac{x\sqrt{3}}{3} - \frac{x^2}{30}$	A1	5	A1: CSO , AG must see the line above final answer, OE
(b)	$y = x - 6$	M1		M1: For seeing $y=x\pm 6$
	$x - 6 = \frac{x\sqrt{3}}{3} - \frac{x^2}{30}$	dM1		dM1: Substituting $x\pm 6$ into the given eqn.
	$x^2 + \left(30 - 10\sqrt{3}\right)x - 180 = 0$	A1		A1: Correct simplified quadratic
	$x = \frac{-\left(30 - 10\sqrt{3}\right) \pm \sqrt{\left(30 - 10\sqrt{3}\right)^2 - 4\times 1(-180)}}{2\times 1}$	dM1		dM1: Solving quadratic
	$x = 8.50$ $\left(x = -21.2 \text{ or exact equivalent not needed}\right)$	A1		A1: CAO, accept 8.5 or AWRT 8.50
	$PQ = \frac{8.499 - 6}{\cos 45^\circ}$	dM1		dM1: Correct exp. for PQ. FT their +ve x-value
	$PQ = 3.53$ or $\frac{5\sqrt{2}}{2}$ (m)	A1	7	A1: CAO, AWRT 3.53 or exact value, allow 3.54
	Total		**12**	

Alternative (b)	Solution	Mark	Total	Comment
	Let $PQ = d$.			
	$x = 6 + d\cos 45^\circ$	M1		M1:Expression for x in terms of d.
	$y = d\sin 45^\circ$	M1		M1:Expression for y in terms of d.
		A1		A1: Both correct
	$d\sin 45^\circ = (6 + d\cos 45^\circ)\tan 30^\circ - \dfrac{(6 + d\cos 45^\circ)^2}{30}$	dM1		dM1: Substituting into given expression
	$d^2\cos^2 45^\circ + (30\sin 45^\circ - 30\cos 45^\circ\tan 30^\circ + 12\cos 45^\circ)d - (180\tan 30^\circ - 36) = 0$	A1		A1: Correct simplified quadratic
	$d = \dfrac{-(30\sin 45^\circ - 30\cos 45^\circ\tan 30^\circ + 12\cos 45^\circ) \pm \sqrt{(30\sin 45^\circ - 30\cos 45^\circ\tan 30^\circ + 12\cos 45^\circ)^2 - 4(\cos^2 45^\circ)(36 - 180\tan 30^\circ)}}{2\cos^2 45^\circ}$	dM1		dM1: Solution of their quadratic eqn.
	$d = 3.53$ m (Allow 3.54 m)	A1		A1: CAO, AWRT 3.53 or exact value, allow 3.54

Q	Solution	Mark	Total	Comment
4 (a)(i)	$8mu + 4mu = mv_A + 4mv_B$ OE	M1 A1		M1: Four non-zero momentum terms, A1: Correct eqn.
	$7eu = v_B - v_A$ OE	M1 A1		M1: Eqn using e. Allow sign errors. A1: Correct eqn.
	$v_B = \frac{u}{5}(12 + 7e)$ OE	A1		A1: Correct vel of B
	$v_A = \frac{4u}{5}(3 - 7e)$ OE	A1	6	A1: Correct vel of A
(ii)	$\frac{4u}{5}(3 - 7e) < 0$	M1		M1: Their vel of A<0
	$e > \frac{3}{7}$	A1	2	A1: CAO, accept AWRT 0.429
(b)	$\frac{u}{5}(12 + 7e) \times \frac{2}{3} > -\frac{4u}{5}(3 - 7e)$	B1 M1		B1: Correct rebound speed of *B* M1: Correct inequality
	$e < \frac{6}{7}$	A1	3	A1: CAO, AWRT 0.857
(c)	$4m \times \frac{u}{5}\left(12 + 7 \times \frac{4}{7}\right) + 4m \times \frac{u}{5}\left(12 + 7 \times \frac{4}{7}\right) \times \frac{2}{3} =$	M1 A1		M1: Two correct momentum terms, allow sign errors. A1: Correct expression for impulse
	$\frac{64}{3}mu$ or $21.\dot{3}mu$	A1	3	A1: CAO (Must be positive)
	Total		**14**	

Q	Solution	Mark	Total	Comment
5 (a)(i)	$^{-}1 = 22\sin 50^\circ t - \frac{1}{2} g\cos 20^\circ t^2$	M1		M1: Perpendicular eqn with correct terms
		A1		A1: Correct equation
	$\frac{1}{2} g\cos 20^\circ t^2 - 22\sin 50^\circ t - 1 = 0$			
	$t = \dfrac{22\sin 50^\circ \pm \sqrt{\left(-22\sin 50^\circ\right)^2 - 4\left(\frac{1}{2} g\cos 20^\circ\right)(-1)}}{2\left(\frac{1}{2} g\cos 20^\circ\right)}$	dM1		dM1:Solution of their 3-term quadratic eqn.
			4	
(ii)	$t = 3.7185....$ or 3.719	A1		A1: CAO, AWRT 3.72
	$\dot{x} = 22\cos 50^\circ - 9.8\sin 20^\circ (3.7185....)$	M1		M1: Parallel component of vel. with their time
	$\dot{x} = 1.678$ ms^{-1}	A1		A1: Correct component, accept AWRT 1.68
			4	
	$\dot{y} = 22\sin 50^\circ - 9.8\cos 20^\circ (3.7185....)$	M1		M1: Perpendicular component of vel. with their time
	$\dot{y} = {}^{-}17.39$ ms^{-1}	A1		A1: Correct component, accept AWRT -17.4
(b)				
	$\gamma < 90^\circ - 20^\circ$	B1		B1: Seeing 90-20 or 70
	$\frac{17.39e}{1.678} < \tan(90^\circ - 20^\circ)$	B1F		B1F: Multiplying their vertical component by e
		M1		M1: Correct inequality
	$e < 0.265$	A1	4	A1: CAO, accept AWRT 0.265
	Total		**12**	

Q	Solution	Mark	Total	Comment
6 (a)	Along the line of centres:			
	CLM: $u\cos 60^\circ = v_A + v_B$ OE	M1		M1: Four non-zero momentum terms,
		A1		A1: Correct eqn.
	Restitution: $eu\cos 60^\circ = v_B - v_A$	M1		M1: Eqn using e. Allow sign errors.
		A1		A1: Correct eqn.
	$2v_B = (1+e)u\cos 60^\circ$ OE			
	$v_B = \frac{1}{4}u(1+e)$	A1		A1: Correct vel of B (AG) from correct working
	$v_A = \frac{1}{4}u(1-e)$ OE	A1		A1: Correct vel of A
	Perpendicular to line of centres: $v'_A = u\cos 30^\circ$ OE	B1	7	B1: Correct perpend. comp.
(b)				
	$v_A\cos 60^\circ + u\cos^2 30^\circ$ ← (A) ---------- Wall	M1		M1: Components of *A* parallel & perp to the wall
	$u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ$ ↓	A1		A1: Both correct
	$v_B\cos 60^\circ$ ← (B) ---------- Wall	M1		M1: Components of *B* parallel & perp to the wall
	$e'v_B\sin 60^\circ$ ↓	A1		A1: Both correct AG above line oe needed
	$\frac{u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ}{v_A\cos 60^\circ + u\cos^2 30^\circ} = \frac{e'v_B\sin 60^\circ}{v_B\cos 60^\circ}$	dM1		dM1: Equal ratios used to create equation
	$\frac{u\times\frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{4}u(1-e)\times\frac{\sqrt{3}}{2}}{\frac{1}{4}u(1-e)\times\frac{1}{2} + u\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{e'\times\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	A1		A1: Correct equation
	$e' = \frac{2u-u+eu}{u-eu+6u}$			
	$e' = \frac{1+e}{7-e}$	A1		A1:CSO (AG)
			7	
	Total		**14**	

Q	Solution	Mark	Total	Comment
7 (a)		B1		B1:For combined velocity triangles (PI)
	$\dfrac{25}{\sin 40^\circ} = \dfrac{{}_T v_Q}{\sin 30^\circ}$	M1		M1: Sine rule to find Vel of *T* rel to *Q*
	${}_T v_Q = \dfrac{25\sin 30^\circ}{\sin 40^\circ} \quad (= 19.4465)$	A1		A1: Correct expression or value
	$v_T = \sqrt{10^2 + \left(\dfrac{25\sin 30^\circ}{\sin 40^\circ}\right)^2 - 2\times 10 \times \dfrac{25\sin 30^\circ}{\sin 40^\circ} \times \cos 110^\circ}$	M1		M1: Cosine rule to find vel of *T*
	$v_T = 24.7(222662)$ ms^{-1}	A1		A1: Correct expression or value
	$\dfrac{\sin\theta}{\left(\dfrac{25\sin 30^\circ}{\sin 40^\circ}\right)} = \dfrac{\sin 110^\circ}{24.7222662}$	M1		M1: Sine rule to find θ
	$\theta = 47.6601^\circ$	A1		A1: Correct θ
	Bearing: 318°	A1	8	A1: Correct bearing (AG)
(b)		B1		B1:Right-angled velocity triangle
	$\cos\alpha = \dfrac{10}{24.7}$	M1		M1: Using trig to find α
		A1		A1: Correct equation to find α
	$\alpha = 66(.11775...)^\circ$	A1		A1: Correct α
	Motor cyclist's bearing: 024°	A1	5	A1: Correct bearing
	Total		**13**	

Q	Solution	Mark	Total	Comment
7(a) Alternative 1		B1		B1: For combined velocity triangles (PI)
	If v_T makes angle β with the north, then			
	$\dfrac{v_T}{\sin 30^\circ} = \dfrac{15}{\sin(60^\circ - \beta)}$	M1		M1: Sine rule to link v_T and β using 15, 30, 60.
	$\dfrac{v_T}{\sin 110^\circ} = \dfrac{10}{\sin(\beta - 20^\circ)}$	M1		M1: Sine rule to link v_T and β using 10, 110 , 20
	$15\sin 30^\circ \sin(\beta - 20^\circ) = 10\sin 110^\circ \sin(60^\circ - \beta)$	dM1		dM1: Correct eqn in β
	$15\sin 30^\circ(\sin\beta\cos 20^\circ - \cos\beta\sin 20^\circ) =$ $10\sin 110^\circ(\sin 60^\circ\cos\beta - \cos 60^\circ\sin\beta)$			
	$\tan\beta = \dfrac{10\sin 110^\circ \sin 60^\circ + 15\sin 30^\circ \sin 20^\circ}{15\sin 30^\circ \cos 20^\circ + 10\sin 110^\circ \cos 60^\circ}$	dM1	8	dM1: Finding tanβ
	$\beta = 42.3^\circ$	A1		A1: Correct β □
	Bearing : 318°	A1		A1: Correct bearing (AG)
	$\dfrac{v_T}{\sin 30^\circ} = \dfrac{15}{\sin(60^\circ - 42.3)}$ $v_T = 24.7$ ms^{-1}	A1		A1: Correct vel of *T*
Alternative 2				
	$v_T = \begin{pmatrix} a \\ b \end{pmatrix}$			
	${}_T v_Q = \begin{pmatrix} a+10 \\ b \end{pmatrix}$	B1		B1: Correct vector
	${}_T v_C = \begin{pmatrix} a-15 \\ b \end{pmatrix}$	B1		B1: Correct vector
	$\dfrac{a+10}{b} = -\tan 20^\circ$ OE	M1		M1: Correct ratio for tan20
	$\dfrac{a-15}{b} = -\tan 60^\circ$ OE	M1	8	M1: Correct ratio for tan60
	$a = -16.65\ldots \quad b = 18.27\ldots$	A1		A1: Both *a* and *b* correct
	Bearing : $270^\circ + \tan^{-1}\left(\dfrac{18.27}{16.65}\right) =$			
	318°	A1		A1: Correct expression for

	$v_T = \sqrt{(16.65...)^2 + (18.27...)^2}$ $v_T = 24.7 \text{ ms}^{-1}$	A1 A1		bearing A1: CAO (AG) A1: Correct vel of T

Unit grade boundaries – June 2016 exams

A-level – specifications that use uniform marks

Code	Title	Maximum Mark	Grade Boundaries and A* Conversion Points					
			A*	A	B	C	D	E
MM03	MATHEMATICS UNIT MM03	75	53	47	41	36	31	26
MM04	MATHEMATICS UNIT MM04	75	66	59	52	45	38	31
MM05	MATHEMATICS UNIT MM05	75	64	56	48	41	34	27
MM1B	MATHEMATICS UNIT MM1B	75	-	51	45	40	35	30
MM2B	MATHEMATICS UNIT MM2B	75	63	58	51	44	37	31
MPC1	MATHEMATICS UNIT MPC1	75	-	62	56	50	45	40
MPC2	MATHEMATICS UNIT MPC2	75	-	59	52	45	38	32
MPC3	MATHEMATICS UNIT MPC3	75	53	47	43	39	36	33
MPC4	MATHEMATICS UNIT MPC4	75	60	55	50	45	41	37
MS03	MATHEMATICS UNIT MS03	75	67	60	53	46	39	32
MS04	MATHEMATICS UNIT MS04	75	67	59	51	43	36	29
MS1A	MATHEMATICS UNIT MS1A	100	no candidates were entered for this unit					
MS1B	MATHEMATICS UNIT MS1B	75	-	59	54	49	44	39
MS2B	MATHEMATICS UNIT MS2B	75	69	65	59	53	47	41
MEST1	MEDIA STUDIES UNIT 1	80	-	55	49	43	37	31
MEST2	MEDIA STUDIES UNIT 2	80	-	65	56	47	38	30
MEST3	MEDIA STUDIES UNIT 3	80	75	70	60	50	40	31
MEST4	MEDIA STUDIES UNIT 4	80	75	70	58	47	36	25
MHEB1	MODERN HEBREW UNIT 1	100	-	70	65	60	55	50
MHEB2	MODERN HEBREW UNIT 2	100	80	72	64	57	50	43