



General Certificate of Education  
Advanced Level Examination  
June 2015

# Mathematics

# MM03

## Unit Mechanics 3

Wednesday 3 June 2015 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.  
You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

- 1 A formula for calculating the lift force acting on the wings of an aircraft moving through the air is of the form

$$F = k v^\alpha A^\beta \rho^\gamma$$

where  $F$  is the lift force in newtons,  
 $k$  is a dimensionless constant,  
 $v$  is the air velocity in  $\text{m s}^{-1}$ ,  
 $A$  is the surface area of the aircraft's wings in  $\text{m}^2$ , and  
 $\rho$  is the density of the air in  $\text{kg m}^{-3}$ .

By using dimensional analysis, find the values of the constants  $\alpha$ ,  $\beta$  and  $\gamma$ .

[6 marks]

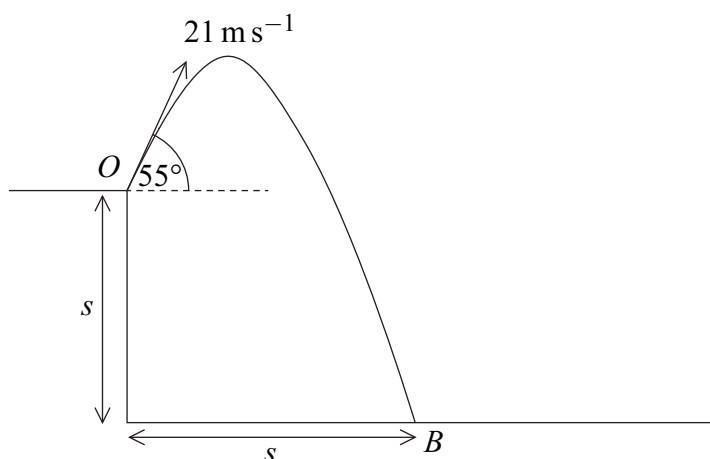
- 2 A projectile is launched from a point  $O$  on top of a cliff with initial velocity  $u \text{ m s}^{-1}$  at an angle of elevation  $\alpha$  and moves in a vertical plane. During the motion, the position vector of the projectile relative to the point  $O$  is  $(x\mathbf{i} + y\mathbf{j})$  metres where  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal and vertical unit vectors respectively.

- (a) Show that, during the motion, the equation of the trajectory of the projectile is given by

$$y = x \tan \alpha - \frac{4.9x^2}{u^2 \cos^2 \alpha}$$

[5 marks]

- (b) When  $u = 21$  and  $\alpha = 55^\circ$ , the projectile hits a small buoy  $B$ . The buoy is at a distance  $s$  metres vertically below  $O$  and at a distance  $s$  metres horizontally from  $O$ , as shown in the diagram.



- (i) Find the value of  $s$ .

[3 marks]

- (ii) Find the acute angle between the velocity of the projectile and the horizontal just before the projectile hits  $B$ , giving your answer to the nearest degree.

[5 marks]



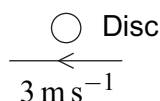
- 3 A disc of mass  $0.5 \text{ kg}$  is moving with speed  $3 \text{ m s}^{-1}$  on a smooth horizontal surface when it receives a horizontal impulse in a direction **perpendicular to its direction of motion**. Immediately after the impulse, the disc has speed  $5 \text{ m s}^{-1}$ .

(a) Find the magnitude of the impulse received by the disc.

[3 marks]

(b) Before the impulse, the disc is moving parallel to a smooth vertical wall, as shown in the diagram.

////// Wall

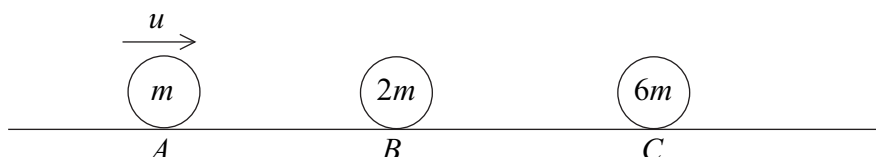


After the impulse, the disc hits the wall and rebounds with speed  $3\sqrt{2} \text{ m s}^{-1}$ .

Find the coefficient of restitution between the disc and the wall.

[4 marks]

- 4 Three uniform smooth spheres,  $A$ ,  $B$  and  $C$ , have equal radii and masses  $m$ ,  $2m$  and  $6m$  respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with  $B$  between  $A$  and  $C$ . The sphere  $A$  is projected with speed  $u$  directly towards  $B$  and collides with it.



The coefficient of restitution between  $A$  and  $B$  is  $\frac{2}{3}$ .

(a) (i) Show that the speed of  $B$  immediately after the collision is  $\frac{5}{9}u$ .

(ii) Find, in terms of  $u$ , the speed of  $A$  immediately after the collision.

[6 marks]

(b) Subsequently,  $B$  collides with  $C$ . The coefficient of restitution between  $B$  and  $C$  is  $e$ .

Show that  $B$  will collide with  $A$  again if  $e > k$ , where  $k$  is a constant to be determined.

[8 marks]

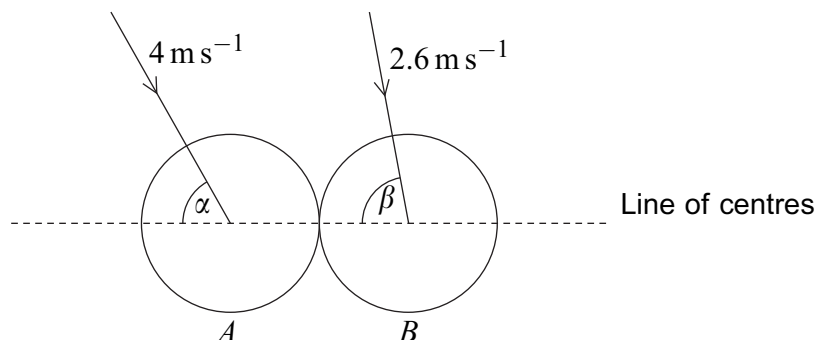
(c) Explain why it is not necessary to model the spheres as particles in this question.

[2 marks]

Turn over ►



- 5 Two smooth spheres,  $A$  and  $B$ , have equal radii and masses  $2\text{ kg}$  and  $1\text{ kg}$  respectively. The spheres move on a smooth horizontal surface and collide. As they collide,  $A$  has velocity  $4\text{ m s}^{-1}$  in a direction inclined at an angle  $\alpha$  to the line of centres, and  $B$  has velocity  $2.6\text{ m s}^{-1}$  in a direction inclined at an angle  $\beta$  to the line of centres, as shown in the diagram.

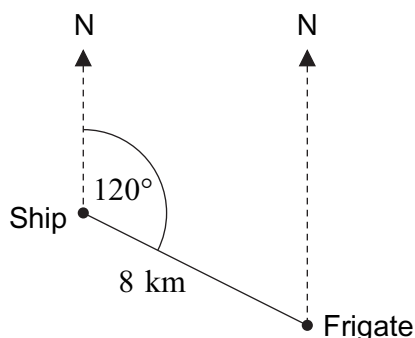


The coefficient of restitution between  $A$  and  $B$  is  $\frac{4}{7}$ .

Given that  $\sin \alpha = \frac{4}{5}$  and  $\sin \beta = \frac{12}{13}$ , find the speeds of  $A$  and  $B$  immediately after the collision.

[11 marks]

- 6 A ship and a navy frigate are a distance of  $8\text{ km}$  apart, with the frigate on a bearing of  $120^\circ$  from the ship, as shown in the diagram.

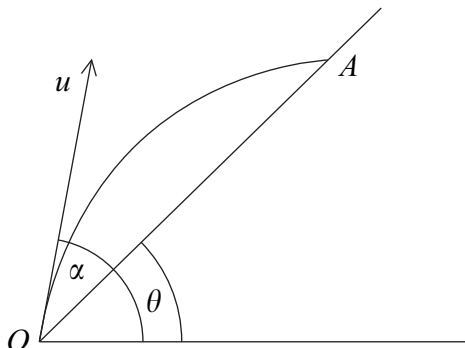


The ship travels due east at a constant speed of  $50\text{ km h}^{-1}$ . The frigate travels at a constant speed of  $35\text{ km h}^{-1}$ .

- (a) (i) Find the bearings, to the nearest degree, of the two possible directions in which the frigate can travel to intercept the ship. [5 marks]
- (ii) Hence find the **shorter** of the two possible times for the frigate to intercept the ship. [5 marks]
- (b) The captain of the frigate would like the frigate to travel at less than  $35\text{ km h}^{-1}$ . Find the minimum speed at which the frigate can travel to intercept the ship. [3 marks]



- 7 A particle is projected from a point  $O$  on a plane which is inclined at an angle  $\theta$  to the horizontal. The particle is projected up the plane with velocity  $u$  at an angle  $\alpha$  **above the horizontal**. The particle strikes the plane for the first time at a point  $A$ . The motion of the particle is in a vertical plane which contains the line  $OA$ .



- (a) Find, in terms of  $u$ ,  $\theta$ ,  $\alpha$  and  $g$ , the time taken by the particle to travel from  $O$  to  $A$ . **[4 marks]**
- (b) The particle is moving horizontally when it strikes the plane at  $A$ .

By using the identity  $\sin(P - Q) = \sin P \cos Q - \cos P \sin Q$ , or otherwise, show that

$$\tan \alpha = k \tan \theta$$

where  $k$  is a constant to be determined.

**[5 marks]**



---

## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

Question	Solution	Marks	Total	Comments
1	$[F] = MLT^{-2}$ $MLT^{-2} = (LT^{-1})^{\alpha} (L^2)^{\beta} (ML^{-3})^{\gamma}$ $= M^{\gamma} L^{\alpha+2\beta-3\gamma} T^{-\alpha}$ $\left. \begin{array}{l} \gamma = 1 \\ \alpha + 2\beta - 3\gamma = 1 \\ -\alpha = -2 \end{array} \right\}$ $\alpha = 2 \quad , \quad \beta = 1$	B1  M1  m1  A1 m1  A1	6	B1: Correct dimensions of $F$ M1: Substituting the dimensions of the quantities into the given equation to obtain RHS correctly. m1: Collecting indices on RHS. Could be implied by later work.  A1: $\gamma = 1$ m1: Two correct equations for $\alpha$ and $\beta$ .  A1: Correct values for $\alpha$ and $\beta$ . Condone use of units instead of dimensions.
	<b>Total</b>		<b>6</b>	

Question	Solution	Marks	Total	Comments
2 (a)	$x = u \cos \alpha t$	M1		M1: Correct expression for horizontal displacement.
	$t = \frac{x}{u \cos \alpha}$	A1		A1: Correct expression for $t$ .
	$y = u \sin \alpha t - \frac{1}{2} g t^2$			
	$y = u \sin \alpha \times \frac{x}{u \cos \alpha} - \frac{1}{2} (9.8) \left( \frac{x}{u \cos \alpha} \right)^2$	M1		M1: Correct expression for vertical displacement. Allow sign errors.
(b)(i)	$y = x \tan \alpha - \frac{4.9x^2}{u^2 \cos^2 \alpha}$	AG		m1: Elimination of $t$ from equation for vertical displacement.
	$-s = s \tan 55^\circ - \frac{4.9s^2}{21^2 \cos^2 55^\circ}$	A1	5	A1: Correct result from correct working. Penalise use of $g = 9.81$ .
(ii)	$s = \frac{(1 + \tan 55^\circ) 21^2 \cos^2 55^\circ}{4.9}$			
	$s = 71.9$	M1		M1: Substituting $\pm s$ for $x$ and $y$ .
	$\dot{x} = 21 \cos 55^\circ = 12.045$	m1		m1: Making $s$ the subject of their equation.
	$\dot{y} = 21 \sin 55^\circ - 9.8 \times \frac{71.895}{21 \cos 55^\circ}$	A1	3	A1: AWRT 71.9 Condone use of $g = 9.81$ which gives 71.8.
	or $\dot{y}^2 = (21 \sin 55^\circ)^2 - 2(9.8)(-71.895)$	B1		B1: Correct expression or value for horizontal component of velocity.
	$\dot{y} = -41.292$			
	$\tan^{-1} \frac{-41.292}{21 \cos 55^\circ}$	M1		M1: Correct expression or value for vertical component of velocity, with their answer to (b)(i).
	$= -74^\circ$		5	
	or $74^\circ$	A1		A1: Correct expression or value.
		m1		m1: Use of tan with their velocity components.
		A1		A1: Correct angle to nearest degree. CAO.
	<b>Total</b>		<b>13</b>	



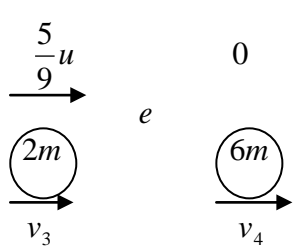
(b)(ii)	Alternative: $y = x \tan \alpha - \frac{4.9x^2}{u^2 \cos^2 \alpha}$ $\frac{dy}{dx} = \tan \alpha - \frac{2(4.9)x}{u^2 \cos^2 \alpha}$ $= \tan 55^\circ - \frac{2(4.9)(71.895)}{21^2 \times \cos^2 55^\circ}$ $= -3.428$ <p>The angle = <math>\tan^{-1}(-3.428)</math></p> $= -74^\circ \text{ or } 74^\circ$	B1 M1 A1 m1 A1	5	B1: Correct derivative. M1: Substituting values. A1: Correct value of the derivative m1: Use of tan to find the angle. A1: Correct angle to nearest degree. CAO.
---------	---	----------------------------	---	--



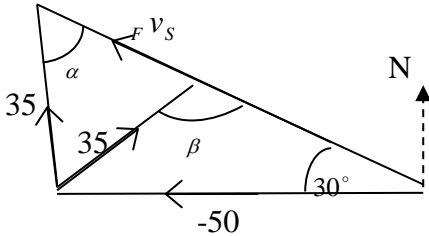
(a)	Alternative: $\vec{I} = 0.5(5 \cos \alpha \mathbf{i} + 5 \sin \alpha \mathbf{j}) - 0.5(3\mathbf{i})$ $2.5 \cos \alpha - 1.5 = 0$ $\cos \alpha = 0.6$ $\sin \alpha = 0.8$ $I = 0.5(5 \times 0.8)$ $I = 2$	B1 M1 A1	3	B1: Correct vector equation. M1: Correct value for $\sin \alpha$ . A1: Correct impulse.
(b)	Alternative: $3 = 3\sqrt{2} \sin \beta$ $\cos \beta = \frac{1}{\sqrt{2}}$ $e = \frac{3\sqrt{2} \left( \frac{1}{\sqrt{2}} \right)}{\frac{2}{0.5}}$ $e = \frac{3}{4} \text{ or } 0.75$	B1 B1 M1 A1	4	B1: Correct equation for motion parallel to B1: Value for $\cos \beta$ or $\beta = 45^\circ$ . M1: Correct expression for $e$ or correct eq A1: Correct impulse.

Question	Solution	Marks	Total	Comments
4 (a) (i)	$mu = mv_1 + 2mv_2$ OE	M1 A1		M1: Equation with three momentum terms. A1: Correct equation.
(ii)	$u = v_1 + 2v_2$ $\frac{2}{3}u = v_2 - v_1$ OE $3v_2 = \frac{5}{3}u$	M1 A1		M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation.
(b)	$v_2 = \frac{5}{9}u$ <b>AG</b> $v_1 = u - \frac{10}{9}u$ $v_1 = -\frac{1}{9}u$ The speed of A is $\frac{1}{9}u$	A1	6	A1: Correct speed of B, from correct working.
	<p style="text-align: center;"> <math>\frac{5}{9}u</math>                      0  <math>\xrightarrow{\hspace{1.5cm}}</math>                      <math>e</math>  <math>\textcircled{2m}</math>                      <math>\textcircled{6m}</math>  <math>\xleftarrow{v_3}</math>                      <math>\xrightarrow{v_4}</math> </p>	M1 A1		M1: Equation with three momentum terms. A1: Correct equation
	$2m\left(\frac{5}{9}u\right) = -2mv_3 + 6mv_4$ OE $\frac{10}{9}u = -2v_3 + 6v_4$	M1 A1		M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation
(c)	$e\left(\frac{5}{9}u\right) = v_3 + v_4$ OE $\frac{10}{9}u = -2v_3 + 6\left(\frac{5}{9}ue - v_3\right)$ $8v_3 = \frac{10}{3}ue - \frac{10}{9}u$ $v_3 = \frac{5}{12}ue - \frac{5}{36}u$ OE	m1 A1F	8	m1: Solving equations to find the speed of B after the second collision. A1F: Correct speed of B after the second collision. FT their equations
			2	

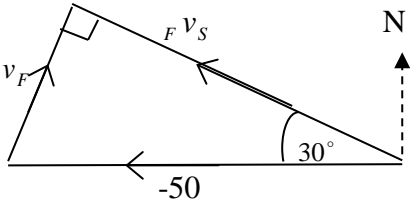
Q	Solution	Marks	Total	Comments
	second collision $\Rightarrow$ $\frac{5}{12}ue - \frac{5}{36}u > \frac{1}{9}u$ $\frac{5}{12}ue > \frac{9}{36}u$ $e > \frac{3}{5} \text{ or } 0.6$ Equal radii $\Rightarrow$ Velocities are parallel to the line of centres	M1  A1F  B1 B1		M1: For the inequality $v_3 > v_1$  A1F: Correct value of $k$ . FT their $v_3 > v_1$ . The value of $k$ must be less than 1 and greater than 0 to score A1F  B1: Comment about equal radii or same size. B1: Comment about the line of centres.
	<b>Total</b>		<b>16</b>	

<p>(b) Alternative:</p>  $2m\left(\frac{5}{9}u\right) = 2mv_3 + 6mv_4$ $\frac{10}{9}u = 2v_3 + 6v_4$ $e\left(\frac{5}{9}u\right) = v_4 - v_3$ $\frac{10}{9}u = 2v_3 + 6\left(\frac{5}{9}ue + v_3\right)$ $8v_3 = \frac{10}{9}u - \frac{10}{3}ue$ $v_3 = \frac{5}{36}u - \frac{5}{12}ue \quad \text{OE}$ <p>second collision <math>\Rightarrow</math></p> $\frac{5}{36}u - \frac{5}{12}ue < -\frac{1}{9}u$ $\frac{5}{12}ue > \frac{9}{36}u$ $e > \frac{3}{5} \text{ or } 0.6$	<p>M1A1</p> <p>M1A1</p> <p>m1A1 F</p> <p>M1</p> <p>A1F</p>		<p>M1: Equation with three momentum terms. A1: Correct equation.</p> <p>M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation.</p> <p>m1: Solving equations to find the velocity of <math>B</math> after the second collision. A1F: Correct velocity of <math>B</math> after the second collision. FT their equations.</p> <p>M1: For the inequality <math>v_3 &lt; v_1</math></p> <p>A1F: Correct value of <math>k</math>. The value of <math>k</math> must be less than 1 and greater than 0 to score A1F</p>
---	--	--	--

Question	Solution	Marks	Total	Comments
5	$\cos \alpha = \frac{3}{5} \text{ or } 0.6 \text{ and } \cos \beta = \frac{5}{13} \text{ or } 0.3846\dots$ $2(4 \cos \alpha) + 1(2.6 \cos \beta) = 2v_A + 1v_B$ $2(2.4) + 1(1) = 2v_A + 1v_B$ $\frac{4}{7}(4 \cos \alpha - 2.6 \cos \beta) = v_B - v_A$ $\frac{4}{7}(2.4 - 1) = v_B - v_A$ $\begin{cases} 5.8 = 2v_A + v_B \\ 0.8 = v_B - v_A \end{cases}$ $v_A = \frac{5}{3} \text{ ms}^{-1}$ $v_B = \frac{37}{15} \text{ ms}^{-1}$ $V_A = \sqrt{\left(\frac{5}{3}\right)^2 + (4 \sin \alpha)^2}$ $V_A = \sqrt{\left(\frac{5}{3}\right)^2 + (3.2)^2} = 3.61 \text{ ms}^{-1}$ $V_B = \sqrt{\left(\frac{37}{15}\right)^2 + (2.6 \sin \beta)^2}$ $V_B = \sqrt{\left(\frac{37}{15}\right)^2 + (2.4)^2} = 3.44 \text{ ms}^{-1}$	<p>B1</p> <p>M1A1</p> <p>M1 A1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p>	11	<p>B1: Correct values for <math>\cos \alpha</math> and <math>\cos \beta</math>.</p> <p>M1: Four term momentum equation along the line of centres.</p> <p>A1: Correct equation. May be in terms of <math>\alpha</math> and <math>\beta</math>.</p> <p>M1: Newton's Law of Restitution. (Allow sign errors.)</p> <p>A1: Correct equation.</p> <p>A1: Correct velocity of A. AWRT 1.67</p> <p>A1: Correct velocity of B. AWRT 2.47</p> <p>m1: Finding speed of A with their <math>v_A</math>. May be in terms of <math>\alpha</math> and <math>\beta</math>.</p> <p>A1: Correct speed. AWRT 3.61</p> <p>m1: Finding speed of B with their <math>v_B</math>. May be in terms of <math>\alpha</math> and <math>\beta</math>.</p> <p>A1: Correct speed. AWRT 3.44</p>
<b>Total</b>			<b>11</b>	

<p>6 (a)(i)</p> <p>(ii)</p> <p>(b)</p>	 <p> <math>\frac{\sin \alpha}{50} = \frac{\sin 30^\circ}{35}</math> or <math>\frac{\sin \beta}{50} = \frac{\sin 30^\circ}{35}</math>  <math>\alpha = 45.58^\circ</math>  <math>\beta = 134.42^\circ</math> </p> <p>                     Bearings: <math>\left. \begin{array}{l} 346^\circ \\ 074^\circ \end{array} \right\}</math> </p> <p>Angle for shorter time : <math>45.58^\circ</math></p> <p> <math>\frac{{}_F v_S}{\sin 104.42^\circ} = \frac{35}{\sin 30^\circ}</math> </p> <p> <math>{}_F v_S = 67.79 \text{ km h}^{-1}</math> </p> <p> <math>t = \frac{8}{67.79}</math>  <math>= 0.118 \text{ h}</math> or <math>7.08 \text{ min}</math> </p>	<p>B1 B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1F</p>	<p>5</p> <p>5</p>	<p>B1: For one velocity triangle, could be implied by later working. B1: For the other velocity triangle drawn together or separately, could be implied by the correct 2<sup>nd</sup> angle</p> <p>M1: Correct use of sine rule to find <math>\alpha</math> or <math>\beta</math>.</p> <p>A1: <b>Either</b> angle correct.</p> <p>A1: <b>Two</b> correct bearings. Accept <math>74^\circ</math>.</p> <p>B1: Selecting the smaller of their two angles from part (a).</p> <p>M1: Using the sine rule to find the speed of the frigate relative to the ship, with their angle. A1: Correct speed.</p> <p>m1: Using distance over speed. A1F: Correct time. FT their speed. Full marks can be scored by using both angles <b>and</b> choosing the shorter time. If both times calculated and none selected do not award final A1 mark.</p>
--	---	---	-------------------	---



	 <p> <math>v_F = 50 \sin 30^\circ</math>      OE  <math>v_F = 25 \text{ kmh}^{-1}</math> </p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>3</p>	<p>B1: Correct right angled velocity triangle. Could be implied by later working.</p> <p>M1: Use of trigonometry to find speed.</p> <p>A1: Correct speed. CAO.</p>
	<p><b>Total</b></p>		<p><b>13</b></p>	

(a)(ii)	<p>Alternative:</p> <p>Angle for shorter time : <math>45.58^\circ</math></p> $t(50 \cos 30^\circ + 35 \cos 45.58^\circ) = 8$ $\left( t = \frac{8}{50 \cos 30^\circ + 35 \cos 45.58^\circ} \right)$ <p><math>t = 0.118 \text{ h}</math> or <math>7.08 \text{ min}</math></p>	B1	5	<p>B1: Selecting the smaller of their two angles from part (a).</p> <p>M1: For <math>50 \cos 30^\circ \pm 35 \cos 46^\circ</math></p> <p>A1: Correct expression.</p> <p>m1: Using distance over speed.</p>
	A1F	<p>Alternative:</p> <p>Angle for shorter time : <math>45.58^\circ</math></p> $\frac{d}{\sin 30^\circ} = \frac{8}{\sin 104.42^\circ}$ <p><math>d = 4.130 \text{ km}</math></p> $\left( t = \frac{4.130}{35} \right)$ <p><math>t = 0.118 \text{ h}</math> or <math>7.08 \text{ min}</math></p>		
		B1		<p>A1F: Correct time. FT their angle.</p> <p>Full marks can be scored by using both angles <b>and</b> choosing the shorter time. If both times calculated and none selected do not award final A1 mark.</p>
		M1		
		A1		<p>B1: Selecting the smaller of their two angles from part (a).</p> <p>M1: Using the sine rule to find the distance travelled by the frigate with their angle.</p> <p>A1: Correct distance</p> <p>m1: Using distance over speed.</p>
		m1		
		A1F	5	<p>A1: Correct time. FT their angle.</p> <p>Full marks can be scored b using both angles <b>and</b> choosing the shorter time. If both times calculated and none selected do not award final A1 mark.</p>

Question	Solution	Marks	Total	Comments
7 (a)	$y = u \sin(\alpha - \vartheta)t - \frac{1}{2} g \cos \vartheta t^2$ $0 = u \sin(\alpha - \vartheta)t - \frac{1}{2} g \cos \vartheta t^2$ $t = \frac{2u \sin(\alpha - \vartheta)}{g \cos \vartheta}$	M1 A1 m1 A1	4	M1: Expression for perpendicular height of particle above the plane. Accept wrong angles for M1 but <b>not</b> sin and cos in wrong places. A1: Correct expression with $y = 0$ . m1: Solving for non-zero $t$ . A1: Correct $t$ .
(b)	$u \sin \alpha - gt = 0$ $t = \frac{u \sin \alpha}{g}$ $\frac{u \sin \alpha}{g} = \frac{2u \sin(\alpha - \vartheta)}{g \cos \vartheta}$ $\sin \alpha \cos \vartheta = 2 \sin(\alpha - \vartheta)$ $\sin \alpha \cos \vartheta = 2 \sin \alpha \cos \vartheta - 2 \cos \alpha \sin \vartheta$ $\left. \begin{aligned} \sin \alpha \cos \vartheta &= 2 \cos \alpha \sin \vartheta \\ \frac{\sin \alpha}{\cos \alpha} &= 2 \frac{\sin \vartheta}{\cos \vartheta} \end{aligned} \right\}$ $\tan \alpha = 2 \tan \vartheta$	M1 A1 m1 M1 A1	5	M1: Velocity equation to find time to A. A1: Correct time. m1: Forming an equation using their time from part (a) and this time. M1: Use of identity to eliminate compound expressions. It is not enough to only expand $\sin(\alpha - \theta)$ in the expression in part (a) without anything else. A1: Seeing required expression derived with $k = 2$ .
	<b>Total</b>		<b>9</b>	
	<b>TOTAL</b>		<b>75</b>	

(b)	<p>Alternative: Taking <math>x</math> and <math>y</math> axes parallel and perpendicular to the plane respectively and using <math>\tan \theta = \frac{-\dot{y}}{\dot{x}}</math> or equivalent,</p> $\left( u \cos(\alpha - \theta) - g \frac{2u \sin(\alpha - \theta)}{g \cos \theta} \sin \theta \right) \tan \theta =$ $-u \sin(\alpha - \theta) + \frac{g 2u \sin(\alpha - \theta)}{g \cos \theta} \cos \theta$ $\cos(\alpha - \theta) \tan \theta = \sin(\alpha - \theta) (2 \tan^2 \theta + 1)$ $\frac{(\cos \alpha \cos \theta + \sin \alpha \sin \theta) \tan \theta}{(\sin \alpha \cos \theta - \sin \theta \cos \alpha)} = (2 \tan^2 \theta + 1)$ $\tan \alpha \tan^2 \theta + \tan \alpha - 2 \tan^3 \theta - 2 \tan \theta = 0$ $\tan \alpha (1 + \tan^2 \theta) = 2 \tan \theta (1 + \tan^2 \theta)$ $\tan \alpha = 2 \tan \theta$	<p>M1</p> <p>A1</p> <p>M1</p> <p>m1</p> <p>A1</p>	<p>5</p>	<p>M1: Correct terms, allow sign errors. A1: All correct</p> <p>M1: Use of identities to eliminate compound expressions.</p> <p>m1: Rearranging to the required form.</p> <p>A1: Seeing required expression derived with <math>k = 2</math>.</p>
-----	---	--	----------	---



Scaled mark unit grade boundaries – June 2015 exams

A-level

Code	Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points					
			A*	A	B	C	D	E
MD01	MATHEMATICS UNIT MD01	75	-	60	55	50	46	42
MD02	MATHEMATICS UNIT MD02	75	67	62	55	48	42	36
MFP1	MATHEMATICS UNIT MFP1	75	-	60	53	46	39	32
MFP2	MATHEMATICS UNIT MFP2	75	61	55	48	41	35	29
MFP3	MATHEMATICS UNIT MFP3	75	67	62	56	51	46	41
MFP4	MATHEMATICS UNIT MFP4	75	68	61	54	47	40	34
<b>MM03</b>	<b>MATHEMATICS UNIT MM03</b>	<b>75</b>	<b>66</b>	<b>59</b>	<b>52</b>	<b>46</b>	<b>40</b>	<b>34</b>
MM04	MATHEMATICS UNIT MM04	75	66	59	52	45	38	31
MM05	MATHEMATICS UNIT MM05	75	64	56	48	41	34	27
MM1B	MATHEMATICS UNIT MM1B	75	-	56	49	42	35	29
MM2B	MATHEMATICS UNIT MM2B	75	65	60	52	44	36	29
MPC1	MATHEMATICS UNIT MPC1	75	-	64	58	52	46	40
MPC2	MATHEMATICS UNIT MPC2	75	-	58	50	42	35	28
MPC3	MATHEMATICS UNIT MPC3	75	64	57	52	47	43	39
MPC4	MATHEMATICS UNIT MPC4	75	63	57	52	47	42	37
MS03	MATHEMATICS UNIT MS03	75	67	60	53	46	39	32
MS04	MATHEMATICS UNIT MS04	75	67	60	52	44	37	30
MS1A	MATHEMATICS UNIT MS1A	100	-	84	75	66	57	48
<i>MS1A/W</i>	<i>MATHEMATICS UNIT MS1A - WRITTEN</i>	75		64				38
<i>MS1A/C</i>	<i>MATHEMATICS UNIT MS1A - COURSEWORK</i>	25		20				10