General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MM03

Unit Mechanics 3

Tuesday 18 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A stone, of mass 2 kg, is moving in a straight line on a smooth horizontal sheet of ice under the action of a single force which acts in the direction of motion. At time t seconds, the force has magnitude $(3t + 1)$ newtons, $0 \leqslant t \leqslant 3$.

When $t = 0$, the stone has velocity $1\,\mathrm{m\,s^{-1}}$.
When $t = T$, the stone has velocity $5\,\mathrm{m\,s^{-1}}$.

Find the value of T. *(6 marks)*

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2 A car has mass m and travels up a slope which is inclined at an angle θ to the horizontal. The car reaches a maximum speed v at a height h above its initial position. A constant resistance force R opposes the motion of the car, which has a maximum engine power output P.

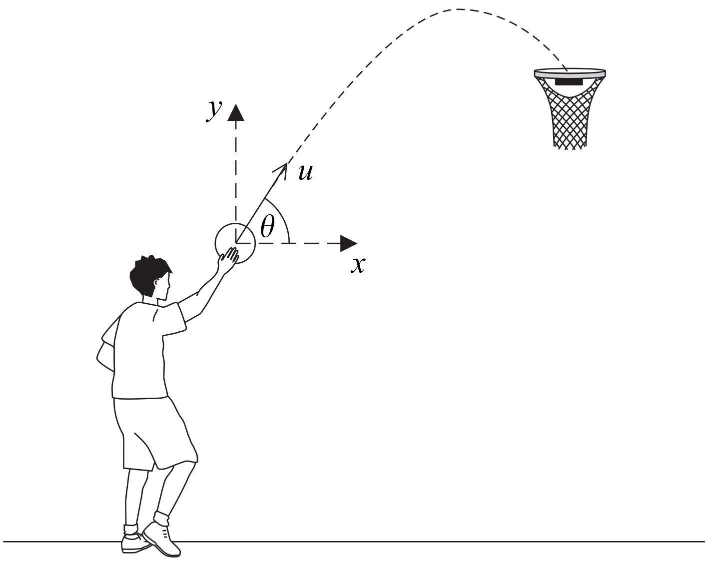
Neda finds a formula for P as

$$P = mgv\sin\theta + Rv + \frac{1}{2}mv^3\frac{\sin\theta}{h}$$

where g is the acceleration due to gravity.

Given that the engine power output may be measured in newton metres per second, determine whether the formula is dimensionally consistent. *(6 marks)*

3 A player projects a basketball with speed $u\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal. The basketball travels in a vertical plane through the point of projection and goes into the basket. During the motion, the horizontal and upward vertical displacements of the basketball from the point of projection are x metres and y metres respectively.



(a) Find an expression for y in terms of x, u, g and $\tan\theta$. *(6 marks)*

(b) The player projects the basketball with speed $8\,\mathrm{m\,s^{-1}}$ from a point 0.5 metres vertically below and 5 metres horizontally from the basket.

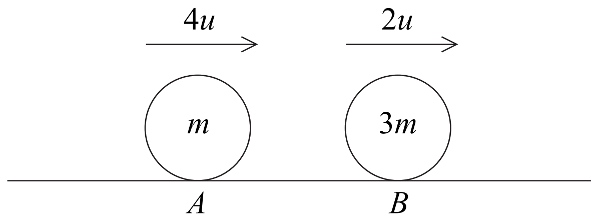
(i) Show that the two possible values of θ are approximately 63.1° and 32.6°, correct to three significant figures. *(5 marks)*

(ii) Given that the player projects the basketball at 63.1° to the horizontal, find the direction of the motion of the basketball as it enters the basket. Give your answer to the nearest degree. *(4 marks)*

(c) State a modelling assumption needed for answering parts **(a)** and **(b)** of this question. *(1 mark)*

Turn over ▶

4 A smooth sphere A, of mass m, is moving with speed $4u$ in a straight line on a smooth horizontal table. A smooth sphere B, of mass $3m$, has the same radius as A and is moving on the table with speed $2u$ in the same direction as A.



The sphere A collides directly with sphere B. The coefficient of restitution between A and B is e.

(a) Find, in terms of u and e, the speeds of A and B immediately after the collision. *(6 marks)*

(b) Show that the speed of B after the collision cannot be greater than $3u$. *(2 marks)*

(c) Given that $e = \frac{2}{3}$, find, in terms of m and u, the magnitude of the impulse exerted on B in the collision. *(3 marks)*

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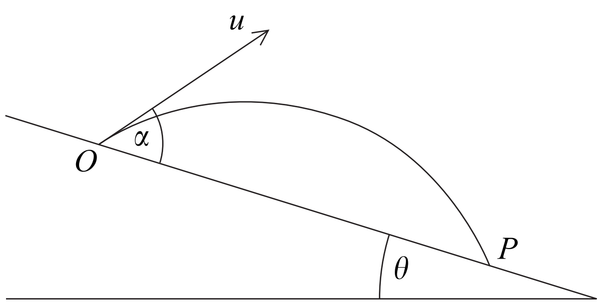
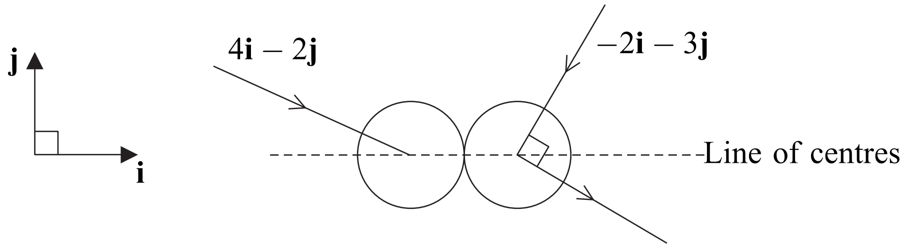
5 A particle is projected from a point O on a plane which is inclined at an angle θ to the horizontal. The particle is projected down the plane with velocity u at an angle α above the plane. The particle first strikes the plane at a point P, as shown in the diagram. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Given that the time of flight from O to P is T, find an expression for u in terms of θ, α, T and g. *(4 marks)*

(b) Using the identity $\cos(X - Y) = \cos X \cos Y + \sin X \sin Y$, show that the distance OP is given by $\dfrac{2u^2 \sin\alpha \cos(\alpha - \theta)}{g \cos^2\theta}$. *(6 marks)*

6 Two smooth spheres, A and B, have equal radii and masses 4 kg and 2 kg respectively. The sphere A is moving with velocity $(4\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the sphere B is moving with velocity $(-2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ on the same smooth horizontal surface. The spheres collide when their line of centres is parallel to unit vector $\mathbf{i}$. The direction of motion of B is changed through 90° by the collision, as shown in the diagram.

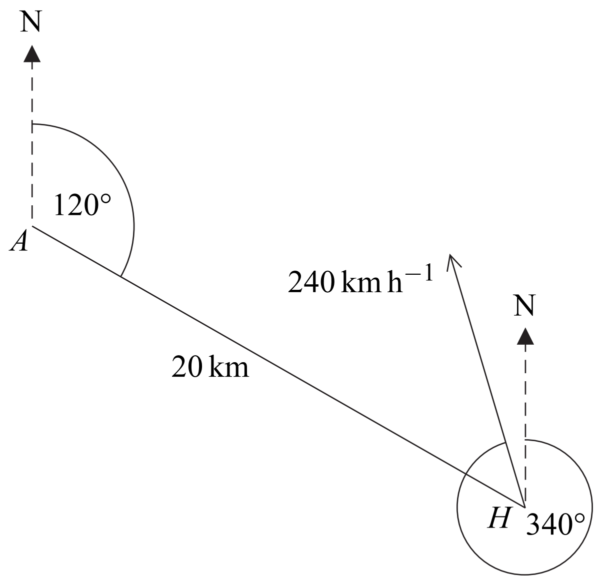


(a) Show that the velocity of B immediately after the collision is $(\frac{9}{2}\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$.
(4 marks)

(b) Find the coefficient of restitution between the spheres. *(5 marks)*

(c) Find the impulse exerted on B during the collision. State the units of your answer.
(3 marks)

7 From an aircraft A, a helicopter H is observed 20 km away on a bearing of 120°. The helicopter H is travelling horizontally with a constant speed $240\,\text{km h}^{-1}$ on a bearing of 340°. The aircraft A is travelling with constant speed $v_A\,\text{km h}^{-1}$ in a straight line and at the same altitude as H.



(a) Given that $v_A = 200$:

(i) find a bearing, to one decimal place, on which A could travel in order to intercept H; *(5 marks)*

(ii) find the time, in minutes, that it would take A to intercept H on this bearing. *(4 marks)*

(b) Given that $v_A = 150$, find the bearing on which A should travel in order to approach H as closely as possible. Give your answer to one decimal place. *(5 marks)*

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

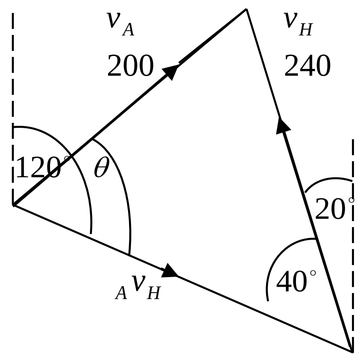
Q	Solution	Marks	Total	Comments
1	Use of Impulse-momentum principle	M1		$\int_{(0)}^{(T)}(3t+1)\,\mathrm{d}t = \pm 2(5) \pm 2(1)$ Condone sign error for M1
	$\int_{(0)}^{(T)}(3t+1)\ \mathrm{d}t = 2(5) - 2(1)$	A1		A1 for all correct
	$\left[\frac{3}{2}t^2 + t\right]_{(0)}^{(T)} = (8)$	A1		Correct integration, PI by the correct quadratic
	$3T^2 + 2T - 16 = 0$	A1		Correct use of correct limits and rearrangement
	$(3T+8)(T-2) = 0$ or $T = \frac{-2 \pm \sqrt{4 - 4(3)(-16)}}{2(3)}$	m1		Solution of their quadratic, **correct** attempt needed
	($T = -\frac{8}{3}$ unacceptable, not in the interval $0 \le t \le 3$)			
	$\underline{T = 2}$	A1	6	
	Total		**6**	
2	$[P]$ = MLT^{-2}. L . T^{-1} = ML^{2}T^{-3}	B1		For correct unsimplified dimensions of quantities
	$[mgv\sin\theta]$ = M . LT^{-2}. LT^{-1} = ML^{2}T^{-3}	B1		
	$[Rv]$ = MLT^{-2}. LT^{-1} = ML^{2}T^{-3}	B1		
	$\left[\frac{1}{2}mv^3\frac{\sin\theta}{h}\right]$ = M . L^{3}T^{-3}. L^{-1} = ML^{2}T^{-3}	B1		
		B1		All simplifications correct
	The formula is dimensionally consistent	E1		Dependent on the last B1
	Total		**6**	

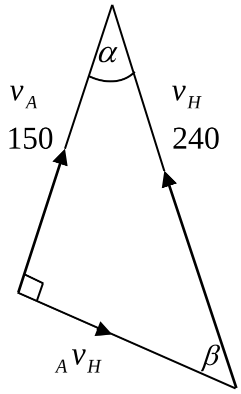
Q	Solution	Marks	Total	Comments
3(a)	$x = ut\cos\theta$	M1		
	$t = \dfrac{x}{u\cos\theta}$	A1		
	$y = -\dfrac{1}{2}gt^2 + ut\sin\theta$	M1		Condone $+g$ for M1
	$y = -\dfrac{1}{2}gt^2 + ut\sin\theta$	A1		
	$y = -\dfrac{1}{2}g(\dfrac{x}{u\cos\theta})^2 + u(\dfrac{x}{u\cos\theta})\sin\theta$	m1		Elimination of *t*, condone $+g$ for m1
	$y = -\dfrac{gx^2}{2u^2\cos^2\theta} + \dfrac{x\sin\theta}{\cos\theta}$			
	$y = -\dfrac{gx^2}{2u^2}(1+\tan^2\theta) + x\tan\theta$	A1	6	OE in terms of x, u, g, $\tan\theta$
(b)(i)	$0.5 = -\dfrac{9.8(5)^2}{2(8)^2}(1+\tan^2\theta) + 5\tan\theta$	M1		Correctly substituting for x, y, u and g into their equation of trajectory
		A1		All correct, condone decimal approximation.
	$245\tan^2\theta - 640\tan\theta + 309 = 0$	A1		OE exact quadratic in $\tan\theta$
	$\tan\theta = \dfrac{640 \pm \sqrt{(-640)^2 - 4(245)(309)}}{2(245)}$	m1		PI by the values of $\tan\theta$
	$\tan\theta = 1.973(004)$, $0.6392(41)$			
	$\theta = 63.12^\circ$, 32.58°			
	$\theta = 63.1^\circ$, 32.6°	A1	5	AG Must see the above or more accurate values
(ii)	$\dot{y} = -9.8(\dfrac{5}{8\cos 63.1^\circ}) + 8\sin 63.1^\circ$ OE	M1		Condone +9.8 for M1.
	$(\dot{y} = -6.4035)$			
	$\dot{x} = 8\cos 63.1^\circ$	M1		
	$(\dot{x} = 3.6195)$			
	$\tan^{-1}\dfrac{6.4(035)}{3.6(195)}$ $(=61^\circ)$ OE	m1		PI by correct angle in a statement
	Direction : 61° to the horizontal or 29° to the vertical	A1	4	Have to see "horizontal" or "vertical" or diagram

Q	Solution	Marks	Total	Comments
3(b)(ii)	**Alternative:**			
	$\frac{dy}{dx} = -\frac{2gx}{2u^2}(1+\tan^2\theta)+\tan\theta$	(M1)		
	$= -\frac{2\times 9.8\times 5}{2\times 8^2}(1+\tan^2 63.1^\circ)+\tan 63.1^\circ$ $= -1.7692$	(A1)		
	$\tan^{-1}(-1.7692) = -60.52368^\circ$	(m1)		
	Direction: 61° to the horizontal or 29° to the vertical	(A1)		
(c)	The ball is a particle, or No air resistance, or The ball does not spin	B1	1	
	Total		**16**	
4(a)	$m(4u)+3m(2u) = mv_A + 3mv_B$	M1 A1		M1 for four correct momentum terms with any signs. A1for all correct
	$\frac{v_B - v_A}{4u-2u} = e$	M1 A1		M1 for correct terms for any signs, A1 for all correct.
	$\left(\begin{array}{l} v_A + 3v_B = 10u \\ v_B - v_A = 2ue \\ 4v_B = 2ue + 10u \end{array}\right)$			
	$v_B = \frac{u}{2}(e+5)$	A1		OE, simplified
	$\left(v_A = \frac{u}{2}(e+5) - 2ue\right)$			
	$v_A = \frac{u}{2}(-3e+5)$	A1	6	OE, simplified
(b)	$e \le 1 \Rightarrow v_B \le \frac{u}{2}(1+5)$	M1		Use of $e \le 1$ (OE) needed FT their v_B
	$\Rightarrow v_B \le 3u$	A1	2	
(c)	$(I =)\ 3m\,.\,\frac{u}{2}(\frac{2}{3}+5) - 3m\,.\,2u$	M1		M1 for a difference of two momentums FT their velocity from part (a)
		A1F		A1F for their '*Final B – Initial B*'
	$= \frac{5mu}{2}$ or $2.5mu$	A1	3	
	Total		**11**	

Q	Solution	Marks	Total	Comments
5(a)	$\perp$ to plane $y = ut\sin\alpha - \frac{1}{2}gt^2\cos\theta$	M1		For M1, $\sin\alpha$ and $\cos\theta$ must be in the correct terms but accept $+g$.
	$y = ut\sin\alpha - \frac{1}{2}gt^2\cos\theta$	A1		
	$uT\sin\alpha - \frac{1}{2}gT^2\cos\theta = 0$	m1		Accept +*g* for m1.
	$u = \dfrac{Tg\cos\theta}{2\sin\alpha}$	A1	4	OE
(b)	t or $T = \dfrac{2u\sin\alpha}{g\cos\theta}$	B1		
	$\parallel$ to plane $x = ut\cos\alpha + \frac{1}{2}gt^2\sin\theta$	M1		For M1, $\cos\alpha$ and $\sin\theta$ must be in the correct terms but accept $-g$.
	$x = ut\cos\alpha + \frac{1}{2}gt^2\sin\theta$	A1		
	$(\overrightarrow{OP} =)\ u(\dfrac{2u\sin\alpha}{g\cos\theta})\cos\alpha + \frac{1}{2}g(\dfrac{2u\sin\alpha}{g\cos\theta})^2\sin\theta$	m1		Elimination of *t* substituting their expression into their equation for *x*.
	$\left(= \dfrac{2u^2\sin\alpha\cos\alpha}{g\cos\vartheta} + \dfrac{2u^2\sin^2\alpha\sin\theta}{g\cos^2\theta}\right)$			
	$= \dfrac{2u^2\sin\alpha(\cos\alpha\cos\theta + \sin\alpha\sin\theta)}{g\cos^2\theta}$	m1		OE single correct fraction in factorised form
	$= \dfrac{2u^2\sin\alpha\cos(\alpha-\theta)}{g\cos^2\theta}$	A1	6	AG Sight of the above line needed
	Total		**10**	

Q	Solution	Marks	Total	Comments
6	(Let $v_B = a\mathbf{i} - b\mathbf{j}$)			
	$\frac{a}{b} = \frac{3}{2}$	M1		Allow sign error
	$\frac{a}{b} = \frac{3}{2}$	A1		OE
	(Squares are smooth ⇒ j component ⇒) $b = 3$	B1		
	$a = \frac{9}{2}$	A1	4	AG
	$\left(v_B = \frac{9}{2}\mathbf{i} - 3\mathbf{j}\right)$			
(b)	(C.L.M. along the line of centres:)			
	$4(4) - 2(2) = 4(v_A) + 2(\frac{9}{2})$	M1		OE, No sign errors
	$v_A = \frac{3}{4}$	A1		
	(Restitution along the line of centres:)			
	$e = \frac{-\frac{3}{4} + \frac{9}{2}}{4+2}$ OE	M1 A1		M1 for correct terms, A0 for sign error
	$e = \frac{5}{8}$	A1	5	
(c)	(I = Change in momentum of B along the line of centres)			
	$= 2\,(\frac{9}{2}\mathbf{i}) - 2\,(-2\mathbf{i})$	M1		Allow sign error and missing **i**
	$= 13\mathbf{i}$	A1		A0 for magnitude or $-13\mathbf{i}$
	Ns or kg m s^{-1}	B1	3	
	Total		**12**	

Q	Solution	Marks	Total	Comments
7(a)(i)		B1 B1		Correct diagram with or without arrows. 40° marked correctly, PI by correct method.
	$\frac{\sin\theta}{240} = \frac{\sin 40}{200}$	M1		Correct sine rule allowing their angle opposite 200 in their diagram.
	$\theta = 50.47483^\circ$ or 50.5°	A1		AWRT 50.5°, PI by correct bearing
	Bearing of $v_A = 069.5^\circ$	A1	5	Allow 69.5°
(a)(ii)	$\frac{_Av_H}{\sin(180^\circ - 40^\circ - 50.5^\circ)} = \frac{200}{\sin 40^\circ}$ or $\frac{240}{\sin 50.5^\circ}$	M1		Allow using their angle from part (a)(i).
	$_Av_H = 311.13408$ or 311	A1F		FT their angle from part (a)(i)
	Time $= \frac{20}{311.13408}$	M1		PI by correct answer. Allow their $_Av_H$.
	$(= 0.0642809 \text{ hours}) = 3.86$ min	A1F	4	3sf required

Q	Solution	Marks	Total	Comments
7(b)		M1 A1		Right-angled triangle with 240 and 150 marked. Correct orientation
	$\cos\alpha = \frac{150}{240}$ or $\sin\beta = \frac{150}{240}$	M1		
	$\alpha = 51.3^\circ$ or $\beta = 38.7^\circ$	A1		PI by correct bearing
	Bearing: 031.3°	A1	5	Allow 31.3°
	Total		**14**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2013 exams

A-level

Code	Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points A*	A	B	C	D	E
LAW02	LAW UNIT 2	94	-	77	69	61	53	45
LAW03	LAW UNIT 3	80	69	63	57	52	47	42
LAW04	LAW UNIT 4	85	73	67	61	56	51	46
MD01	MATHEMATICS UNIT MD01	75	-	64	59	54	50	46
MD02	MATHEMATICS UNIT MD02	75	69	64	56	48	41	34
MFP1	MATHEMATICS UNIT MFP1	75	-	55	49	43	37	32
MFP2	MATHEMATICS UNIT MFP2	75	65	61	54	47	40	34
MFP3	MATHEMATICS UNIT MFP3	75	67	64	56	49	42	35
MFP4	MATHEMATICS UNIT MFP4	75	64	60	52	44	36	28
MM1B	MATHEMATICS UNIT MM1B	75	-	56	49	42	35	29
MM2B	MATHEMATICS UNIT MM2B	75	67	61	55	49	43	37
MM03	MATHEMATICS UNIT MM03	75	70	65	58	51	44	37
MM04	MATHEMATICS UNIT MM04	75	67	59	51	44	37	30
MM05	MATHEMATICS UNIT MM05	75	68	61	52	44	36	28
MPC1	MATHEMATICS UNIT MPC1	75	-	59	53	47	41	36
MPC2	MATHEMATICS UNIT MPC2	75	-	61	55	49	43	37
MPC3	MATHEMATICS UNIT MPC3	75	66	60	54	49	44	39
MPC4	MATHEMATICS UNIT MPC4	75	60	55	50	45	40	35
MS1A	MATHEMATICS UNIT MS1A	100	-	76	67	59	51	43
MS/SS1A/W	*MATHEMATICS UNIT S1A - WRITTEN*	*75*		*56*				*33*
MS/SS1A/C	*MATHEMATICS UNIT S1A - COURSEWORK*	*25*		*20*				*10*
MS1B	MATHEMATICS UNIT MS1B	75	-	56	50	44	39	34
MS2B	MATHEMATICS UNIT MS2B	75	71	67	60	53	46	39