Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MM03

Unit Mechanics 3

Friday 22 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

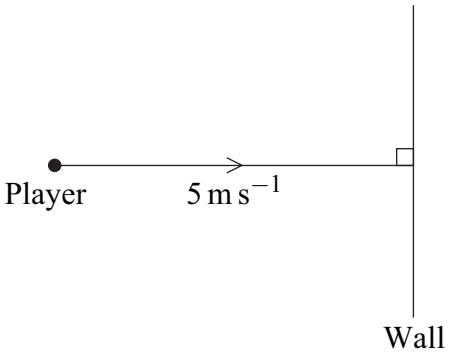
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 An ice-hockey player has mass 60 kg. He slides in a straight line at a constant speed of $5\,\text{m}\,\text{s}^{-1}$ on the horizontal smooth surface of an ice rink towards the vertical perimeter wall of the rink, as shown in the diagram.



The player collides directly with the wall, and remains in contact with the wall for 0.5 seconds.

At time t seconds after coming into contact with the wall, the force exerted by the wall on the player is $4 \times 10^4 t^2 (1 - 2t)$ newtons, where $0 \leqslant t \leqslant 0.5$.

(a) Find the magnitude of the impulse exerted by the wall on the player. *(4 marks)*

(b) The player rebounds from the wall. Find the player's speed immediately after the collision. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 1

2 A pile driver of mass m_1 falls from a height h onto a pile of mass m_2, driving the pile a distance s into the ground. The pile driver remains in contact with the pile after the impact. A resistance force R opposes the motion of the pile into the ground.

Elizabeth finds an expression for R as

$$R = \frac{g}{s}\left[s(m_1 + m_2) + \frac{h(m_1)^2}{m_1 + m_2}\right]$$

where g is the acceleration due to gravity.

Determine whether the expression is dimensionally consistent. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 2

3 **(In this question, take $g = 10\,\text{m}\,\text{s}^{-2}$.)**

A projectile is fired from a point O with speed u at an angle of elevation α above the horizontal so as to pass through a point P. The projectile travels in a vertical plane through O and P. The point P is at a horizontal distance $2k$ from O and at a vertical distance k above O.

(a) Show that α satisfies the equation

$$20k\tan^2\alpha - 2u^2\tan\alpha + u^2 + 20k = 0 \qquad \textit{(7 marks)}$$

(b) Deduce that

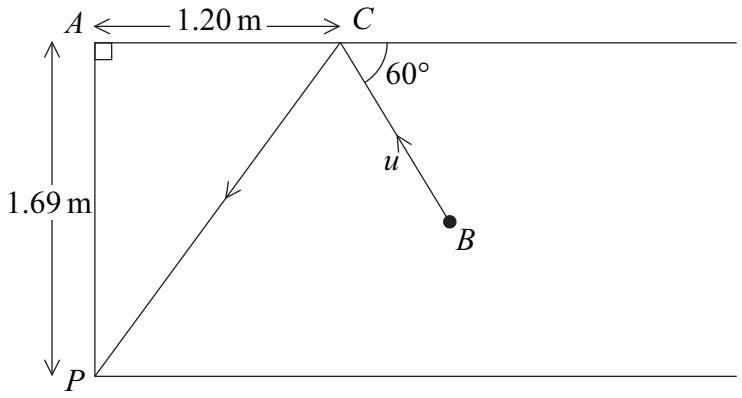
$$u^4 - 20ku^2 - 400k^2 \geqslant 0 \qquad \textit{(3 marks)}$$

QUESTION PART REFERENCE

Answer space for question 3

4 The diagram shows part of a horizontal snooker table of width 1.69 m.

A player strikes the ball B directly, and it moves in a straight line. The ball hits the cushion of the table at C before rebounding and moving to the pocket at P at the corner of the table, as shown in the diagram. The point C is 1.20 m from the corner A of the table. The ball has mass 0.15 kg and, immediately before the collision with the cushion, it has velocity u in a direction inclined at 60° to the cushion. The **table** and the **cushion** are modelled as smooth.



(a) Find the coefficient of restitution between the ball and the cushion. *(5 marks)*

(b) Show that the magnitude of the impulse on the cushion at C is approximately $0.236u$. *(4 marks)*

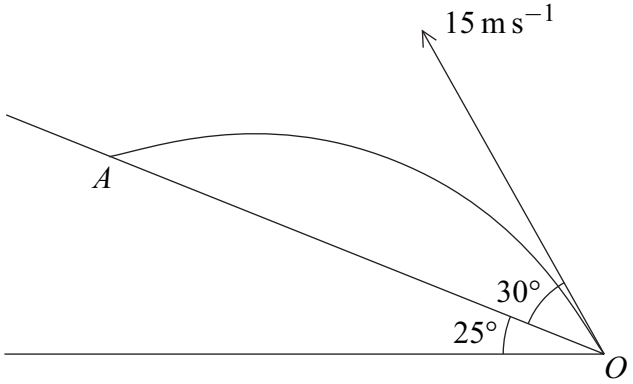
(c) Find, in terms of u, the time taken between the ball hitting the cushion at C and entering the pocket at P. *(3 marks)*

(d) Explain how you have used the assumption that the cushion is smooth in your answers. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

5 A particle is projected from a point O on a smooth plane, which is inclined at 25° to the horizontal. The particle is projected up the plane with velocity $15\,\mathrm{m\,s^{-1}}$ at an angle 30° above the plane. The particle strikes the plane for the first time at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Find the time taken by the particle to travel from O to A. *(4 marks)*

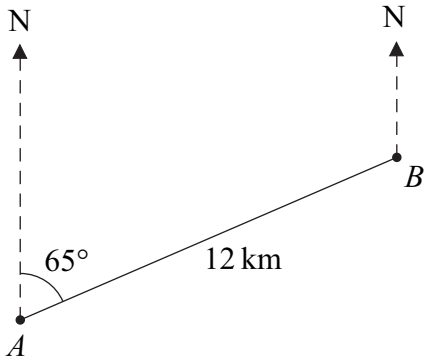
(b) The coefficient of restitution between the particle and the inclined plane is $\frac{2}{3}$.

Find the speed of the particle as it rebounds from the inclined plane at A. *(8 marks)*

QUESTION PART REFERENCE

Answer space for question 5

6 At noon, two ships, A and B, are a distance of 12 km apart, with B on a bearing of 065° from A. The ship B travels due north at a constant speed of $10\,\text{km}\,\text{h}^{-1}$. The ship A travels at a constant speed of $18\,\text{km}\,\text{h}^{-1}$.



(a) Find the direction in which A should travel in order to intercept B. Give your answer as a bearing. *(4 marks)*

(b) In fact, the ship A actually travels on a bearing of 065°.

(i) Find the distance between the ships when they are closest together. *(7 marks)*

(ii) Find the time when the ships are closest together. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 6

7 Two smooth spheres, A and B, have equal radii and masses $2m$ kg and m kg respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(3\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it collides with the sphere B, which has velocity $(2\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$. Immediately after the collision, the velocity of the sphere B is $(2\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$.

(a) Find the velocity of A immediately after the collision. *(3 marks)*

(b) Show that the impulse exerted on B in the collision is $(6m\mathbf{j})\,\text{Ns}$. *(3 marks)*

(c) Find the coefficient of restitution between the two spheres. *(4 marks)*

(d) After the collision, each sphere moves in a straight line with constant speed. Given that the radius of each sphere is 0.05 m, find the time taken, from the collision, until the centres of the spheres are 1.10 m apart. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 7

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

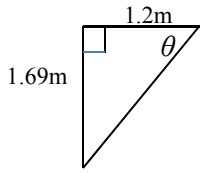
MM03

Q	Solution	Marks	Total	Comments
1(a)	$I = \int_0^{0.5} 4\times10^4 t^2(1-2t)\ \mathrm{d}t$	M1		Attempt to integrate
		A1		Use of correct limits, PI
	$= 4\times10^4\left[\frac{1}{3}t^3 - \frac{1}{2}t^4\right]_0^{0.5}$	A1F		Correct integration
	$= 417$ (or $\frac{1250}{3}$) Ns	A1F	4	Accept $416.\dot{6}$ or 416.7
(b)	$416.\dot{6} = 60v + 60\times5$	M1A1F		A1F correct sign
	$v = 1.94$	A1F	3	AWRT 1.94, accept 1.95 ISW
	Total		**7**	
2	Dimension of g is LT^{-2} Dimension of s is L Dimension of h is L Dimension of m_1 and m_2 is M	B 1		B1 for dimensions of the five quantities
	Dimension of $\frac{g}{s}[s(m_1+m_2)+\frac{hm_1^2}{m_1+m_2}]$ is			
	$\frac{LT^{-2}}{L}[LM+\frac{LM^2}{M}] \cong MLT^{-2}+MLT^{-2}$	M1		Correct substitution of dimensions
	$\cong MLT^{-2}$	A1		
	which is a force	B1	4	
	Total		**4**	

MM03

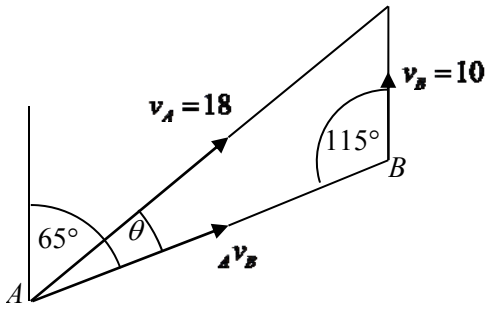
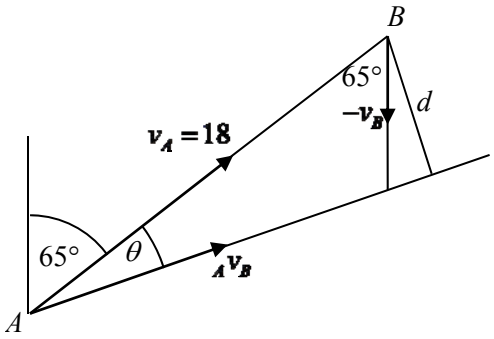
Q	Solution	Marks	Total	Comments
3(a)	$x = ut\cos\alpha$	M1		
	$t = \dfrac{x}{u\cos\alpha}$	A1		
	$y = -\dfrac{1}{2}gt^2 + ut\sin\alpha$	M1		Must have correct signs
	$y = -\dfrac{1}{2}g\left(\dfrac{x}{u\cos\alpha}\right)^2 + u\left(\dfrac{x}{u\cos\alpha}\right)\sin\alpha$	M1		
	$y = -\dfrac{gx^2}{2u^2\cos^2\alpha} + \dfrac{x\sin\alpha}{\cos\alpha}$			
	$y = -\dfrac{gx^2}{2u^2}(1+\tan^2\alpha) + x\tan\alpha$	A1		
	$k = -\dfrac{10(2k)^2}{2u^2}(1+\tan^2\alpha) + 2k\tan\alpha$	M1		
	$u^2 = -20k(1+\tan^2\alpha) + 2u^2\tan\alpha$			
	$20k\tan^2\alpha - 2u^2\tan\alpha + u^2 + 20k = 0$	A1	7	AG
(b)	Pass through $P \Longrightarrow$ Discriminant ≥ 0			
	$(-2u^2)^2 - 4(20k)(u^2+20k) \geq 0$	M1A1		OE must be seen
	$4u^4 - 80ku^2 - 1600k^2 \geq 0$			
	$u^4 - 20ku^2 - 400k^2 \geq 0$	A1	3	AG
	Total		**10**	

MM03

Q	Solution	Marks	Total	Comments
4(a)				
	$\theta = \tan^{-1}\dfrac{1.69}{1.2} = 54.623^\circ$	B1		AWRT 55°
	$u\cos 60^\circ = v\cos 54.623^\circ$	M1		$v = 0.864u$
	$eu\sin 60^\circ = v\sin 54.623^\circ$	M1		
	$e = \dfrac{v\sin 54.623^\circ}{\dfrac{v\cos 54.623^\circ}{\cos 60^\circ}\times\sin 60^\circ}$	m1		OE, dependent on both M1s
	$e = 0.813$ or 0.812	A1	5	ISW
(b)	$I = 0.15u\sin 60^\circ + 0.15v\sin 54.623^\circ$	M1A1		Single angle values needed for A1
	$= 0.15u\sin 60^\circ + 0.15\times\dfrac{u\cos 60^\circ}{\cos 54.623^\circ}\times\sin 54.623^\circ$	m1		
	$= 0.236u$	A1	4	AG (condone 0.2355 or negative result)
(c)	Attempt at considering motion parallel or perpendicular to AC	M1		
	$t = \dfrac{1.2}{u\cos 60^\circ}$	M1		
	$t = \dfrac{12}{5u}$ or $\dfrac{2.4}{u}$	A1	3	OE, No ISW
	Alternative:			
	$CP = \dfrac{1.2}{\cos 54.623^\circ}$ $(= 2.072703844$ m)	(M1)		
	$t = \dfrac{\dfrac{1.2}{\cos 54.623^\circ}}{\dfrac{u\cos 60^\circ}{\cos 54.623^\circ}}$	(M1)		
	$= \dfrac{12}{5u}$ or $\dfrac{2.4}{u}$	(A1)	(3)	(OE), No ISW
(d)	Velocity (momentum) parallel to the cushion is unchanged, or, Restitution only affects motion perpendicular to the cushion	E1	1	Accept 'horizontal component of velocity is unchanged'
	Total		**13**	

Q	Solution	Marks	Total	Comments
5(a)	$0 = 15t\sin 30 - \frac{1}{2}g\cos 25t^2$	M1A1		Accept wrong angle(s) for M1 but **not** sin and cos in wrong places
	$t = \frac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$t = 1.69$ sec.	A1F	4	AWRT 1.69
(b)	$\perp$ to plane $\dot{y} = 15\sin 30 - g\cos 25 \times \frac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$\dot{y} = -7.5$ ms^{-1}	A1F		Or –7.51, ft from their answer in (a)
	$\parallel$ to plane $\dot{x} = 15\cos 30 - g\sin 25 \times \frac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$\dot{x} = 5.995766$ or 6.00 ms^{-1}	A1F		Accept 5.99
	Restitution: Rebound $\dot{y} = \frac{2}{3} \times 7.5 = 5$ ms^{-1}	M1		Or 5.01
	$\dot{x}$ unchanged	B1		PI, dependent on the last M1
	Speed of rebound $= \sqrt{5.995766^2 + 5^2}$	m1		Dependent on the last three M1s
	$= 7.81$ ms^{-1}	A1F	8	
	Total		**12**	

MM03

Q	Solution	Marks	Total	Comments
6(a)		B1		For any appropriate diagram PI by correct method
	$\frac{\sin\theta}{10} = \frac{\sin 115^\circ}{18}$	M1		
	$\theta = 30.2^\circ$	A1		
	Bearing = 035°	A1	4	Accept 034.8°
(b)(i)		B1		For any appropriate diagram PI by correct method
	$_Av_B{}^2 = 18^2 + 10^2 - 2(18)(10)\cos 65^\circ$	M1		
	$_Av_B = 16.4881$ ms^{-1}	A1		OE
	$\frac{\sin 65^\circ}{16.4881} = \frac{\sin\theta}{10}$	M1		
	$\theta = 33.3446^\circ$	A1F		
	$d = 12 \times \sin 33.3446^\circ$	m1		OE
	$d = 6.60$ km	A1F	7	Dependent on the previous two M1s (AWRT 6.6 km)
(ii)	$t = \frac{12 \times \cos 33.3446^\circ}{16.4881} = 0.607987$ hours	M1		Or 0.608 hours
		A1F		LHS values
	(= 36.5 min)	A1F	3	Correct time
	Total		**14**	

MM03

Q6 (b)(i) Alternative:

$r_A = [(18\cos 25)\mathbf{i} + (18\sin 25)\mathbf{j}]t$

$r_B = [(12\cos 25)\mathbf{i} + (12\sin 25)\mathbf{j}] + 10\mathbf{j}t$ M1 for both

${}_A r_B = (-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}$ m1 A1

$\left|{}_A r_B\right|^2 = (-12\cos 25 + 18t\cos 25)^2 + (-12\sin 25 + 18t\sin 25 - 10t)^2$ A1

$\frac{\mathrm{d}\left|{}_A r_B\right|^2}{\mathrm{d}t} = (36\cos 25)(-12\cos 25 + 18t\cos 25) + (36\sin 25 - 20)(-12\sin 25 + 18t\sin 25 - 10t) = 0$ m1

$t = 0.608$ A1

$d = 6.60$ km or 6.6 km A1

The corresponding marks awarded for finding the closest approach time:

$\frac{\mathrm{d}\left|{}_A r_B\right|^2}{\mathrm{d}t} = (36\cos 25)(-12\cos 25 + 18t\cos 25) + (36\sin 25 - 20)(-12\sin 25 + 18t\sin 25 - 10t) = 0$ M1 A1

$t = 0.608$ (or better) A1

(b)(i) Alternative (**Not in the specification**):

${}_A r_B = (-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}$ M1 A1

$[(-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}] \cdot [(18\sin 65)\mathbf{i} + (18\cos 65 - 10)\mathbf{j}] = 0$ m1

$(-12\cos 25 + 18t\cos 25)(18\sin 65) + (-12\sin 25 + 18t\sin 25 - 10t)(18\cos 65 - 10) = 0$ A1

$271.85\,t = 165.27$ m1

$t = 0.608$ (or better) A1

$d = 6.60$ km or 6.6 km A1

The corresponding marks awarded for finding the closest approach time:

$(-12\cos 25 + 18t\cos 25)(18\sin 65) + (-12\sin 25 + 18t\sin 25 - 10t)(18\cos 65 - 10) = 0$ M1

$271.85\,t = 165.27$ A1

$t = 0.608$ (or better) A1

(b)(ii) FT from their answers in part (b)(i)

MM03

Q	Solution	Marks	Total	Comments
7(a)	$2m(3i+j)+m(2i-5j)=2mv_A+m(2i+j)$	M1A1		
	$8i-3j=2v_A+(2i+j)$			
	$v_A=3i-2j$	A1	3	
(b)	$I=m(2i+j)-m(2i-5j)$	M1A1		
	$I=6mj$	A1	3	AG
(c)	$I=6mj \Rightarrow$ Line of centres along j	B1		PI
	Restitution along j : $1+2=e(5+1)$	M1A1		
	$e=0.5$	A1	4	
	Accept energy methods			
(d)	${}_Av_B=i-3j$			
	${}_Ar_B=-0.1j+(i-3j)t$	M1A1		
	$1.1^2=t^2+(-0.1-3t)^2$	M1		OE
	$10t^2+0.6t-1.2=0$			
	$t=\dfrac{-0.6\pm\sqrt{0.6^2-4(10)(-1.2)}}{2(10)}$ $(=0.31770677)$	m1		Dependent on both M1s
	$t=0.318$ or 0.317 sec.	A1	5	
	Total		**15**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2012 exams

A-level

Code	Title	Max. Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points					
			A*	A	B	C	D	E
MD01	MATHEMATICS UNIT MD01	75	-	60	55	50	45	40
MD02	MATHEMATICS UNIT MD02	75	68	61	53	45	38	31
MFP1	MATHEMATICS UNIT MFP1	75	-	61	54	47	41	35
MFP2	MATHEMATICS UNIT MFP2	75	68	63	56	49	42	35
MFP3	MATHEMATICS UNIT MFP3	75	70	65	57	49	41	33
MFP4	MATHEMATICS UNIT MFP4	75	61	55	48	41	34	28
MM1A	MATHEMATICS UNIT MM1A	100	-	79	69	59	49	39
MM1A/W	*MATHEMATICS UNIT MM1A - WRITTEN*	*75*		*59*				*29*
MM1A/C	*MATHEMATICS UNIT MM1A - COURSEWORK*	*25*		*20*				*10*
MM1B	MATHEMATICS UNIT MM1B	75	-	57	49	41	33	26
MM2B	MATHEMATICS UNIT MM2B	75	69	63	55	48	41	34
MM03	MATHEMATICS UNIT MM03	75	62	55	48	41	34	27
MM04	MATHEMATICS UNIT MM04	75	67	60	52	44	37	30
MM05	MATHEMATICS UNIT MM05	75	67	60	52	44	37	30
MPC1	MATHEMATICS UNIT MPC1	75	-	58	51	44	37	30
MPC2	MATHEMATICS UNIT MPC2	75	-	51	46	41	36	31
MPC3	MATHEMATICS UNIT MPC3	75	67	61	55	49	43	38
MPC4	MATHEMATICS UNIT MPC4	75	59	53	47	41	36	31
MS1A	MATHEMATICS UNIT MS1A	100	-	76	67	59	51	43