General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MM03

Unit Mechanics 3

Wednesday 22 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 A ball of mass 0.2 kg is hit directly by a bat. Just before the impact, the ball is travelling horizontally with speed $18\,\mathrm{m\,s^{-1}}$. Just after the impact, the ball is travelling horizontally with speed $32\,\mathrm{m\,s^{-1}}$ in the opposite direction.

(a) Find the magnitude of the impulse exerted on the ball. *(2 marks)*

(b) At time t seconds after the ball first comes into contact with the bat, the force exerted by the bat on the ball is $k(0.9t - 10t^2)$ newtons, where k is a constant and $0 \leqslant t \leqslant 0.09$. The bat stays in contact with the ball for 0.09 seconds.

Find the value of k. *(4 marks)*

---

2 The time, t, for a single vibration of a piece of taut string is believed to depend on

the length of the taut string, l,
the tension in the string, F,
the mass per unit length of the string, q, and
a dimensionless constant, k,

such that

$$t = kl^{\alpha}F^{\beta}q^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, find the values of α, β and γ. *(5 marks)*

---

3 **(In this question, use $g = 10\,\mathrm{m\,s^{-2}}$.)**

A golf ball is hit from a point O on a horizontal golf course with a velocity of $40\,\mathrm{m\,s^{-1}}$ at an angle of elevation θ. The golf ball travels in a vertical plane through O. During its flight, the horizontal and upward vertical distances of the golf ball from O are x and y metres respectively.

(a) Show that the equation of the trajectory of the golf ball during its flight is given by

$$x^2\tan^2\theta - 320x\tan\theta + (x^2 + 320y) = 0$$ *(6 marks)*

(b) (i) The golf ball hits the top of a tree, which has a vertical height of 8 m and is at a horizontal distance of 150 m from O.

Find the two possible values of θ. *(5 marks)*

(ii) Which value of θ gives the shortest possible time for the golf ball to travel from O to the top of the tree? Give a reason for your choice of θ. *(2 marks)*

4 The unit vectors **i**, **j** and **k** are directed due east, due north and vertically upwards respectively.

A helicopter, A, is travelling in the direction of the vector $-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ with constant speed $140\,\text{km h}^{-1}$. Another helicopter, B, is travelling in the direction of the vector $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ with constant speed $60\,\text{km h}^{-1}$.

(a) Find the velocity of A relative to B. *(5 marks)*

(b) Initially, the position vectors of A and B are $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})\,\text{km}$ and $(-3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})\,\text{km}$ respectively, relative to a fixed origin.

Write down the position vector of A relative to B, t hours after they leave their initial positions. *(2 marks)*

(c) Find the distance between A and B when they are closest together. *(8 marks)*

---

5 A ball is dropped from a height of 2.5 m above a horizontal floor. The ball bounces repeatedly on the floor.

(a) Find the speed of the ball when it first hits the floor. *(2 marks)*

(b) The coefficient of restitution between the ball and the floor is e.

(i) Show that the time taken between the first contact of the ball with the floor and the second contact of the ball with the floor is $\frac{10e}{7}$ seconds. *(3 marks)*

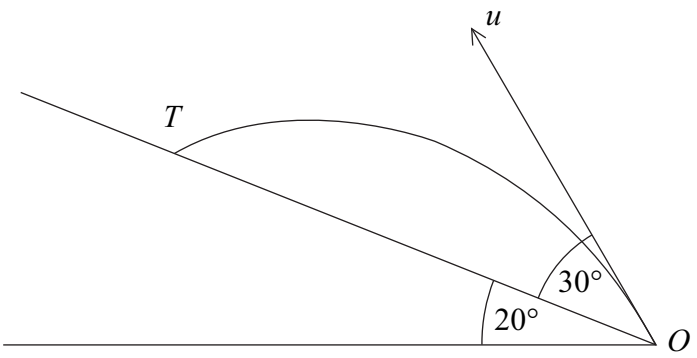
(ii) Find, in terms of e, the time taken between the second contact and the third contact of the ball with the floor. *(1 mark)*

(c) Find, in terms of e, the total vertical distance travelled by the ball from when it is dropped until its third contact with the floor. *(5 marks)*

(d) State a modelling assumption for answering this question, other than the ball being a particle. *(1 mark)*

6 A projectile is fired from a point O on a plane which is inclined at an angle of 20° to the horizontal. The projectile is fired up the plane with velocity $u\,\mathrm{m\,s^{-1}}$ at an angle of 30° to the inclined plane. The projectile travels in a vertical plane containing a line of greatest slope of the inclined plane.

The projectile hits a target T on the inclined plane.



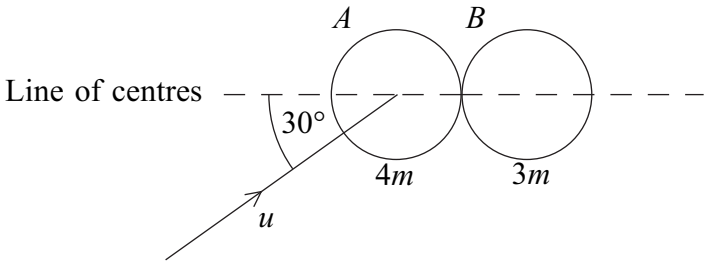
(a) Given that $OT = 200\,\mathrm{m}$, determine the value of u. *(7 marks)*

(b) Find the greatest perpendicular distance of the projectile from the inclined plane. *(4 marks)*

7 Two smooth spheres, A and B, have equal radii and masses $4m$ and $3m$ respectively. The sphere A is moving on a smooth horizontal surface and collides with the sphere B, which is stationary on the same surface.

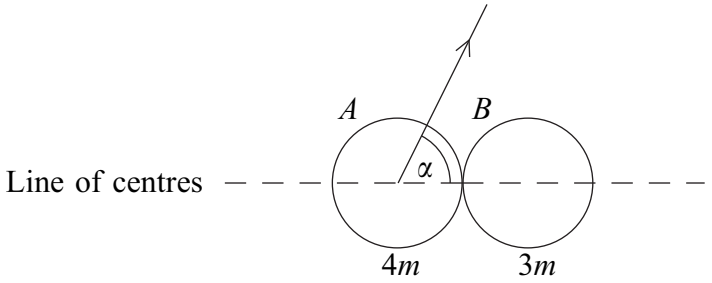
Just before the collision, A is moving with speed u at an angle of 30° to the line of centres, as shown in the diagram below.

Before collision



Immediately after the collision, the direction of motion of A makes an angle α with the line of centres, as shown in the diagram below.

After collision



The coefficient of restitution between the spheres is $\frac{5}{9}$.

(a) Find the value of α. *(10 marks)*

(b) Find, in terms of m and u, the magnitude of the impulse exerted on B during the collision. *(3 marks)*

END OF QUESTIONS

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1 (a)	I = 0.2(32) + 0.2(18) I = 10 Ns	M1 A1	2	Condone +10
(b)	$\int_0^{0.09} k(0.9t-10t^2)\mathrm{d}t = 10$	M1		Condone limits
	$k\left[0.45t^2 - \frac{10}{3}t^3\right]_0^{0.09} = 10$	A1F		Condone limits
	$1.215\times10^{-3}k = 10$	m1		For substituting 0.09
	$k = 8230$	A1F	4	
			6	
2	$T^1 = L^{\alpha}(MLT^{-2})^{\beta}(ML^{-1})^{\gamma}$	M1 A1		
	$\alpha+\beta-\gamma=0$ $\beta+\gamma=0$ $-2\beta=1$	m1		Getting three equations
	$\beta=-\frac{1}{2}$	m1		
	$\gamma=\frac{1}{2}$ $\alpha=1$	A1F	5	Solution
			5	

Q	Solution	Marks	Total	Comments
3 (a)	$x = 40\cos\theta.t$	M1		
	$y = -\frac{1}{2}(10)t^2 + 40\sin\theta.t$	M1 A1		
	$y = -\frac{1}{2}(10)(\frac{x}{40\cos\theta})^2 + 40\sin\theta.(\frac{x}{40\cos\theta})$	m1		Dependent on both M1s
	$y = -\frac{x^2}{320\cos^2\theta} + x\tan\theta$			
	$320y = -x^2(1+\tan^2\theta) + 320x\tan\theta$	m1		
	$x^2\tan^2\theta - 320x\tan\theta + (x^2 + 320y) = 0$	A1	6	Answer Given (Condone missing brackets)
(b)(i)	$150^2\tan^2\theta - 320(150)\tan\theta + (150^2 + 320\times 8) = 0$	M1		
	$1125\tan^2\theta - 2400\tan\theta + 1253 = 0$	A1		Correct quadratic
	$\tan\theta = \frac{2400 \pm \sqrt{2400^2 - 4(1125)(1253)}}{2(1125)}$	m1		
	$\tan\theta = 1.22\ ,\ 0.912$	A1F		PI
	$\theta = 50.7^\circ\ ,\ 42.4^\circ$	A1F	5	
(b)(ii)	$\theta = 42.4^\circ$	B1F		For the smaller angle
	$t = \frac{150}{40\cos\theta}$ and $\cos 42.4 > \cos 50.7$	E1	2	OE
			13	

Q	Solution	Marks	Total	Comments
4 (a)	$u_A = \dfrac{(-2\mathbf{i}+3\mathbf{j}+6\mathbf{k})140}{\sqrt{(2)^2+(3)^2+(6)^2}} = -40\mathbf{i}+60\mathbf{j}+120\mathbf{k}$	M1 A1		Simplification not needed
	$u_B = \dfrac{(2\mathbf{i}-\mathbf{j}+2\mathbf{k})60}{\sqrt{(2)^2+(1)^2+(2)^2}} = 40\mathbf{i}-20\mathbf{j}+40\mathbf{k}$	A1		Simplification not needed
	${}_Au_B = (-40\mathbf{i}+60\mathbf{j}+120\mathbf{k})-(40\mathbf{i}-20\mathbf{j}+40\mathbf{k})$ $= -80\mathbf{i}+80\mathbf{j}+80\mathbf{k}$	M1 A1F	5	Subtracting *B* from *A*
(b)	${}_Ar_B = (4\mathbf{i}-2\mathbf{j}+3\mathbf{k})-(-3\mathbf{i}+6\mathbf{j}+3\mathbf{k})+ t(-80\mathbf{i}+80\mathbf{j}+80k)$ **or** $(7\mathbf{i}-8\mathbf{j})+t(-80\mathbf{i}+80\mathbf{j}+80\mathbf{k})$	M1 A1F	2	A difference of initial p.v. $+t\times{}_Au_B$
(c)	${}_Ar_B = (7-80t)\mathbf{i}+(-8+80t)\mathbf{j}+(80t)\mathbf{k}$	B1F		
	$s^2 = (7-80t)^2+(-8+80t)^2+(80t)^2$	B1F		
	$2s\dfrac{\mathrm{d}s}{\mathrm{d}t} = 2(7-80t)(-80)+2(-8+80t)(80)+ 2(80t)(80)=0$	M1 A1F		Differentiation
	$240t = 15$	m1		Solving
	$t = 0.0625$ or $\dfrac{1}{16}$	A1F		
	$s^2 = (7-80\times0.0625)^2+(-8+80\times0.0625)^2+ (80\times0.0625)^2$	M1		
	$s = 6.16$ km or $\sqrt{38}$ km	A1F	8	
			15	
	<u>Alternative</u> (Not in the specification) *A* and *B* are closest $\Rightarrow$ ${}_Ar_B \cdot {}_Av_B = 0$ $[(7-80t)\mathbf{i}+(-8+80t)\mathbf{j}+(80t)\mathbf{k}]$ **.** $[-80\mathbf{i}+80\mathbf{j}+80\mathbf{k}]=0$ $-80(7-80t)+80(-8+80t)+80(80t)=0$ $240t=15$ $t=0.0625$	B1 M1 A1 A1 M1 A1		

Q	Solution	Marks	Total	Comments
5(a)	$v^2 = u^2 + 2as$			
	$v^2 = 0^2 + 2(9.8)(2.5)$	M1		
	$v = 7$	A1	2	
(b)(i)	$\frac{w}{7} = e$	M1		
	$w = 7e$			
	$0 = 7et - \frac{9.8}{2}t^2$ **or** ($0 = 7e - 9.8t$)	M1		
	$t = \frac{10e}{7}$ ($t = 2 \times \frac{7e}{9.8}$)	A1	3	Answer given
(ii)	$w' = 7e^2$			
	$0 = 7e^2 t' - \frac{9.8}{2}t'^2$			
	$t' = \frac{10e^2}{7}$	B1	1	OE
(c)	$0^2 = (7e)^2 + 2(-9.8)h_2$	M1		Or for correct method to find h_4
	$h_2 = 2.5e^2$	A1		
	$h_3 = 2.5e^2$			
	$0^2 = (7e^2)^2 + 2(-9.8)h_4$			
	$h_4 = 2.5e^4$	A1		
	$h_5 = 2.5e^4$			
	Total distance $= 2.5 + 2(2.5e^2) + 2(2.5e^4)$	m1		
	$= 2.5 + 5e^2 + 5e^4$	A1	5	
	Alternative (not in the specification)			
	K.E. after each bounce $= e^2 \times$ K.E. before the bounce			
	P.E. at max. height after each bounce = $e^2 \times$ P.E. at max. height before the bounce	(M1)		
	Height after first bounce $= 2.5e^2$	(A1)		
	Height after second bounce $= 2.5e^4$	(A1)		
	Total $= 2.5 + 2(2.5e^2 + 2(2.5e^4)$	(m1)		
	$= 2.5 + 5e^2 + 5e^4$	(A1)		
(d)	Motion in vertical line, No air resistance, No energy loss, Instantaneous bounce	B1	1	
			12	

Q	Solution	Marks	Total	Comments
6 (a)	Perpendicular to the plane:			
	$y = -\frac{1}{2}gt^2\cos 20 + ut\sin 30$	M1		
	$0 = -4.9t^2\cos 20 + ut\sin 30$	M1		
	$t = 0.108589568u$ or $\frac{2u\sin 30}{g\cos 20}$	A1		
	Parallel to the plane:			
	$x = -\frac{1}{2}gt^2\sin 20 + ut\cos 30$	M1		
	$200 = -4.9(0.108589568u)^2\sin 20 + u(0.108589568u)\cos 30$	m1		
	$u^2 = 2693$	A1F		
	$u = 51.9$ or 51.894	A1F	7	Do not accept $\sqrt{2693}$
(b)	$\dot{y} = -gt\cos 20 + u\sin 30 = 0$	M1		
	$t = 2.817899$ or 2.817580214 or $\frac{51.9\sin 30}{g\cos 20}$	A1F		Accept 3 significant fig.
	The greatest $\perp$ distance = $-\frac{1}{2}9.8(2.817899)^2\cos 20 + 51.9(2.817899)\sin 30$ or $-\frac{1}{2}9.8\left(\frac{51.894\sin 30}{9.8\cos 20}\right)^2\cos 20 + 51.9\left(\frac{51.894\sin 30}{9.8\cos 20}\right)\sin 30$	m1		
	$= 36.5622$ m or 36.5538 $= 36.6$ 3sf	A1F	4	
			11	
6 (a)	**Alternative:**			
	$x = 200\cos 20$	B1		
	$y = 200\sin 30$	B1		
	$200\cos 20 = u\cos 50t$	M1		
	$t = \frac{292.4}{u}$	A1		
	$200\sin 30 = \frac{1}{2}(-9.8)\left(\frac{292.4}{u}\right)^2 + u\sin 50\left(\frac{292.4}{u}\right)$	M1		
	$u^2 = 2693$	A1		
	$u = 51.9$	A1		
(b)	**Alternative:**			
	$0 = (u\sin 30)^2 - 2g\cos 20.s$	M1		
	$s = \frac{(51.9\sin 30)^2}{2(9.8)\cos 20}$	m1A1		
	$s = 36.6$	A1		

Q	Solution	Marks	Total	Comments
7 (a)	Momentum of A is unchanged $\perp$ to the line of centres			
	$4mu\sin 30 = 4mv_A \sin\alpha$	M1		
	$v_A = \dfrac{u}{2\sin\alpha}$ .....(1)	A1		
	C.L.M.:			
	$4mu\cos 30 = 4mv_A\cos\alpha + 3mv_B$	M1A1		
	$2\sqrt{3}u = 4v_A\cos\alpha + 3v_B$ .....(2)	A1F		OE
	Restitution along the line of centres:			
	$\dfrac{v_B - v_A\cos\alpha}{u\cos 30} = \dfrac{5}{9}$	M1A1		
	$v_B = v_A\cos\alpha + \dfrac{5\sqrt{3}u}{18}$ .....(3)	B1		Or equivalent, could be in part (b)
	$2\sqrt{3}u = 4\dfrac{u}{2\sin\alpha}\cos\alpha + 3\dfrac{u}{2\sin\alpha}\cos\alpha + \dfrac{15\sqrt{3}u}{18}$	m1		Solving (1), (2) and (3) Dependent on three M1s
	$\dfrac{7\sqrt{3}}{6} = \dfrac{7}{2\tan\alpha}$			
	$\tan\alpha = \sqrt{3}$			
	$\alpha = 60^\circ$ or $\dfrac{\pi}{3}$	A1F	10	
(b)	Impulse on B = Change in momentum of B along the line of centres			
	$v_B = \dfrac{u}{2\sin 60}\cos 60 + \dfrac{5\sqrt{3}u}{18}$			
	$v_B = \dfrac{u}{2\sqrt{3}} + \dfrac{5\sqrt{3}u}{18}$ $\left(= \dfrac{4\sqrt{3}}{9}\right)$	M1		
	$\mathrm{I} = 3m\left(\dfrac{u}{2\sqrt{3}} + \dfrac{5\sqrt{3}u}{18}\right) - 3m(0)$	M1		
	$\mathrm{I} = \dfrac{4mu}{\sqrt{3}}$ or $2.31mu$	A1F	3	
			13	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2011 exams

A-level

Code	Title	Max. Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points					
			A*	A	B	C	D	E
MS1B	GCE MATHEMATICS UNIT S1B	75	-	59	52	46	40	34
MD02	GCE MATHEMATICS UNIT D02	75	69	64	56	49	42	35
MFP2	GCE MATHEMATICS UNIT FP2	75	62	55	48	41	34	28
MM2B	GCE MATHEMATICS UNIT M2B	75	68	62	55	48	41	34
MPC2	GCE MATHEMATICS UNIT PC2	75	-	54	47	41	35	29
MS2B	GCE MATHEMATICS UNIT S2B	75	62	54	46	38	30	23
XMCA2	GCE MATHEMATICS UNIT XMCA2	125	88	76	66	57	48	39
MFP3	GCE MATHEMATICS UNIT FP3	75	69	64	55	46	38	30
MM03	GCE MATHEMATICS UNIT M03	75	67	59	51	43	36	29
MPC3	GCE MATHEMATICS UNIT PC3	75	68	59	52	46	40	34
MS03	GCE MATHEMATICS UNIT S03	75	68	62	54	46	39	32
MFP4	GCE MATHEMATICS UNIT FP4	75	68	61	53	46	39	32
MM04	GCE MATHEMATICS UNIT M04	75	63	57	51	45	39	33
MPC4	GCE MATHEMATICS UNIT PC4	75	58	51	46	41	36	31
MS04	GCE MATHEMATICS UNIT S04	75	67	60	52	44	37	30
MM05	GCE MATHEMATICS UNIT M05	75	62	55	48	41	34	28
MEST1	GCE MEDIA STUDIES UNIT 1	80	-	55	47	40	33	26
MEST2	GCE MEDIA STUDIES UNIT 2	80	-	63	54	45	36	28
MEST3	GCE MEDIA STUDIES UNIT 3	80	70	61	50	39	28	18
MEST4	GCE MEDIA STUDIES UNIT 4	80	74	68	56	45	34	23
MHEB1	GCE MODERN HEBREW UNIT 1	100	-	61	54	47	40	34
MHEB2	GCE MODERN HEBREW UNIT 2	100	80	71	62	54	46	38
MUSC1	GCE MUSIC UNIT 1	80	-	57	51	45	39	34
MUS2A	GCE MUSIC UNIT 2A	60	-	45	40	35	30	26
MUS2B	GCE MUSIC UNIT 2B	60	-	49	43	37	32	27
MUS2C	GCE MUSIC UNIT 2C	60	-	49	44	39	34	29