General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MM03

Unit Mechanics 3

Tuesday 22 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 A tank containing a liquid has a small hole in the bottom through which the liquid escapes. The speed, $u\,\mathrm{m\,s^{-1}}$, at which the liquid escapes is given by

$$u = CV\rho g$$

where $V\ \mathrm{m}^3$ is the volume of the liquid in the tank, $\rho\,\mathrm{kg\,m^{-3}}$ is the density of the liquid, g is the acceleration due to gravity and C is a constant.

By using dimensional analysis, find the dimensions of C. *(5 marks)*

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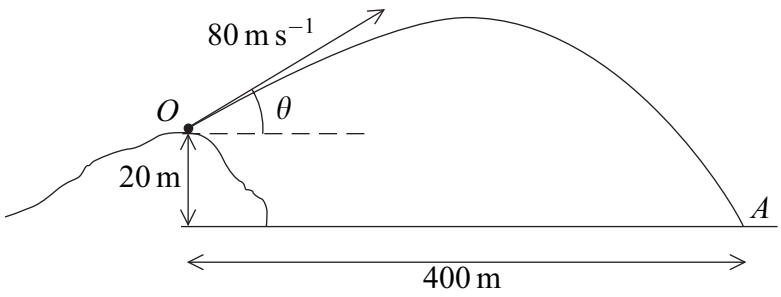
2 A projectile is fired from a point O on top of a hill with initial velocity $80\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal and moves in a vertical plane. The horizontal and upward vertical distances of the projectile from O are x metres and y metres respectively.

(a) (i) Show that, during the flight, the equation of the trajectory of the projectile is given by

$$y = x\tan\theta - \frac{gx^2}{12\,800}(1 + \tan^2\theta)$$ *(5 marks)*

(ii) The projectile hits a target A, which is 20 m vertically below O and 400 m horizontally from O.



Show that

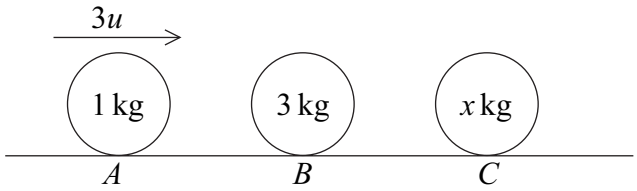
$$49\tan^2\theta - 160\tan\theta + 41 = 0$$ *(2 marks)*

(b) (i) Find the two possible values of θ. Give your answers to the nearest 0.1°. *(3 marks)*

(ii) Hence find the shortest possible time of the flight of the projectile from O to A. *(2 marks)*

(c) State a necessary modelling assumption for answering part **(a)(i)**. *(1 mark)*

3 Three smooth spheres, A, B and C, of equal radii have masses 1 kg, 3 kg and x kg respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with B between A and C. The sphere A is projected with speed $3u$ directly towards B and collides with it.



The coefficient of restitution between each pair of spheres is $\frac{1}{3}$.

(a) Show that A is brought to rest by the impact and find the speed of B immediately after the collision in terms of u. *(6 marks)*

(b) Subsequently, B collides with C.

Show that the speed of C immediately after the collision is $\dfrac{4u}{3+x}$.

Find the speed of B immediately after the collision in terms of u and x. *(6 marks)*

(c) Show that B will collide with A again if $x > 9$. *(2 marks)*

(d) Given that $x = 5$, find the magnitude of the impulse exerted on C by B in terms of u. *(2 marks)*

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4 The unit vectors **i**, **j** and **k** are directed east, north and vertically upwards respectively.

At time $t = 0$, the position vectors of two small aeroplanes, A and B, relative to a fixed origin O are $(-60\mathbf{i} + 30\mathbf{k})$ km and $(-40\mathbf{i} + 10\mathbf{j} - 10\mathbf{k})$ km respectively.

The aeroplane A is flying with constant velocity $(250\mathbf{i} + 50\mathbf{j} - 100\mathbf{k})\,\text{km h}^{-1}$ and the aeroplane B is flying with constant velocity $(200\mathbf{i} + 25\mathbf{j} + 50\mathbf{k})\,\text{km h}^{-1}$.

(a) Write down the position vectors of A and B at time t hours. *(3 marks)*

(b) Show that the position vector of A relative to B at time t hours is $((-20 + 50t)\mathbf{i} + (-10 + 25t)\mathbf{j} + (40 - 150t)\mathbf{k})$ km. *(2 marks)*

(c) Show that A and B do not collide. *(4 marks)*

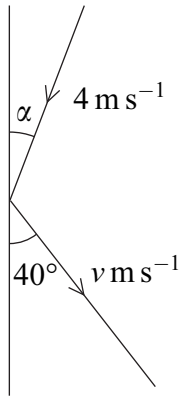
(d) Find the value of t when A and B are closest together. *(6 marks)*

5 A smooth sphere is moving on a smooth horizontal surface when it strikes a smooth vertical wall and rebounds.

Immediately before the impact, the sphere is moving with speed $4\,\text{m s}^{-1}$ and the angle between the sphere's direction of motion and the wall is α.

Immediately after the impact, the sphere is moving with speed $v\,\text{m s}^{-1}$ and the angle between the sphere's direction of motion and the wall is $40°$.

The coefficient of restitution between the sphere and the wall is $\frac{2}{3}$.



(a) Show that $\tan\alpha = \frac{3}{2}\tan 40°$. *(3 marks)*

(b) Find the value of v. *(3 marks)*

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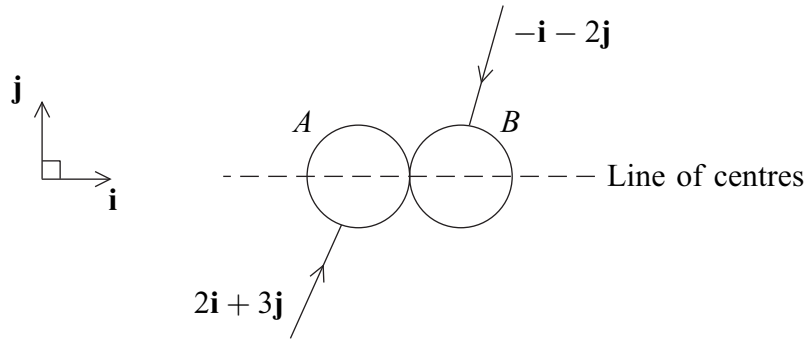
6 Two smooth spheres, A and B, have equal radii and masses 1 kg and 2 kg respectively.

The sphere A is moving with velocity $(2\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and the sphere B is moving with velocity $(-\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$ on the same smooth horizontal surface.

The spheres collide when their line of centres is parallel to the unit vector $\mathbf{i}$, as shown in the diagram.



(a) Briefly state why the components of the velocities of A and B parallel to the unit vector **j** are not changed by the collision. *(1 mark)*

(b) The coefficient of restitution between the spheres is 0.5 .

Find the velocities of A and B immediately after the collision. *(6 marks)*

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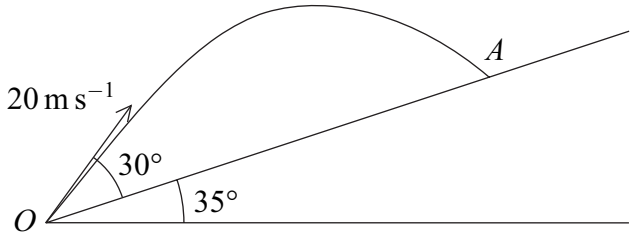
7 A ball is projected from a point O on a smooth plane which is inclined at an angle of 35° above the horizontal. The ball is projected with velocity $20\,\text{m s}^{-1}$ at an angle of 30° above the plane, as shown in the diagram. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane at the point A.



(a) Find the components of the velocity of the ball, parallel and perpendicular to the plane, as it strikes the inclined plane at A. *(7 marks)*

(b) On striking the plane at A, the ball rebounds. The coefficient of restitution between the plane and the ball is $\frac{4}{5}$.

Show that the ball next strikes the plane at a point lower down than A. *(6 marks)*

END OF QUESTIONS

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1	LT^{-1}	B1		For dimensions of u
	$LT^{-1} = M^{\alpha}L^{\beta}T^{\gamma} \times L^3 \times ML^{-3} \times LT^{-2}$	M1 A1		M1 for equation with five components
	$1 = \beta + 1$			
	$-1 = \gamma - 2$			
	$0 = \alpha + 1$	m1		Forming and solving equations (PI)
	$\beta = 0,\ \alpha = -1,\ \gamma = 1$			
	The dimensions of C are $M^{-1}T$	A1F	5	
	Alternative :			
	LT^{-1}	(B1)		For dimensions of u
	$LT^{-1} = C \times L^3 \times ML^{-3} \times LT^{-2}$	(M1A1)		M1 for equation with five components
	$LT^{-1} = C \times L\, MT^{-2}$	(m1)		
	The dimensions of C are $M^{-1}T$	(A1F)	5	
	Total		**5**	
2(a)(i)	$x = 80\cos\theta .\ t$	B1		
	$t = \dfrac{x}{80\cos\theta}$	B1		
	$y = 80\sin\theta .t - \dfrac{1}{2}gt^2$	B1		
	$y = 80\sin\theta \dfrac{x}{80\cos\theta} - \dfrac{1}{2}g\left(\dfrac{x}{80\cos\theta}\right)^2$	M1		
	$y = x\tan\theta - \dfrac{gx^2}{12800}(1+\tan^2\theta)$	A1	5	Answer given
(ii)	$-20 = 400\tan\theta - \dfrac{9.8\times 400^2}{12800}(1+\tan^2\theta)$	M1		Condone + 20
	$122.5\tan^2\theta - 400\tan\theta + 102.5 = 0$			
	$49\tan^2\theta - 160\tan\theta + 41 = 0$	A1	2	Answer given
(b)(i)	$tan\theta = \dfrac{160 \pm \sqrt{25600 - 4(49)(41)}}{2\times 49}$	M1		
	$= 2.9850,\ 0.2803$	A1		PI
	$\theta = 71.5^{\circ},\ 15.7^{\circ}$	A1F	3	
(ii)	For the shortest time			
	$400 = 80\cos 15.7^{\circ}.t$	M1		
	$t = 5.19$	AIF	2	
(c)	• The projectile is a particle • The air resistance is negligible	E1	1	
	Total		**13**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	C.L.M. $(1)3u = (1)v_A + (3)v_B$ Restitution : $\frac{1}{3}\times 3u = v_B - v_A$ $v_B = u$ $v_A = 0$	M1 A1 M1 A1 m1 A1	6	M1 for three non-zero terms Accept $v_A - v_B$ Solution A1 for both answers
(b)	C.L.M. $3u = 3w_B + xw_c$ Restitution : $\frac{1}{3}u = w_C - w_B$ $w_C = \frac{4u}{3+x}$ $w_B = \frac{u(9-x)}{3(3+x)}$ OE	 M1 A1 M1 A1 m1 A1	6	 Solution attempt, dep. on both M1s AG A1 for both
(c)	For further collision $\frac{u(9-x)}{3(3+x)} < 0$ $9u - xu < 0$ $x > 9$	M1 A1	2	 AG
(d)	$I = 5(\frac{4u}{3+5})$ $I = \frac{5u}{2}$ **Alternative:** $I = 3u - 3\times\frac{u(9-5)}{3(3+5)}$ $I = \frac{5u}{2}$	M1 A1 (M1) (A1F)	2	 Accept $-\frac{5u}{2}$ Follow through on their w_B
	Total		**16**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$r_A = (-60\mathbf{i}+30\mathbf{k})+(250\mathbf{i}+50\mathbf{j}-100\mathbf{k})t$	M1		For correct form
	$r_B = (-40\mathbf{i}+10\mathbf{j}-10\mathbf{k})+(200\mathbf{i}+25\mathbf{j}+50\mathbf{k})t$	A1,2	3	A1 for each
(b)	${}_B r_A = [\,(-60\mathbf{i}+30\mathbf{k})+(250\mathbf{i}+50\mathbf{j}-100\mathbf{k})t\,]-$ $[(-40\mathbf{i}+10\mathbf{j}-10\mathbf{k})+(200\mathbf{i}+25\mathbf{j}+50\mathbf{k})t]$	M1		Attempt at the difference using their answers
	${}_B r_A = (-20+50t)\mathbf{i}+(-10+25t)\mathbf{j}+(40-150t)\mathbf{k}$	A1	2	AG
(c)	For collision			
	$(-20+50t)\mathbf{i}+(-10+25t)\mathbf{j}+(40-150t)\mathbf{k}=0$	M1		
	$-20+50t=0 \quad\Rightarrow\quad t=\frac{2}{5}$			
	$-10+25t=0 \quad\Rightarrow\quad t=\frac{2}{5}$	m1 A1F		
	$40-150t=0 \quad\Rightarrow\quad t=\frac{4}{15}$			
	The relative position vector cannot be zero.			
	Therefore A and B do not collide	E1	4	
(d)	$S^2=(-20+50t)^2+(-10+25t)^2+(40-150t)^2$	M1A1		
	For minimum S			
	$\frac{\mathrm{d}S^2}{\mathrm{d}t}=100(-20+50t)+50(-10+25t)-$ $300(40-150t)=0$	M1 A1F		
	$51250t-14500=0$	m1		Solution
	$t=0.283$	A1F	6	
	Total		**15**	
	Alternative:			
	$\begin{pmatrix}-20+50t\\-10+25t\\40-150t\end{pmatrix}.\begin{pmatrix}50\\25\\-150\end{pmatrix}=0$	(M1) (A1)		
	$-1000+2500t-250+625t-6000+22500t=0$	(m1) (A1F)		
	$25625t-7250=0$	(A1F)		
	$t=0.283$	(A1F)		

MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Parallel to the wall			
	$4\cos\alpha = v\cos 40^\circ$	M1		Correct trigonometric ratios
	Perpendicular to the wall			
	$v\sin 40^\circ = \frac{2}{3}\times 4\sin\alpha$	M1		Correct trigonometric ratios
	$\tan\alpha = \frac{3}{2}\tan 40^\circ$	A1	3	AG
(b)	$\alpha = 51.5^\circ$	M1		
	$v = \frac{4\cos 51.5^\circ}{\cos 40^\circ}$	M1		
	$v = 3.25$ ms^{-1}	A1	3	OE
	Total		**6**	
6(a)	The spheres are smooth, no force acting in **j** direction	E1	1	Any valid reason
(b)	$v_A = a\mathbf{i} + b\mathbf{j}$			
	$v_B = c\mathbf{i} + d\mathbf{j}$			
	C.L.M. along **i**: $1(2) + 2(-1) = 1(a) + 2(c)$	M1A1		
	a+ 2c= 0			
	Restitution along **i** : $c - a = 0.5(2 - (-1))$	M1A1		
	$c - a = 1.5$			
	$c = 0.5$			
	$a = -1$			
	$v_A = -\mathbf{i} + 3\mathbf{j}$	A1F		
	$v_B = 0.5\mathbf{i} - 2\mathbf{j}$	A1F	6	
	Total		**7**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	On striking A:			
	$20\sin30^\circ.t - \frac{1}{2}(9.8)\cos35^\circ.t^2 = 0$	M1A1		
	$t = 2.49$	A1		AWRT OE
	Components of Velocity :			
	$u_x = 20\cos30^\circ - 9.8\sin35^\circ(2.49)$	M1		
	$u_x = 3.32$	A1F		AWRT
	$u_y = 20\sin30^\circ - 9.8\cos35^\circ(2.49)$	M1		
	$u_y = -10$ (or -9.99)	A1F	7	
(b)	On Rebounding			
	$v_x = 3.32$			
	$v_y = \frac{4}{5}\times10$	B1F		For $\frac{4}{5}\times$ their u_y
	$v_y = 8$ (or 7.99)			
	The rebound angle $= \tan^{-1}\frac{8}{3.32}$	M1		
	$= 67.5^\circ$ (or 67.4°)	A1F		
	$35^\circ + 67.5^\circ = 102.5^\circ$	M1		
		A1F		
	$102.5^\circ > 90^\circ$, therefore the second strike will be at a point lower down than A.	E1	6	Dependent on the two M1s
	Alternative:			
	$\frac{4}{5}\times10 = 8$	(B1)		Condone negative sign
	$0 = 8t - \frac{1}{2}g\cos35.t^2$	(M1)		
	$t = 1.9931$	(A1)		OE
	$x = 3.32t - \frac{1}{2}g\sin35.t^2$	(M1)		
	$x = -4.55$ or -4.56	(A1)		
	The second strike will be at a point lower down than A.	(E1)		
	Total		**13**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2010 exams

A-level

Code	Title	Max. Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points					
			A*	A	B	C	D	E
MS1B	GCE MATHEMATICS UNIT S1B	75	-	59	52	45	39	33
MD02	GCE MATHEMATICS UNIT D02	75	69	64	56	48	40	32
MFP2	GCE MATHEMATICS UNIT FP2	75	70	65	57	49	41	33
MM2B	GCE MATHEMATICS UNIT M2B	75	63	55	47	39	31	23
MPC2	GCE MATHEMATICS UNIT PC2	75	-	61	54	47	41	35
MS2B	GCE MATHEMATICS UNIT S2B	75	65	58	51	44	37	31
XMCA2	GCE MATHEMATICS UNIT XMCA2	125	102	92	80	68	57	46
MFP3	GCE MATHEMATICS UNIT FP3	75	69	64	56	49	42	35
MM03	GCE MATHEMATICS UNIT M03	75	71	68	61	54	48	42
MPC3	GCE MATHEMATICS UNIT PC3	75	67	62	54	46	39	32
MS03	GCE MATHEMATICS UNIT S03	75	60	54	48	42	36	31
MFP4	GCE MATHEMATICS UNIT FP4	75	67	60	52	44	36	29
MM04	GCE MATHEMATICS UNIT M04	75	68	61	53	45	37	29
MPC4	GCE MATHEMATICS UNIT PC4	75	67	62	56	50	44	39
MS04	GCE MATHEMATICS UNIT S04	75	68	61	53	45	38	31
MM05	GCE MATHEMATICS UNIT M05	75	63	56	49	42	35	29
MEST1	GCE MEDIA STUDIES UNIT 1	80	-	52	45	38	31	25
MEST2	GCE MEDIA STUDIES UNIT 2	80	-	63	54	45	36	28
MEST3	GCE MEDIA STUDIES UNIT 3	80	69	59	48	38	28	18
MEST4	GCE MEDIA STUDIES UNIT 4	80	72	65	53	42	31	20
MHEB1	GCE MODERN HEBREW UNIT 1	100	-	66	59	52	45	38
MHEB2	GCE MODERN HEBREW UNIT 2	100	80	71	62	53	45	37
MUSC1	GCE MUSIC UNIT 1	80	-	57	51	45	39	34
MUS2A	GCE MUSIC UNIT 2A	60	-	46	41	36	31	26
MUS2B	GCE MUSIC UNIT 2B	60	-	48	42	36	31	26