General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS MM03
Unit Mechanics 3

Wednesday 17 June 2009 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A ball of mass m is travelling vertically downwards with speed u when it hits a horizontal floor. The ball bounces vertically upwards to a height h.

It is thought that h depends on m, u, the acceleration due to gravity g, and a dimensionless constant k, such that

$$h = km^{\alpha}u^{\beta}g^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, find the values of α, β and γ. *(5 marks)*

2 A particle is projected from a point O on a horizontal plane and has initial velocity components of $2\,\mathrm{m\,s^{-1}}$ and $10\,\mathrm{m\,s^{-1}}$ parallel to and perpendicular to the plane respectively. At time t seconds after projection, the horizontal and upward vertical distances of the particle from the point O are x metres and y metres respectively.

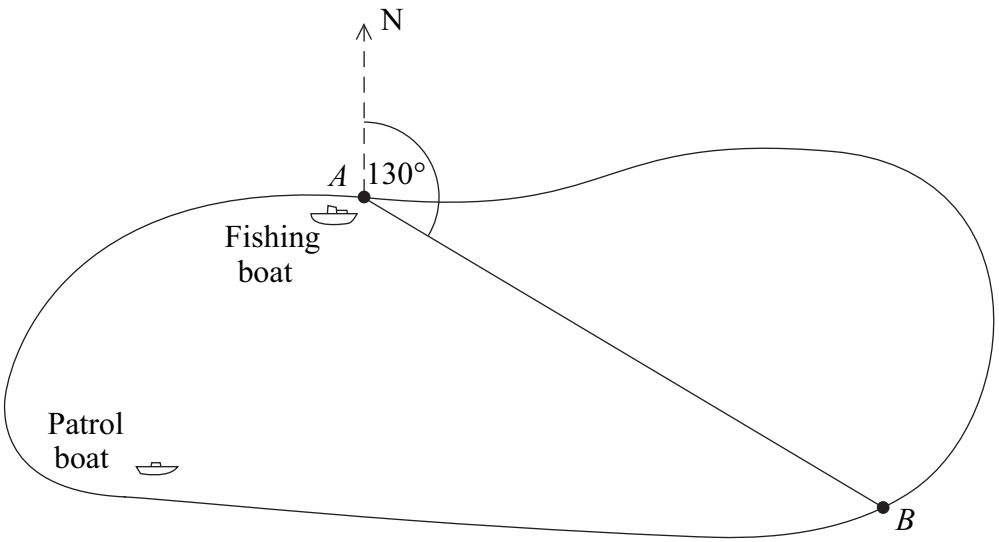
(a) Show that x and y satisfy the equation

$$y = -\frac{g}{8}x^2 + 5x$$ *(4 marks)*

(b) By using the equation in part (a), find the horizontal distance travelled by the particle whilst it is more than 1 metre above the plane. *(4 marks)*

(c) Hence find the time for which the particle is more than 1 metre above the plane. *(2 marks)*

3 A fishing boat is travelling between two ports, A and B, on the shore of a lake. The bearing of B from A is 130°. The fishing boat leaves A and travels directly towards B with speed $2\,\text{m s}^{-1}$. A patrol boat on the lake is travelling with speed $4\,\text{m s}^{-1}$ on a bearing of 040°.



(a) Find the velocity of the fishing boat relative to the patrol boat, giving your answer as a speed together with a bearing. *(5 marks)*

(b) When the patrol boat is 1500 m due west of the fishing boat, it changes direction in order to intercept the fishing boat in the shortest possible time.

(i) Find the bearing on which the patrol boat should travel in order to intercept the fishing boat. *(4 marks)*

(ii) Given that the patrol boat intercepts the fishing boat before it reaches B, find the time, in seconds, that it takes the patrol boat to intercept the fishing boat after changing direction. *(4 marks)*

(iii) State a modelling assumption necessary for answering this question, other than the boats being particles. *(1 mark)*

4 A particle of mass 0.5 kg is initially at rest. The particle then moves in a straight line under the action of a single force. This force acts in a constant direction and has magnitude $(t^3 + t)\,\text{N}$, where t is the time, in seconds, for which the force has been acting.

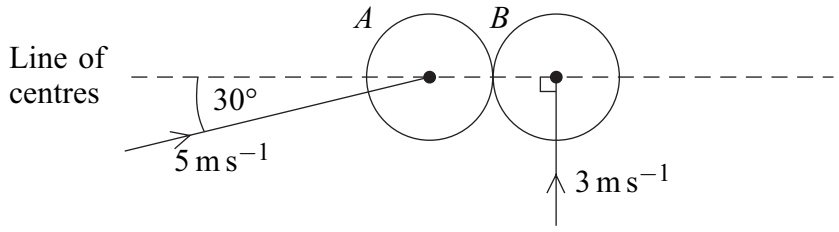
(a) Find the magnitude of the impulse exerted by the force on the particle between the times $t = 0$ and $t = 4$. *(3 marks)*

(b) Hence find the speed of the particle when $t = 4$. *(2 marks)*

(c) Find the time taken for the particle to reach a speed of $12\,\text{m s}^{-1}$. *(5 marks)*

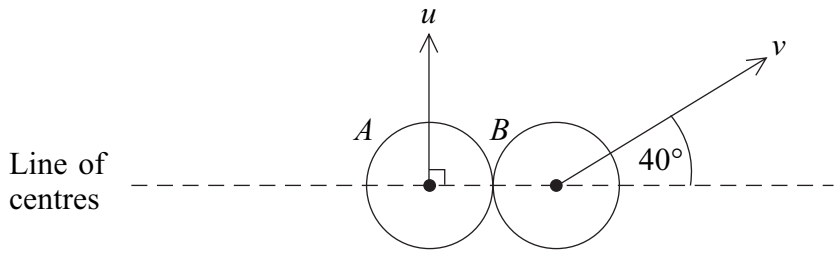
5 Two smooth spheres, A and B, of equal radii and different masses are moving on a smooth horizontal surface when they collide.

Just before the collision, A is moving with speed $5\,\mathrm{m\,s^{-1}}$ at an angle of 30° to the line of centres of the spheres, and B is moving with speed $3\,\mathrm{m\,s^{-1}}$ perpendicular to the line of centres, as shown in the diagram below.



Before collision

Immediately after the collision, A and B move with speeds u and v in directions which make angles of 90° and 40° respectively with the line of centres, as shown in the diagram below.



After collision

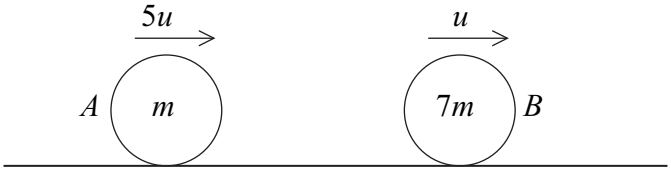
(a) Show that $v = 4.67\,\mathrm{m\,s^{-1}}$, correct to three significant figures. *(3 marks)*

(b) Find the coefficient of restitution between the spheres. *(3 marks)*

(c) Given that the mass of A is 0.5 kg, show that the magnitude of the impulse exerted on A during the collision is 2.17 N s, correct to three significant figures. *(3 marks)*

(d) Find the mass of B. *(3 marks)*

6 A smooth sphere A of mass m is moving with speed $5u$ in a straight line on a smooth horizontal table. The sphere A collides directly with a smooth sphere B of mass $7m$, having the same radius as A and moving with speed u in the same direction as A. The coefficient of restitution between A and B is e.



Before collision

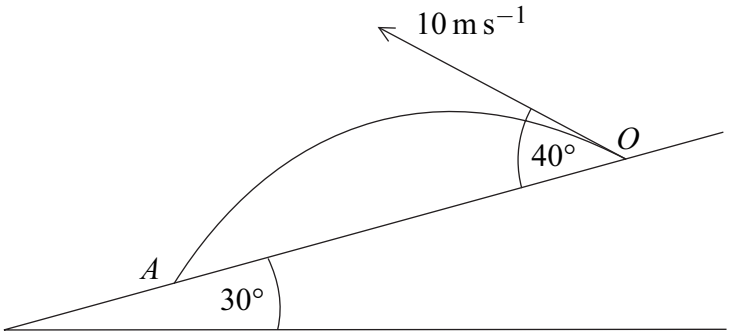
(a) Show that the speed of B after the collision is $\frac{u}{2}(e+3)$. *(5 marks)*

(b) Given that the direction of motion of A is reversed by the collision, show that $e > \frac{3}{7}$. *(4 marks)*

(c) Subsequently, B hits a wall fixed at right angles to the direction of motion of A and B. The coefficient of restitution between B and the wall is $\frac{1}{2}$. Given that after B rebounds from the wall both spheres move in the same direction and collide again, show also that $e < \frac{9}{13}$. *(4 marks)*

Turn over for the next question

7 A particle is projected from a point O on a smooth plane which is inclined at 30° to the horizontal. The particle is projected down the plane with velocity $10\,\mathrm{m\,s^{-1}}$ at an angle of 40° above the plane and first strikes it at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Show that the time taken by the particle to travel from O to A is

$$\frac{20\sin 40°}{g\cos 30°}$$

(3 marks)

(b) Find the components of the velocity of the particle parallel to and perpendicular to the slope as it hits the slope at A. *(4 marks)*

(c) The coefficient of restitution between the slope and the particle is 0.5. Find the speed of the particle as it rebounds from the slope. *(4 marks)*

END OF QUESTIONS

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1	$L = M^{\alpha}(LT^{-1})^{\beta}(LT^{-2})^{\gamma}$	M1A1		
	$\beta + \gamma = 1$ $-\beta - 2\gamma = 0$			
	$\alpha = 0$	m1		Getting three equations
	$\gamma = -1$	m1		Solution
	$\beta = 2$	A1F	5	
	Total		**5**	
2(a)	$x = 2t$	M1		
	$y = -\frac{1}{2}gt^2 + 10t$	M1		
	$t = \frac{x}{2}$			
	$y = -\frac{1}{2}g\left(\frac{x}{2}\right)^2 + 10\left(\frac{x}{2}\right)$	m1		
	$y = -\frac{g}{8}x^2 + 5x$	A1	4	AG
(b)	$1 = -\frac{g}{8}x^2 + 5x$	M1		
	$gx^2 - 40x + 8 = 0$			
	$x = \frac{40 \pm \sqrt{(-40)^2 - 4\times 8g}}{2g}$	M1		
	$x = 3.871,\ 0.211$	A1		A1 for both answers
	Distance $= 3.66\,\text{m}$	A1	4	
(c)	$t = \frac{3.66}{2}$	M1		
	$t = 1.83$ sec	A1	2	
	Total		**10**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	${}_{\mathrm{P}}v_{\mathrm{F}} = \sqrt{4^2 + 2^2}$	M1		
	$= 4.47 \text{ m s}^{-1}$ or $2\sqrt{5}\ ms^{-1}$ or $\sqrt{20}\ ms^{-1}$	A1		
	$\theta = \tan^{-1}\frac{2}{4}$	M1		
	$\theta = 26.6°$	A1F		
	Bearing $= 40° + 180° - 26.6°$			
	$= 193°$	A1F	5	
	Alternative:			
	Comp. due west $= 4\sin 40° - 2\sin 50° = 1.04 \text{ m s}^{-1}$ Comp. due south $= 2\cos 50° + 4\cos 40° = 4.35 \text{ m s}^{-1}$	(M1)		OE; resolving in two directions
	${}_{\mathrm{p}}v_{\mathrm{F}} = \sqrt{1.04^2 + 4.35^2} = 4.47 \text{ m s}^{-1}$	(A1)		
	$\theta = \tan^{-1}\frac{1.04}{4.35}$ or $\tan^{-1}\frac{4.35}{1.04}$	(M1)		
	$\theta = 13.4°$ or $76.6°$	(A1F)		
	Bearing $= 13.4° + 180°$ or $270° - 76.6°$			
	$= 193°$	(A1F)		
	Alternative:			
	Correct triangle	(M1)		Any orientation
	${}_{P}v_{F} = \sqrt{1.04^2 + 4.35^2} = 4.47 \text{ms}^{-1}$	(A1)		
	Rel. Vel. Triangle angle $26.6°$ or $63.4°$	(A1)		
	Bearing $= 40° + 180° - 26.6°$ or $63.4° + 40° + 90°$	(M1)		
	$= 193°$	(A1F)		
(b)(i)	$v_{\mathrm{F}} = v_{\mathrm{P}} + {}_{\mathrm{P}}v_{\mathrm{F}}$			
	$\frac{\sin\alpha}{2} = \frac{\sin 140°}{4}$	M1A1		
	$\alpha = 18.7°$	A1F		
	Bearing $= 90° + 18.7°$			
	$= 109°$	A1F	4	
	Alternative:			
	$2\sin 40° = 4\sin\alpha$	(M1)		
	$\alpha = \sin^{-1}\left(\frac{1}{2}\sin 40°\right)$	(A1)		
	$\alpha = 18.7°$	(A1F)		
	Bearing $= 109°$	(A1F)		

MM03 (cont)

Q	Solution	Marks	Total	Comments
3(b)(ii)	$\beta = 180° - (140° + 18.7°)$ $= 21.3°$	B1F		
	$\frac{{}_{\mathrm{P}}v_{\mathrm{F}}}{\sin 21.3°} = \frac{4}{\sin 140°}$	M1		
	${}_{\mathrm{P}}v_{\mathrm{F}} = 2.2568\,\mathrm{m\,s^{-1}}$	A1F		
	$t = \frac{1500}{2.2568}$ $= 665$ sec	A1F	4	
	Alternative:			o.e. resolving in two directions
	${}_{\mathrm{F}}v_{\mathrm{P}} = 4\cos 18.7 - 2\cos 40 = 2.2568$	(M1) (A2,1,0)		
	$t = \frac{1500}{2.2568} = 665$ sec	(A1F)		
(iii)	No cross wind, calm lake, instantaneous change of direction by the patrol boat	B1	1	Any sensible assumption
	Total		**14**	
4(a)	$\mathrm{I} = \int_0^4 (t^3 + t)\,\mathrm{d}t$	M1		
	$= \left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^4$	m1		
	$= 72\,\mathrm{N\,s}$	A1	3	
(b)	$72 = 0.5v - 0.5(0)$	M1		Condone $-5(0)$
	$v = 144$	A1F	2	
(c)	$\int_0^T (t^3 + t)\,\mathrm{d}t = 0.5(12) - 0.5(0)$	M1		Condone $-5(0)$
	$\left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^T = 6$			
	$T^4 + 2T^2 - 24 = 0$	A1		
	$T^2 = \frac{-2 \pm \sqrt{2^2 - 4(1)(-24)}}{2(1)}$	m1 A1F		
	or $(T^2 - 4)(T^2 + 6) = 0$			
	$T^2 = 4$ $T = 2$	A1F	5	
	Total		**10**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Momentum of B perpendicular to the line of centres is unchanged			
	$m_B v \sin 40° = 3m_B$	M1A1		
	$v = 4.667 \text{ ms}^{-1} = 4.67 \text{ ms}^{-1}$ (3sf)	A1	3	AG
(b)	$e = \dfrac{4.67\cos 40°}{5\cos 30°}$	M1A1		
	$e = 0.826$	A1F	3	
(c)	Impulse on A = change in momentum of A along the line of centres			
	$= 0.5 \times 5\cos 30° = 2.165$	M1A1		
	$= 2.17$ Ns	A1	3	AG
(d)	$2.165 = m_B(4.667)\cos 40°$	M1A1		
	$m_B = 0.6056 = 0.606$ kg (3sf)	A1F	3	Condone use of premature rounding giving 0.605kg or 0.607 kg
	Total		**12**	
6(a)	$5mu + 7mu = mv_A + 7mv_B$	M1A1		Allow consistent use of positive or negative sign for v_A.
	$12u = v_A + 7v_B$			
	$e = \dfrac{-v_A + v_B}{4u}$	M1		
	$-v_A + v_B = 4eu$			
	$8v_B = 12u + 4eu$	m1		
	$v_B = \dfrac{u}{2}(e+3)$	A1	5	AG
(b)	$v_A = \dfrac{u}{2}(e+3) - 4eu$	M1		
	$v_A = \dfrac{u}{2}(3-7e)$	A1F		
	$\dfrac{u}{2}(3-7e) < 0$	M1		
	$3 - 7e < 0$			
	$e > \dfrac{3}{7}$	A1	4	AG
(c)	$w_B = \dfrac{u}{4}(e+3)$	M1		
	$\dfrac{u}{2}(7e-3) < \dfrac{u}{4}(e+3)$	M1		
	$2(7e-3) < e+3$			
	$13e < 9$	m1		
	$e < \dfrac{9}{13}$	A1	4	AG
	Total		**13**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$y = 10t\sin 40° - \frac{1}{2}gt^2\cos 30°$	M1A1		
	$y = 0 \Rightarrow t = \frac{20\sin 40°}{g\cos 30°}$	A1	3	AG
(b)	$\dot{x} = 10\cos 40° + g\sin 30°\left(\frac{20\sin 40°}{g\cos 30°}\right)$	M1		
	$\dot{x} = 15.08\ \text{m s}^{-1}$	A1		
	$\dot{y} = 10\sin 40° - g\cos 30°\left(\frac{20\sin 40°}{g\cos 30°}\right)$	M1		
	$\dot{y} = -6.427\ \text{m s}^{-1}$	A1	4	Allow 3 sf
(c)	$\dot{x}$ will be unchanged	B1		
	Rebound $\dot{y} = 6.427 \times 0.5 = 3.214$	M1		Allow using 3 sf
	Rebound speed $= \sqrt{15.08^2 + 3.214^2}$	m1		
	$= 15.4\ \text{m s}^{-1}$	A1F	4	
	Total		**11**	
	TOTAL		**75**	

Scaled mark component grade boundaries - June 2009 exams

GCE (legacy)

Component Code	Component Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries A	B	C	D	E
ICT4	INFORMATION AND COMM TECH UNIT 4	90	67	61	55	49	44
ICT5	INFORMATION AND COMM TECH UNIT 5	90	66	60	55	50	45
ICT6	INFORMATION AND COMM TECH UNIT 6	90	59	51	43	36	29
LAW1	LAW UNIT 1	65	48	43	39	35	31
LAW2	LAW UNIT 2	65	42	38	34	30	26
LAW3	LAW UNIT 3	65	45	40	35	31	27
LAW4	LAW UNIT 4	85	58	53	48	44	40
LAW5	LAW UNIT 5	85	57	53	49	45	42
LAW6	LAW UNIT 6	70	48	43	38	34	30
MD01	MATHEMATICS UNIT MD01	75	60	53	46	39	32
MD02	MATHEMATICS UNIT MD02	75	63	55	48	41	34
MFP1	MATHEMATICS UNIT MFP1	75	61	53	45	37	29
MFP2	MATHEMATICS UNIT MFP2	75	59	51	43	36	29
MFP3	MATHEMATICS UNIT MFP3	75	62	54	46	38	30
MFP4	MATHEMATICS UNIT MFP4	75	60	52	44	36	28
MM03	MATHEMATICS UNIT MM03	75	59	51	43	35	28
MM04	MATHEMATICS UNIT MM04	75	55	47	39	31	23
MM05	MATHEMATICS UNIT MM05	75	59	51	43	35	28
MM1A/W	MATHEMATICS UNIT MM1A WRITTEN	75	58	51	44	37	30
MM1A/C	MATHEMATICS UNIT MM1A CWK	25	20	17	14	12	10
MM1B	MATHEMATICS UNIT MM1B	75	60	52	44	36	29
MM2B	MATHEMATICS UNIT MM2B	75	58	50	42	35	28
MPC1	MATHEMATICS UNIT MPC1	75	63	55	48	41	34
MPC2	MATHEMATICS UNIT MPC2	75	60	52	44	37	30
MPC3	MATHEMATICS UNIT MPC3	75	61	53	46	39	32
MPC4	MATHEMATICS UNIT MPC4	75	63	56	49	42	36