General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Friday 23 May 2008 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The speed, $v\,\mathrm{m\,s^{-1}}$, of a wave travelling along the surface of a sea is believed to depend on

the depth of the sea, d m,
the density of the water, $\rho\,\mathrm{kg\,m^{-3}}$,
the acceleration due to gravity, g, and
a dimensionless constant, k

so that

$$v = kd^{\alpha}\rho^{\beta}g^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, show that $\beta = 0$ and find the values of α and γ. *(6 marks)*

2 The unit vectors **i** and **j** are directed due east and due north respectively.

Two runners, Albina and Brian, are running on level parkland with constant velocities of $(5\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$ and $(3\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ respectively. Initially, the position vectors of Albina and Brian are $(-60\mathbf{i} + 160\mathbf{j})$ m and $(40\mathbf{i} - 90\mathbf{j})$ m respectively, relative to a fixed origin in the parkland.

(a) Write down the velocity of Brian relative to Albina. *(2 marks)*

(b) Find the position vector of Brian relative to Albina t seconds after they leave their initial positions. *(3 marks)*

(c) Hence determine whether Albina and Brian will collide if they continue running with the same velocities. *(3 marks)*

3 A particle of mass 0.2 kg lies at rest on a smooth horizontal table. A horizontal force of magnitude F newtons acts on the particle in a constant direction for 0.1 seconds. At time t seconds,

$$F = 5 \times 10^3 t^2, \quad 0 \leqslant t \leqslant 0.1$$

Find the value of t when the speed of the particle is $2\,\mathrm{m\,s^{-1}}$. *(4 marks)*

4 Two smooth spheres, A and B, have equal radii and masses m and $2m$ respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(4\mathbf{i} + 3\mathbf{j})$ when it collides with the sphere B which has velocity $(-2\mathbf{i} + 2\mathbf{j})$. After the collision, the velocity of B is $(\mathbf{i} + \mathbf{j})$.

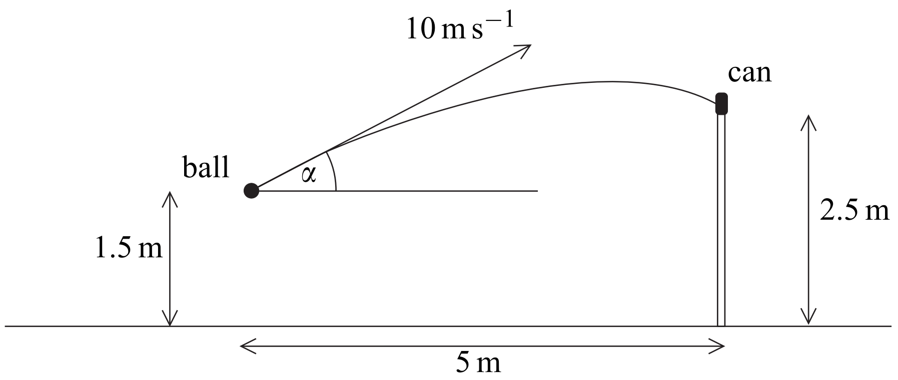
(a) Find the velocity of A immediately after the collision. *(3 marks)*

(b) Find the angle between the velocities of A and B immediately after the collision. *(3 marks)*

(c) Find the impulse exerted by B on A. *(3 marks)*

(d) State, as a vector, the direction of the line of centres of A and B when they collide. *(1 mark)*

5 A boy throws a small ball from a height of 1.5 m above horizontal ground with initial velocity $10\,\text{m s}^{-1}$ at an angle α above the horizontal. The ball hits a small can placed on a vertical wall of height 2.5 m, which is at a horizontal distance of 5 m from the initial position of the ball, as shown in the diagram.



(a) Show that α satisfies the equation

$$49\tan^2\alpha - 200\tan\alpha + 89 = 0$$ *(7 marks)*

(b) Find the **two** possible values of α, giving your answers to the nearest 0.1°. *(3 marks)*

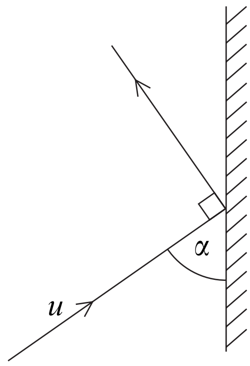
(c) (i) To knock the can off the wall, the horizontal component of the velocity of the ball must be greater than $8\,\text{m s}^{-1}$.

Show that, for one of the possible values of α found in part (b), the can will be knocked off the wall, and for the other, it will **not** be knocked off the wall. *(3 marks)*

(ii) Given that the can is knocked off the wall, find the direction in which the ball is moving as it hits the can. *(4 marks)*

6 A small smooth ball of mass m, moving on a smooth horizontal surface, hits a smooth vertical wall and rebounds. The coefficient of restitution between the wall and the ball is $\frac{3}{4}$.

Immediately before the collision, the ball has velocity u and the angle between the ball's direction of motion and the wall is α. The ball's direction of motion immediately after the collision is at right angles to its direction of motion before the collision, as shown in the diagram.



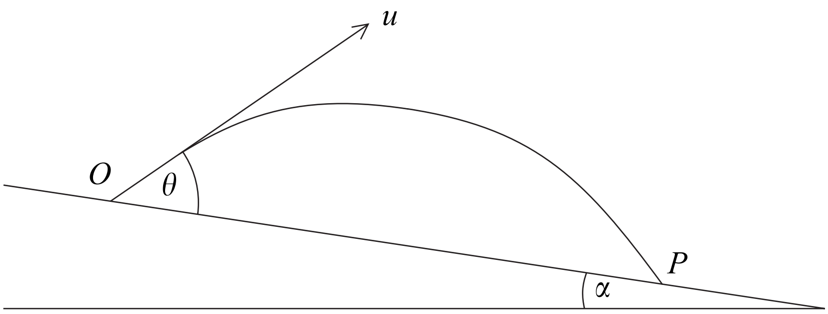
(a) Show that $\tan \alpha = \dfrac{2}{\sqrt{3}}$. *(5 marks)*

(b) Find, in terms of u, the speed of the ball immediately after the collision. *(2 marks)*

(c) The force exerted on the ball by the wall acts for 0.1 seconds.

Given that $m = 0.2\,\text{kg}$ and $u = 4\,\text{m s}^{-1}$, find the average force exerted by the wall on the ball. *(6 marks)*

7 A projectile is fired with speed u from a point O on a plane which is inclined at an angle α to the horizontal. The projectile is fired at an angle θ to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point P, lower down the inclined plane, as shown in the diagram.



(a) Find, in terms of u, g, θ and α, the greatest perpendicular distance of the projectile from the plane. *(4 marks)*

(b) (i) Find, in terms of u, g, θ and α, the time of flight from O to P. *(2 marks)*

(ii) By using the identity $\cos A \cos B + \sin A \sin B = \cos(A - B)$, show that the distance OP is given by $\dfrac{2u^2 \sin\theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$. *(6 marks)*

(iii) Hence, by using the identity $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ or otherwise, show that, as θ varies, the maximum possible distance OP is $\dfrac{u^2}{g(1 - \sin\alpha)}$. *(5 marks)*

END OF QUESTIONS

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1	$LT^{-1} = L^{\alpha} \times (ML^{-3})^{\beta}(LT^{-2})^{\gamma}$	M1		
	There is no M on the left hand side, so $\beta = 0$.	E1		
	$LT^{-1} = L^{\alpha+\gamma}T^{-2\gamma}$	m1		Dependent on M1
	$\alpha + \gamma = 1$ $-2\gamma = -1$	m1		Equating corresponding indices
	$\gamma = \frac{1}{2}$	A1		
	$\alpha = \frac{1}{2}$	A1	6	
	Total		**6**	
2(a)	${}_Av_B = v_B - v_A$ $= (3i + 4j) - (5i - j)$ $= -2i + 5j$	M1 A1	2	
(b)	${}_Ar_{0B} = (40i - 90j) - (-60i + 160j)$ $= 100i - 250j$	M1		Simplification not necessary
	${}_Ar_B = (100i - 250j) + (-2i + 5j)\text{t}$	m1 A1F	3	**ALTERNATIVE :** $r_A = (60i + 160j) + (5i - j)\text{t}$ M1 $r_B = (40i - 90j) + (3i + 4j)\text{t}$ ${}_Ar_B = \left[(40i - 90j) + (3i + 4j)\text{t}\right] - \left[(60i + 160j) + (5i\ \ -j)\text{t}\right]$ m1A1
(c)	${}_Ar_B = (100 - 2\text{t})i + (-250 + 5t)\text{j}$	M1		Collecting i and j terms
	$(100\ \text{-2t}) = 0 \quad \Leftrightarrow \quad \text{t} = 50$ $(-250 + 5\text{t}) = 0 \quad \Leftrightarrow \quad \text{t} = 50$	A1F		
	$\therefore$ A and B would collide.	E1	3	
				ALTERNATIVE : $\left[(100 - 2\text{t})i + (-250 + 5\text{t})j\right].(-2i + 5j) = 0$ M1 $-200 + 4\text{t} - 1250 + 25\text{t} = 0 \Rightarrow \text{t} = 50$ A1 $\lvert {}_Ar_B \rvert \sqrt{(100 - 2\times 50)^2 + (-250 + 5\times 50)^2} = 0$ $\therefore$ A and B would collide E1
	Total		**8**	

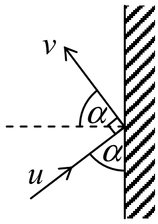
MM03 (cont)

Q	Solution	Marks	Total	Comments
3	$\int_0^t 5\times10^3 t^2 \, dt = 0.2(2) - 0.2(0)$	M1A1		Impulse-Momentum principle
	$\frac{5\times10^3}{3} t^3 = 0.4$	A1F		
	$t = 0.0621$	A1F	4	At least 3 sig. fig. required
	Total		**4**	
4(a)	C.L.M. $m(4\mathbf{i} + 3\mathbf{j}) + 2m(-2\mathbf{i} + 2\mathbf{j}) = mv + 2m(\mathbf{i} + \mathbf{j})$	M1		
	$7\mathbf{j} = v + (2\mathbf{i} + 2\mathbf{j})$ $v = -2\mathbf{i} + 5\mathbf{j}$	A2,1,0	3	A1 for one slip
(b)	The angle with **j** direction : $A: \quad \tan^{-1}\frac{2}{5} = 21.8^\circ$ $B: \quad \tan^{-1}\frac{1}{1} = 45^\circ$	M1		OE. in **i** direction M1 for two inverse tan and addition of angles
	The angle $= 21.8^\circ + 45^\circ = 67^\circ$	A1F	3	AWRT. **Alternative (not in the specification)** $(-2\mathbf{i}+5\mathbf{j}).(\mathbf{i} + \mathbf{j}) = \sqrt{29}\times\sqrt{2}\cos\theta$ (M1) $\cos\theta = \frac{3}{\sqrt{58}}$ (A1) $\theta = 67^\circ$ (A1F) awrt
(c)	The impulse = Gain in momentum of *A* $= m(-2\mathbf{i} + 5\mathbf{j}) - m(4\mathbf{i} + 3\mathbf{j})$ $= -6m\mathbf{i} + 2m\mathbf{j}$	M1 A1F A1F	3	
(d)	$-3\mathbf{i} + \mathbf{j}$ or any scalar multiple of $-3\mathbf{i} + \mathbf{j}$	B1	1	
	Total		**10**	

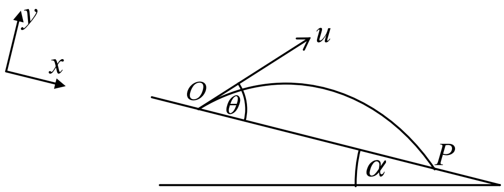
MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$5 = 10\cos\alpha.t$	M1		
	$t = \dfrac{5}{10\cos\alpha}$	A1		
	$1 = -\dfrac{1}{2}(9.8)t^2 + 10\sin\alpha.t$	M1A1		
	$1 = -\dfrac{1}{2}(9.8)\dfrac{25}{100\cos^2\alpha} + 10\sin\alpha\dfrac{5}{10\cos\alpha}$	m1		Dependent on both M1s
	$1 = -\dfrac{1}{2}(9.8)\dfrac{25}{100}(1+\tan^2\alpha) + 10\sin\alpha\dfrac{5}{10\cos\alpha}$	A1		
	$49\tan^2\alpha - 200\tan\alpha + 89 = 0$	A1	7	Answer given
(b)	$\tan\alpha = \dfrac{200 \pm \sqrt{40000 - 4(49)(89)}}{2\times 49}$	M1		
	$= 3.57, 0.508$	A1		AWRT
	$\alpha = 74.4^\circ, 26.9^\circ$	A1F	3	
(c)(i)	$10\cos 26.9^\circ = 8.92$ (or 8.91) > 8			
	⇒ The can will be knocked off the wall	M1		Both values checked
	$10\cos 74.4^\circ = 2.69 < 8$	A1F		Acc. of both results
	⇒ The can will not be knocked off the wall	E1	3	Correct conclusions
		ALTERNATIVE The can will be knocked off the wall if $10\cos\alpha > 8$ $\cos\alpha > 0.8$ $\alpha < 36.9°$ M1A1 So, for $\alpha = 26.9°$ the can will be knocked off and for $\alpha = 74.4°$, the can will not be knocked off E1		
5(c)(ii)	$x = ut$			
	$t = \dfrac{5}{10\cos 26.9^\circ}$			
	$v = 10\sin 26.9^\circ - 9.8\left(\dfrac{5}{10\cos 26.9^\circ}\right)$	M1		Any correct use of equations
	$v = -0.970$	A1F		
	$\tan\theta = \dfrac{-0.970}{8.92}$	M1		
	$\theta = -6.2^\circ$			
	At an angle of depression of 6.2°	A1F	4	AWRT 6°
	Total		**17**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
6(a)				
	Parallel to the wall : velocity is unchanged $u\cos\alpha = v\sin\alpha$	M1		
	Perpendicular to the wall : Law of Restitution $\frac{v\cos\alpha}{u\sin\alpha} = \frac{3}{4}$	M1		
	$\frac{v\cos\alpha}{v\tan\alpha\sin\alpha} = \frac{3}{4}$	m1		Dependent on both M1s
	$\frac{\cos^2\alpha}{\sin^2\alpha} = \frac{3}{4}$	m1		Dependent on both M1s
	$\tan^2\alpha = \frac{4}{3}$			
	$\tan\alpha = \frac{2}{\sqrt{3}}$	A1	5	Answer given
(b)	$v = \frac{u}{\tan\alpha}$	M1		
	$v = \frac{\sqrt{3}}{2}u$ or $0.866u$	A1	2	
(c)	Magnitude of Impulse = Change in momentum perpendicular to the wall	M1		
	$= 0.2\times v\cos\alpha - (-0.2\times 4\sin\alpha)$	A1 A1		
	$= 0.2\times\frac{\sqrt{3}}{2}\times 4\cos\alpha + 0.2\times 4\sin\alpha$	m1		
	= 1.06 Ns	A1F		
	Average Force = $\frac{1.06}{0.1} = 10.6$ N	A1F	6	
	Total		**13**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7				
(a)	$v_y^2 = u^2\sin^2\theta - 2g\cos\alpha . y$	M1 A1		
	$0 = u^2\sin^2\theta - 2g\cos\alpha . y_{max}$	m1		
	$y_{max} = \dfrac{u^2\sin^2\theta}{2g\cos\alpha}$	A1F	4	
(b)(i)	$u\sin\theta\ t - \frac{1}{2}g\cos(\alpha)t^2 = 0$	M1		
	$t = \dfrac{2u\sin\theta}{g\cos\alpha}$	A1	2	
(ii)	$x = u\cos\theta t - \frac{1}{2}g\sin(-\alpha)t^2$	M1 A1		
	$R = u\cos\theta(\frac{2u\sin\theta}{g\cos\alpha}) + \frac{1}{2}g\sin\alpha(\frac{2u\sin\theta}{g\cos\alpha})^2$	M1		
	$= \dfrac{2u^2\cos\theta\sin\theta\cos\alpha + 2u^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$	m1		Dependent on both M1s
	$= \dfrac{2u^2\sin\theta(\cos\theta\cos\alpha + \sin\theta\sin\alpha)}{g\cos^2\alpha}$	A1F		
	$= \dfrac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$	A1	6	Answer given
(iii)	$\overline{OP} = \dfrac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$			
	$= \dfrac{2u^2\frac{1}{2}[\sin(2\theta-\alpha)+\sin\alpha]}{g\cos^2\alpha}$	M1A1		
	$\overline{OP}$ is max when $\sin(2\theta-\alpha) = 1$	M1		
	$\overline{OP}_{max} = \dfrac{u^2(1+\sin\alpha)}{g\cos^2\alpha}$	A1F		
	$\overline{OP}_{max} = \dfrac{u^2(1+\sin\alpha)}{g(1-\sin^2\alpha)}$			
	$\overline{OP}_{max} = \dfrac{u^2}{g(1-\sin\alpha)}$	A1	5	Answer given
	Total		**17**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	**ALTERNATIVE**			
	$0 = u\sin\theta - g\cos a\ t$	M1		
	$t = \dfrac{u\sin\theta}{g\cos a}$	A1		
	$y_{max} = u\sin\theta\left(\dfrac{u\sin\theta}{g\cos a}\right) - \dfrac{1}{2}g\cos a\left(\dfrac{u\sin\theta}{g\cos a}\right)^2$	m1		
	$y_{max} = \dfrac{u^2\sin^2\theta}{2g\cos a}$	A1F	4	
	Total		**4**	

Scaled mark component grade boundaries - June 2008 exams

GCE

Component Code	Component Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries A	B	C	D	E
ICT4	GCE INFO AND COMM TECH UNIT 4	90	61	55	49	44	39
ICT5	GCE INFO AND COMM TECH UNIT 5	90	69	63	57	52	47
ICT6	GCE INFO AND COMM TECH UNIT 6	90	59	51	43	36	29
LAW1	GCE LAW UNIT 1	65	50	45	41	37	33
LAW2	GCE LAW UNIT 2	65	46	40	35	30	25
LAW3	GCE LAW UNIT 3	65	45	40	35	30	26
LAW4	GCE LAW UNIT 4	85	58	53	48	43	39
LAW5	GCE LAW UNIT 5	85	57	53	49	45	41
LAW6	GCE LAW UNIT 6	70	48	43	39	35	31
MD01	GCE MATHEMATICS UNIT D01	75	60	52	45	38	31
MD02	GCE MATHEMATICS UNIT D02	75	58	50	43	36	29
MFP1	GCE MATHEMATICS UNIT FP1	75	63	55	48	41	34
MFP2	GCE MATHEMATICS UNIT FP2	75	58	51	44	37	30
MFP3	GCE MATHEMATICS UNIT FP3	75	63	55	47	39	31
MFP4	GCE MATHEMATICS UNIT FP4	75	66	58	51	44	37
MM03	GCE MATHEMATICS UNIT M03	75	56	48	40	33	26
MM04	GCE MATHEMATICS UNIT M04	75	54	46	39	32	25
MM05	GCE MATHEMATICS UNIT M05	75	60	52	44	36	29
MM1A/C	GCE MATHEMATICS UNIT M1A - COURSEWORK	25	20	18	15	13	10
MM1A/W	GCE MATHEMATICS UNIT M1A - WRITTEN	75	60	51	43	35	28
MM1B	GCE MATHEMATICS UNIT M1B	75	61	52	43	34	25
MM2A/C	GCE MATHEMATICS UNIT M2A - COURSEWORK	25	20	18	15	13	10
MM2A/W	GCE MATHEMATICS UNIT M2A - WRITTEN	75	55	48	40	34	28
MM2B	GCE MATHEMATICS UNIT M2B	75	53	46	39	33	27
MPC1	GCE MATHEMATICS UNIT PC1	75	59	51	43	35	28
MPC2	GCE MATHEMATICS UNIT PC2	75	60	52	44	37	30