General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Monday 11 June 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The magnitude of the gravitational force, F, between two planets of masses m_1 and m_2 with centres at a distance x apart is given by

$$F = \frac{Gm_1m_2}{x^2}$$

where G is a constant.

(a) By using dimensional analysis, find the dimensions of G. *(3 marks)*

(b) The lifetime, t, of a planet is thought to depend on its mass, m, its initial radius, R, the constant G and a dimensionless constant, k, so that

$$t = km^{\alpha}R^{\beta}G^{\gamma}$$

where α, β and γ are constants.

Find the values of α, β and γ. *(5 marks)*

2 The unit vectors **i**, **j** and **k** are directed due east, due north and vertically upwards respectively.

Two helicopters, A and B, are flying with constant velocities of $(20\mathbf{i} - 10\mathbf{j} + 20\mathbf{k})\,\mathrm{m\,s^{-1}}$ and $(30\mathbf{i} + 10\mathbf{j} + 10\mathbf{k})\,\mathrm{m\,s^{-1}}$ respectively. At noon, the position vectors of A and B relative to a fixed origin, O, are $(8000\mathbf{i} + 1500\mathbf{j} + 3000\mathbf{k})\,\mathrm{m}$ and $(2000\mathbf{i} + 500\mathbf{j} + 1000\mathbf{k})\,\mathrm{m}$ respectively.

(a) Write down the velocity of A relative to B. *(2 marks)*

(b) Find the position vector of A relative to B at time t seconds after noon. *(3 marks)*

(c) Find the value of t when A and B are closest together. *(5 marks)*

3 A particle P, of mass 2 kg, is initially at rest at a point O on a smooth horizontal surface. The particle moves along a straight line, OA, under the action of a horizontal force. When the force has been acting for t seconds, it has magnitude $(4t + 5)$ N.

(a) Find the magnitude of the impulse exerted by the force on P between the times $t = 0$ and $t = 3$. *(3 marks)*

(b) Find the speed of P when $t = 3$. *(2 marks)*

(c) The speed of P at A is $37.5\,\mathrm{m\,s^{-1}}$. Find the time taken for the particle to reach A. *(4 marks)*

4 Two small smooth spheres, A and B, of equal radii have masses 0.3 kg and 0.2 kg respectively. They are moving on a smooth horizontal surface directly towards each other with speeds $3\,\text{m}\,\text{s}^{-1}$ and $2\,\text{m}\,\text{s}^{-1}$ respectively when they collide. The coefficient of restitution between A and B is 0.8.

(a) Find the speeds of A and B immediately after the collision. *(6 marks)*

(b) Subsequently, B collides with a fixed smooth vertical wall which is at right angles to the path of the sphere. The coefficient of restitution between B and the wall is 0.7.

Show that B will collide again with A. *(3 marks)*

5 A ball is projected with speed $u\,\text{m}\,\text{s}^{-1}$ at an angle of elevation α above the horizontal so as to hit a point P on a wall. The ball travels in a vertical plane through the point of projection. During the motion, the horizontal and upward vertical displacements of the ball from the point of projection are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$$ *(6 marks)*

(b) The ball is projected from a point 1 metre vertically below and R metres horizontally from the point P.

(i) By taking $g = 10\,\text{m}\,\text{s}^{-2}$, show that R satisfies the equation

$$5R^2\tan^2\alpha - u^2R\tan\alpha + 5R^2 + u^2 = 0$$ *(2 marks)*

(ii) Hence, given that u and R are constants, show that, for $\tan\alpha$ to have real values, R must satisfy the inequality

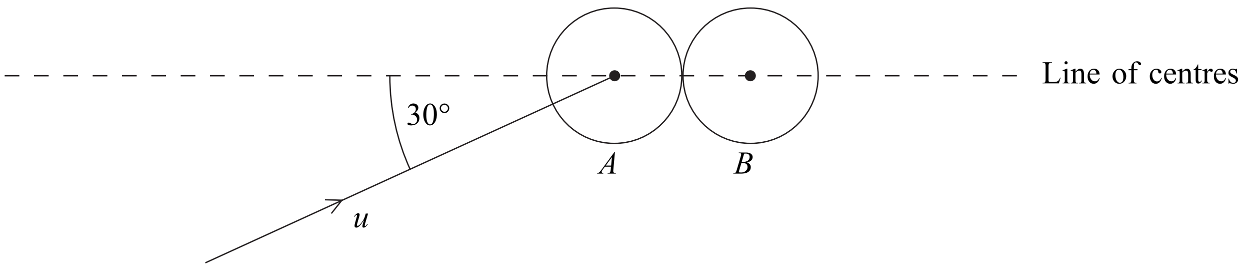
$$R^2 \leqslant \frac{u^2(u^2 - 20)}{100}$$ *(2 marks)*

(iii) Given that $R = 5$, determine the minimum possible speed of projection. *(3 marks)*

Turn over ▶

6 A smooth spherical ball, A, is moving with speed u in a straight line on a smooth horizontal table when it hits an identical ball, B, which is at rest on the table.

Just before the collision, the direction of motion of A makes an angle of 30° with the line of the centres of the two balls, as shown in the diagram.



The coefficient of restitution between A and B is e.

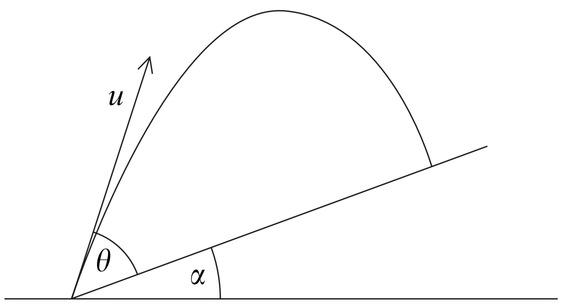
(a) Given that $\cos 30° = \frac{\sqrt{3}}{2}$, show that the speed of B immediately after the collision is

$$\frac{\sqrt{3}}{4}u(1+e)$$ *(5 marks)*

(b) Find, in terms of u and e, the components of the velocity of A, parallel and perpendicular to the line of centres, immediately after the collision. *(3 marks)*

(c) Given that $e = \frac{2}{3}$, find the angle that the velocity of A makes with the line of centres immediately after the collision. Give your answer to the nearest degree. *(3 marks)*

7 A particle is projected from a point on a plane which is inclined at an angle α to the horizontal. The particle is projected up the plane with velocity u at an angle θ above the plane. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Using the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, show that the range up the plane is

$$\frac{2u^2 \sin\theta \cos(\theta+\alpha)}{g\cos^2\alpha}$$ *(8 marks)*

(b) Hence, using the identity $2\sin A \cos B = \sin(A+B) + \sin(A-B)$, show that, as θ varies, the range up the plane is a maximum when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$. *(3 marks)*

(c) Given that the particle strikes the plane at right angles, show that

$$2\tan\theta = \cot\alpha$$ *(4 marks)*

END OF QUESTIONS

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1(a)	$\mathrm{MLT}^{-2} = \dfrac{[G]\mathrm{MM}}{\mathrm{L}^2}$	M1 A1		
	$[G] = \mathrm{L}^3\mathrm{M}^{-1}\mathrm{T}^{-2}$	A1F	3	
(b)	$t = km^{\alpha}R^{\beta}G^{\gamma}$			
	$\mathrm{T} = \mathrm{M}^{\alpha}\mathrm{L}^{\beta}\mathrm{M}^{-\gamma}\mathrm{L}^{3\gamma}\mathrm{T}^{-2\gamma}$	M1 A1F		L, M, T for *G* are needed to gain M1
	$-2\gamma = 1 \quad \Rightarrow \quad \gamma = -\dfrac{1}{2}$			
	$\alpha - \gamma = 0 \quad \Rightarrow \quad \alpha = -\dfrac{1}{2}$	m1 m1		Getting 3 equations Solution
	$\beta + 3\gamma = 0 \quad \Rightarrow \quad \beta = \dfrac{3}{2}$	A1F	5	Finding α, β, γ
	Total		**8**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
2 (a)	${}_B\mathbf{v}_A = \mathbf{v}_A - \mathbf{v}_B$ $= (20\mathbf{i} - 10\mathbf{j} + 20\mathbf{k}) - (30\mathbf{i} + 10\mathbf{j} + 10\mathbf{k})$ $= -10\mathbf{i} - 20\mathbf{j} + 10\mathbf{k}$	M1A1	2	Simplification not necessary
(b)	${}_B\mathbf{r}_{0A} = (8000\mathbf{i} + 1500\mathbf{j} + 3000\mathbf{k}) - (2000\mathbf{i} + 500\mathbf{j} + 1000\mathbf{k})$ $= 6000\mathbf{i} + 1000\mathbf{j} + 2000\mathbf{k}$	M1		
	${}_B\mathbf{r}_A = (6000\mathbf{i} + 1000\mathbf{j} + 2000\mathbf{k}) + (-10\mathbf{i} - 20\mathbf{j} + 10\mathbf{k})t$	M1 A1F	3	Simplification not necessary
	${}_B\mathbf{r}_A = (6000 - 10t)\mathbf{i} + (1000 - 20t)\mathbf{j} + (2000 + 10t)\mathbf{k}$			
(c)	$\lvert {}_B\mathbf{r}_A \rvert^2 = (6000 - 10t)^2 + (1000 - 20t)^2 + (2000 + 10t)^2$	M1 A1F		
	The helicopters are closest when $\lvert {}_B\mathbf{r}_A \rvert^2$ is minimum. $y = (6000 - 10t)^2 + (1000 - 20t)^2 + (2000 + 10t)^2$			
	$\frac{dy}{dt} = 2(-10)(6000 - 10t) + 2(-20)(1000 - 20t) + 2(10)(2000 + 10t) = 0$	m1 A1F		
	$t = 100$	A1F	5	
	Alternative: $\begin{pmatrix} 6000 - 10t \\ 1000 - 20t \\ 2000 + 10t \end{pmatrix} \bullet \begin{pmatrix} -10 \\ -20 \\ 10 \end{pmatrix} = 0$	(M1) (A1F)		
	$-60000 + 100t - 20000 + 400t + 20000 + 100t = 0$	(m1) (A1F)		
	$600t = 60000$ $t = 100$	(A1F)	(5)	
	Total		**10**	

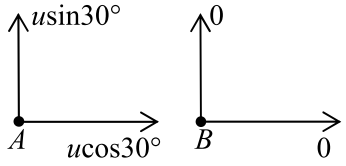
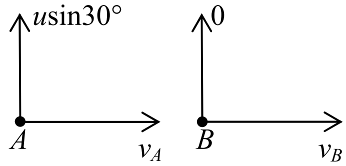
MM03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$I = \int_0^3 (4t+5)\mathrm{d}t$	M1		
	$= \left[2t^2+5t\right]_0^3$	m1		Or evaluation of constant
	$= 33$ Ns	A1	3	
	Alternative:			
	I = Area under the Force–Time graph	(M1)		
	$= \dfrac{17+5}{2}\times 3$	(m1)		
	$= 33$ Ns	(A1)	(3)	
(b)	$I = mv - mu$			
	$33 = 2v - 2(0)$	M1		
	$v = 16.5$ ms^{-1}	A1F	2	
(c)	$I = \int_0^t (4t+5)\mathrm{d}t = 2(37.5) - 2(0)$	M1		
	$2t^2 + 5t - 75 = 0$	A1		
	$t = \dfrac{-5 \pm \sqrt{25 + 8\times 75}}{4}$	m1		
	$t = 5$	A1F	4	For one value of t identified only
	Total		**9**	
4(a)	Conservation of momentum :			
	$0.3(3) - 0.2(2) = 0.3v_A + 0.2v_B$	M1A1		
	$3v_A + 2v_B = 5$ ----------------(1)			
	Newton's experimental law :			
	$0.8 = \dfrac{v_B - v_A}{5}$	M1		
	$v_B - v_A = 4$ -----------------(2)	A1		For both (1) and (2)
	Solving (1) and (2)	m1		Dependent on both M1s
	$v_B = 3.4$			
	$v_A = -0.6$	A1F	6	For both solutions
(b)	$0.7 = \dfrac{v}{3.4}$	M1		
	$v = 2.38$	A1F		
	Speed of B (2.38) $\succ$ Speed of A (0.6)			
	$\therefore$ B collides again with A	E1	3	Cannot be gained without A1F
	Total		**9**	

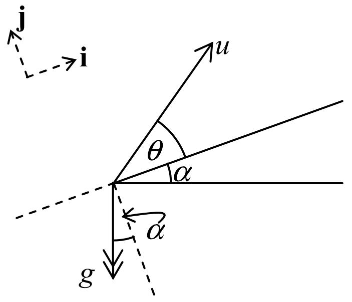
MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$y = ut\sin\alpha - \frac{1}{2}gt^2$	M1 A1		
	$x = ut\cos\alpha$	M1		
	$t = \frac{x}{u\cos\alpha}$	A1		
	$y = u\left(\frac{x}{u\cos\alpha}\right)\sin\alpha - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$	M1		
	$y = x\tan\alpha - \frac{gx^2}{u^2\cos^2\alpha}$			
	$y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$	A1	6	Answer given
(b)(i)	$1 = R\tan\alpha - \frac{10R^2}{2u^2}(1+\tan^2\alpha)$	M1		
	$5R^2\tan^2\alpha - u^2R\tan\alpha + 5R^2 + u^2 = 0$	A1	2	Answer given
(ii)	For real solutions of the quadratic :			
	$u^4R^2 - 20R^2(5R^2+u^2) \geq 0$	M1		
	$R^2 \leq \frac{u^4 - 20u^2}{100}$			
	$R^2 \leq \frac{u^2(u^2-20)}{100}$	A1	2	Answer given
(iii)	$5^2 \leq \frac{u^2(u^2-20)}{100}$			
	$u^4 - 20u^2 - 2500 \geq 0$	M1		Condone equation
	${u_{min}}^2 = 61.0 \quad (\text{or } 10+\sqrt{2600})$	A1		
	$u_{min} = 7.81$	A1F	3	3 sf required
	Total		**13**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Before: 			
	After: 			
	Con. of Mom. along the line of centres: $mu\cos30° = mv_A + mv_B$	M1		
	$v_A + v_B = \frac{\sqrt{3}}{2}u$ ----------(1)	A1		
	Newton's experimental law : $e = \frac{v_B - v_A}{u\cos30° - 0}$	M1		
	$v_B - v_A = \frac{\sqrt{3}}{2}ue$ ----------(2)	A1		
	Solving (1) and (2) : $v_B = \frac{\sqrt{3}}{4}u(1+e)$	A1	5	Answer given
(b)	$\perp$ $u\sin30° = \frac{1}{2}u$	B1		$u$sin30 accepted
	$\parallel$ $v_A = \frac{\sqrt{3}}{2}u - \frac{\sqrt{3}}{4}u(1+e)$ $v_A = \frac{\sqrt{3}}{4}u(1-e)$	M1 A1F	3	Simplification not needed
(c)	$\alpha = \tan^{-1}\frac{\frac{1}{2}u}{\frac{\sqrt{3}}{4}u\left(1-\frac{2}{3}\right)}$	M1 A1F		
	$\alpha = \tan^{-1}\frac{6}{\sqrt{3}}$			
	$\alpha = 74°$	A1F	3	To the nearest degree required
	Total		**11**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)				
	$y = ut\sin\theta - \frac{1}{2}gt^2\cos\theta$	M1A1		
	$y = 0 \quad \Rightarrow \quad t = \frac{2u\sin\theta}{g\cos\alpha}$	A1F		
	$x = ut\cos\theta - \frac{1}{2}gt^2\sin\alpha$	M1A1		
	$R = u\frac{2u\sin\theta}{g\cos\alpha}\cos\theta - \frac{1}{2}g\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2\sin\alpha$	M1		
	$R = \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}$	m1 A1	8	Dependent on M1s Answer given
(b)	$R = \frac{2u^2 \times \frac{1}{2}[\sin(2\theta+\alpha)+\sin(-\alpha)]}{g\cos^2\alpha}$	B1		
	R is maximum when $\sin(2\theta+\alpha)=1$	M1		
	or $2\theta+\alpha = \frac{\pi}{2}$			
	$\therefore \quad \theta = \frac{\pi}{4} - \frac{\alpha}{2}$	A1	3	Answer given
(c)	$y = 0 \quad \Rightarrow \quad t = \frac{2u\sin\theta}{g\cos\alpha}$	M1 A2,1		For using y=0 and $\dot{x} = 0$ A2 for both correct
	$\dot{x} = 0 \quad \Rightarrow \quad t = \frac{u\cos\theta}{g\sin\alpha}$			
	$\frac{2u\sin\theta}{g\cos\alpha} = \frac{u\cos\theta}{g\sin\alpha}$			
	$2\tan\theta = \cot\alpha$	A1	4	Answer given N.B. A problem arose which ultimately affected the marking of part 7(c). Please see the Report on the Examination for details.
	Total		**15**	
	TOTAL		**75**	

Scaled mark component grade boundaries - June 2007 exams

GCE

Component Code	Component Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries A	B	C	D	E
ICT3	GCE INFO AND COMM TECH UNIT 3	60	43	37	31	26	21
ICT4	GCE INFO AND COMM TECH UNIT 4	90	69	63	57	51	46
ICT5	GCE INFO AND COMM TECH UNIT 5	90	62	55	49	43	37
ICT6	GCE INFO AND COMM TECH UNIT 6	90	59	51	43	36	29
LAW1	GCE LAW UNIT 1	65	47	43	39	35	31
LAW2	GCE LAW UNIT 2	65	47	42	37	32	27
LAW3	GCE LAW UNIT 3	65	43	38	33	28	23
LAW4	GCE LAW UNIT 4	85	56	51	47	43	39
LAW5	GCE LAW UNIT 5	85	56	52	48	45	42
LAW6	GCE LAW UNIT 6	70	47	43	39	35	32
MD01	GCE MATHEMATICS UNIT D01	75	63	56	49	42	35
MD02	GCE MATHEMATICS UNIT D02	75	59	51	44	37	30
MFP1	GCE MATHEMATICS UNIT FP1	75	64	56	48	41	34
MFP2	GCE MATHEMATICS UNIT FP2	75	56	49	42	35	29
MFP3	GCE MATHEMATICS UNIT FP3	75	60	53	46	39	32
MFP4	GCE MATHEMATICS UNIT FP4	75	62	54	46	38	31
MM03	GCE MATHEMATICS UNIT M03	75	64	56	48	40	33
MM04	GCE MATHEMATICS UNIT M04	75	62	54	46	39	32
MM05	GCE MATHEMATICS UNIT M05	75	58	50	42	34	27
MM1A/C	GCE MATHEMATICS UNIT M1A - COURSEWORK	25	20	18	15	13	10
MM1A/W	GCE MATHEMATICS UNIT M1A - WRITTEN	75	60	51	43	35	28
MM1B	GCE MATHEMATICS UNIT M1B	75	58	49	40	31	23
MM2A/C	GCE MATHEMATICS UNIT M2A - COURSEWORK	25	20	18	15	13	10
MM2A/W	GCE MATHEMATICS UNIT M2A - WRITTEN	75	63	54	46	39	31
MM2B	GCE MATHEMATICS UNIT M2B	75	65	57	49	41	33
MPC1	GCE MATHEMATICS UNIT PC1	75	60	52	44	37	30