General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Wednesday 21 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The time T taken for a simple pendulum to make a single small oscillation is thought to depend only on its length l, its mass m and the acceleration due to gravity g.

By using dimensional analysis:

(a) show that T does **not** depend on m; *(3 marks)*

(b) express T in terms of l, g and k, where k is a dimensionless constant. *(4 marks)*

2 Three smooth spheres A, B and C of equal radii and masses m, m and $2m$ respectively lie at rest on a smooth horizontal table. The centres of the spheres lie in a straight line with B between A and C. The coefficient of restitution between any two spheres is e.

The sphere A is projected directly towards B with speed u and collides with B.

(a) Find, in terms of u and e, the speed of B immediately after the impact between A and B. *(5 marks)*

(b) The sphere B subsequently collides with C. The speed of C immediately after this collision is $\frac{3}{8}u$. Find the value of e. *(7 marks)*

3 A ball of mass 0.45 kg is travelling horizontally with speed $15\,\mathrm{m\,s^{-1}}$ when it strikes a fixed vertical bat directly and rebounds from it. The ball stays in contact with the bat for 0.1 seconds.

At time t seconds after first coming into contact with the bat, the force exerted on the ball by the bat is $1.4 \times 10^5(t^2 - 10t^3)$ newtons, where $0 \leqslant t \leqslant 0.1$.

In this simple model, ignore the weight of the ball and model the ball as a particle.

(a) Show that the magnitude of the impulse exerted by the bat on the ball is 11.7 N s, correct to three significant figures. *(4 marks)*

(b) Find, to two significant figures, the speed of the ball immediately after the impact. *(4 marks)*

(c) Give a reason why the speed of the ball immediately after the impact is different from the speed of the ball immediately before the impact. *(1 mark)*

4 The unit vectors **i** and **j** are directed due east and due north respectively.

Two cyclists, Aazar and Ben, are cycling on straight horizontal roads with constant velocities of $(6\mathbf{i} + 12\mathbf{j})\,\text{km h}^{-1}$ and $(12\mathbf{i} - 8\mathbf{j})\,\text{km h}^{-1}$ respectively. Initially, Aazar and Ben have position vectors $(5\mathbf{i} - \mathbf{j})$ km and $(18\mathbf{i} + 5\mathbf{j})$ km respectively, relative to a fixed origin.

(a) Find, as a vector in terms of **i** and **j**, the velocity of Ben relative to Aazar. *(2 marks)*

(b) The position vector of Ben relative to Aazar at time t hours after they start is **r** km.

Show that

$$\mathbf{r} = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}$$ *(4 marks)*

(c) Find the value of t when Aazar and Ben are closest together. *(6 marks)*

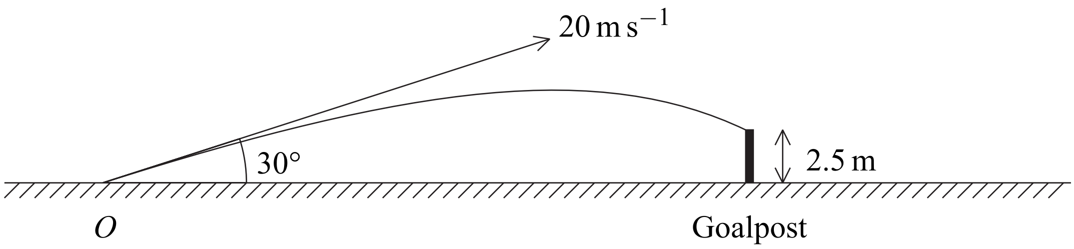
(d) Find the closest distance between Aazar and Ben. *(2 marks)*

5 A football is kicked from a point O on a horizontal football ground with a velocity of $20\,\text{m s}^{-1}$ at an angle of elevation of 30°. During the motion, the horizontal and upward vertical displacements of the football from O are x metres and y metres respectively.

(a) Show that x and y satisfy the equation

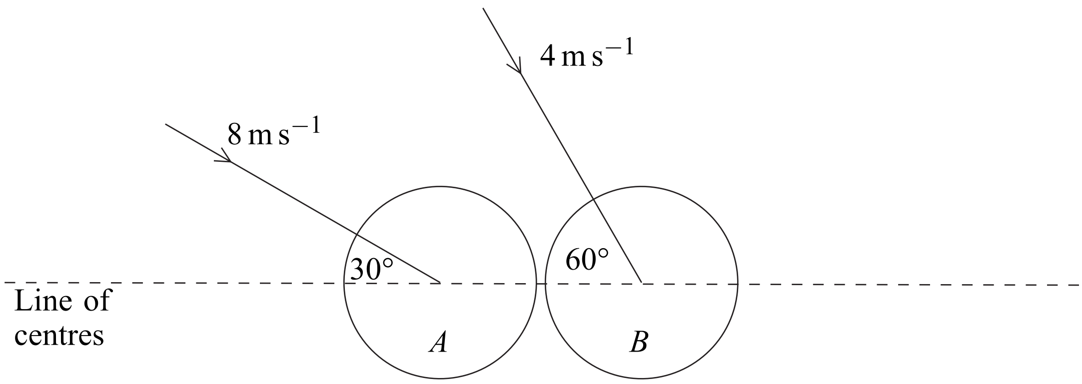
$$y = x \tan 30° - \frac{gx^2}{800 \cos^2 30°}$$ *(6 marks)*

(b) On its downward flight the ball hits the horizontal crossbar of the goal at a point which is 2.5 m above the ground. Using the equation given in part (a), find the horizontal distance from O to the goal. *(4 marks)*



(c) State **two** modelling assumptions that you have made. *(2 marks)*

6 Two smooth billiard balls A and B, of identical size and equal mass, move towards each other on a horizontal surface and collide. Just before the collision, A has velocity $8\,\text{m s}^{-1}$ in a direction inclined at 30° to the line of centres of the balls, and B has velocity $4\,\text{m s}^{-1}$ in a direction inclined at 60° to the line of centres, as shown in the diagram.



The coefficient of restitution between the balls is $\frac{1}{2}$.

(a) Find the speed of B immediately after the collision. *(9 marks)*

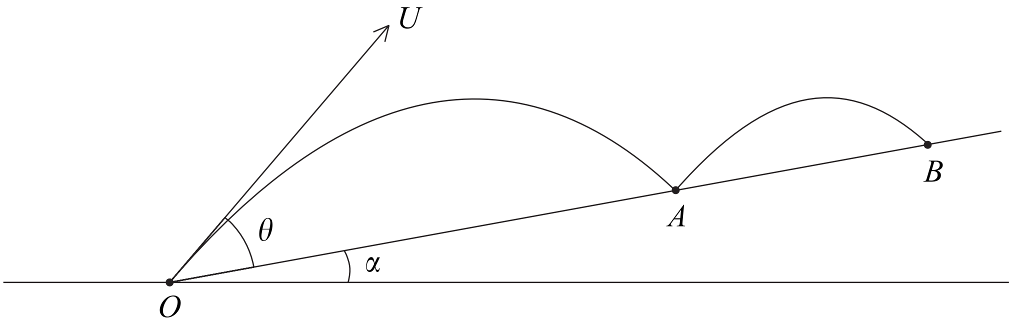
(b) Find the angle between the velocity of B and the line of centres of the balls immediately after the collision. *(2 marks)*

7 A projectile is fired from a point O on the slope of a hill which is inclined at an angle α to the horizontal. The projectile is fired up the hill with velocity U at an angle θ above the hill and first strikes it at a point A. The projectile is modelled as a particle and the hill is modelled as a plane with OA as a line of greatest slope.

(a) (i) Find, in terms of U, g, α and θ, the time taken by the projectile to travel from O to A. *(3 marks)*

(ii) Hence, or otherwise, show that the magnitude of the component of the velocity of the projectile perpendicular to the hill, when it strikes the hill at the point A, is the same as it was initially at O. *(3 marks)*

(b) The projectile rebounds and strikes the hill again at a point B. The hill is smooth and the coefficient of restitution between the projectile and the hill is e.



Find the ratio of the time of flight from O to A to the time of flight from A to B. Give your answer in its simplest form. *(4 marks)*

END OF QUESTIONS

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1(a)(i)	$T^1 = L^a \times M^b \times (LT^{-2})^c$	M1A1		
	There is no M on the left, so $b = 0$	E1	3	
(ii)	$T^1 = L^{a+c} \times M^0 \times T^{-2}$	M1		
	$\begin{cases} -2c = 1 \\ a + c = 0 \end{cases}$	m1		equating corresponding indices
	$a = \frac{1}{2},\ c = -\frac{1}{2}$	m1		solution
	$\therefore$ Period $= kl^{\frac{1}{2}}g^{-\frac{1}{2}}$	A1F	4	constant needed
	Total		**7**	
2(a)	conservation of momentum			
	$mu = mv_A + mv_B$	M1		
	$u = v_A + v_B$	A1		
	restitution			
	$eu = v_B - v_A$	M1A1		OE
	$v_B = \frac{1}{2}u(1+e)$	A1F	5	OE
(b)	$mv_B = mw_B + 2m\frac{3u}{8}$	M1A1		
	$ev_B = \frac{3u}{8} - w_B$	M1A1		OE
	Elimination of w_B	m1		dependent on both M1s
	$4e^2 + 8e - 5 = 0$	A1F		simplified quadratic equation in e only
	$e = \frac{1}{2}$	A1F	7	stated as the only value $(0 < e < 1$ for follow through$)$
	Total		**12**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$I = 1.4 \times 10^5 \int_0^{0.1} (t^2 - 10t^3)\mathrm{d}t$	M1A1		
	$= 1.4 \times 10^5 \left[\frac{1}{3}t^3 - \frac{10}{4}t^4 \right]_0^{0.1}$	m1		
	$= 11.7$ Ns	A1	4	AG
(b)	initial momentum $= 0.45(-15)$ $= -6.75$ Ns	M1		
	final momentum $= 11.7 - 6.75$ $= 4.95$ Ns	M1		
	velocity after impact $= \frac{4.95}{0.45}$	m1		dependent on both previous M1s
	$= 11 \text{ ms}^{-1}$	A1	4	
(c)	The ball is not perfectly elastic or $e \neq 1$ or energy loss	E1	1	
	Total		**9**	

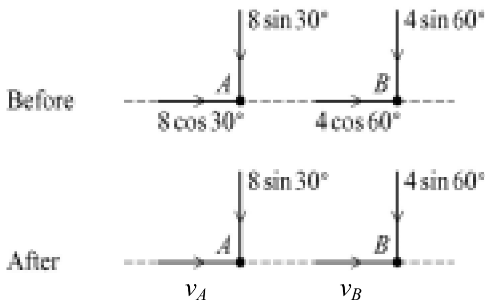
MM03 (cont)

Q	Solution	Marks	Total	Comments
4(a)	${}_A\mathbf{v}_B = (12\mathbf{i} - 8\mathbf{j}) - (6\mathbf{i} + 12\mathbf{j})$	M1		
	$= 6\mathbf{i} - 20\mathbf{j}$	A1	2	needs to be in terms of **i** and **j**
(b)	${}_A\mathbf{r}_B = \mathbf{r}_0 + {}_A\mathbf{v}_B t$	M1A1		attempted use
	${}_A\mathbf{r}_B = (18\mathbf{i} + 5\mathbf{j}) - (5\mathbf{i} - \mathbf{j}) + (6\mathbf{i} - 20\mathbf{j})t$	A1F		
	${}_A\mathbf{r}_B = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}$	A1	4	AG (not penalised if not in terms of **i** and **j**)
	Alternative			
	$\mathbf{r}_A = 5\mathbf{i} - \mathbf{j} + (6\mathbf{i} + 12\mathbf{j})t$			
	$\mathbf{r}_B = 18\mathbf{i} + 5\mathbf{j} + (12\mathbf{i} - 8\mathbf{j})t$	M1A1		A1 for each of $\mathbf{r}_A$ and $\mathbf{r}_B$
	${}_A\mathbf{r}_B = 18\mathbf{i} + 5\mathbf{j} + (12\mathbf{i} - 8\mathbf{j})t$ $-\left[5\mathbf{i} - \mathbf{j} + (6\mathbf{i} + 12\mathbf{j})t\right]$	A1		
	${}_A\mathbf{r}_B = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}$	A1F		
(c)	$s^2 = (13 + 6t)^2 + (6 - 20t)^2$	M1A1F		attempt for squaring and tidying up
	A and B are closest when $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ or	M1		
	$\frac{\mathrm{d}s^2}{\mathrm{d}t} = 0$			
	$2s\frac{\mathrm{d}s}{\mathrm{d}t} = 2(13 + 6t)6 - 2(6 - 20t)20 = 0$	M1 A1		accuracy of differentiation
	$t = 0.0963$ $\left(\text{or } 0.096 \text{ or } \frac{21}{218}\right)$	A1F	6	
	Alternative			
	${}_A\mathbf{r}_B \cdot {}_A\mathbf{v}_B = 0$	M1		
	$[(13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}] \cdot [6\mathbf{i} - 20\mathbf{j}] = 0$	M1		
	$6(13 + 6t) - 20(6 - 20t) = 0$	M1A1		
	$436t - 42 = 0$	A1F		
	$t = 0.0963$ (or 0.096 or $21/218$)	A1F		
(d)	$s = \sqrt{(13 + 6 \times 0.0963)^2 + (6 - 20 \times 0.0963)^2}$	m1		dependent on M1s in part (c)
	$s = 14.2$ km	A1F	2	AWRT
	Total		**14**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$y = -\frac{1}{2}gt^2 + 20\sin 30.t$	M1A1		
	$x = 20\cos 30.t$	M1		
	$t = \frac{x}{20\cos 30}$	A1		
	$y = -\frac{1}{2}g\frac{x^2}{400\cos^2 30} + 20\sin 30\frac{x}{20\cos 30}$	M1		
	$y = x\tan 30 - \frac{gx^2}{800\cos^2 30°}$	A1	6	AG
(b)	$2.5 = x\tan 30 - \frac{9.8x^2}{800\cos^2 30}$			
	$9.8x^2 - 346x + 1500 = 0$	M1A1		substituting and tidying up
	$x = \frac{346 \pm \sqrt{119716 - 58800}}{19.6}$	M1		
	=30.3 (or 30.2) & 5.06 (or 5.05)			
	answer: 30.3m (or 30.2m)	A1F	4	at least 3 s.f. required
(c)	no air resistance,	B1		
	the ball is a particle	B1	2	
	etc.			
	Total		**12**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Components of velocities:			
				
	conservation of linear momentum along the line of centres:			
	$m \times 8\cos 30 + m \times 4\cos 60 = mv_A + mv_B$	M1A1		OE unsimplified
	$v_A + v_B = 8.93$			
	Law of restitution along the line of centre:			
	$\dfrac{v_B - v_A}{8\cos 30 - 4\cos 60} = \dfrac{1}{2}$	M1A1		OE unsimplified
	$v_B - v_A = 2.46$			
	$v_B = 5.70$	m1		dependent on both M1s
		A1F		AWRT $\left(\text{or } 3\sqrt{3} + \dfrac{1}{2}\right)$
	momentum of B perpendicular to the line of centres is unchanged	B1		PI (can also be gained in part (b))
	Speed of $B = \sqrt{{u_B}^2 + {v_B}^2}$			
	$= \sqrt{(4\sin 60)^2 + (5.70)^2}$	m1		dependent on both M1s
	$= 6.67$	A1F	9	
(b)	direction of $B = \tan^{-1}\dfrac{4\sin 60}{5.70} = 31.3°$	m1 A1F	 2	dependent on both M1s and B1
	Total		**11**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	the projectile hits the plane again when			
	$(Ut\sin\theta - \frac{1}{2}gt^2\cos\alpha) = 0$	M1A1		
	$\therefore t = \dfrac{2U\sin\theta}{g\cos\alpha}$	A1F	3	need to be simplified
(ii)	the component of velocity perpendicular to plane $=$			
	$U\sin\theta - g\dfrac{2U\sin\theta}{g\cos\alpha}\cos\alpha =$	M1A1F		
	$-U\sin\theta =$ the initial magnitude	A1	3	AG
(b)	Newton's law of restitution perpendicular to plane:			
	$u = eU\sin\theta$	M1		
	$a = -g\cos\alpha$			
	$s = 0$			
	$0 = eU\sin\theta . T - \frac{1}{2}g\cos\alpha . T^2$	M1 A1		
	$T = \dfrac{2eU\sin\theta}{g\cos\alpha} = et$			
	$t : T = 1 : e$	A1F	4	
	Total		**10**	
	TOTAL		**75**	

AQA June Examinations 2006

Scaled Mark Unit Grade Boundaries (GCE Specifications)

Unit Code	Unit Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries A	B	C	D	E
MD01	GCE MATHEMATICS UNIT D01	75	61	54	47	40	33
MD02	GCE MATHEMATICS UNIT D02	75	64	56	48	41	34
MFP1	GCE MATHEMATICS UNIT FP1	75	60	52	44	36	29
MFP2	GCE MATHEMATICS UNIT FP2	75	58	51	44	37	30
MFP3	GCE MATHEMATICS UNIT FP3	75	62	54	46	39	32
MFP4	GCE MATHEMATICS UNIT FP4	75	59	51	44	37	30
MM03	GCE MATHEMATICS UNIT M03	75	59	51	44	37	30
MM04	GCE MATHEMATICS UNIT M04	75	59	51	43	36	29
MM05	GCE MATHEMATICS UNIT M05	75	57	49	41	33	26
MM1A	GCE MATHEMATICS UNIT M1A	100	79	69	59	49	40
MM1B	GCE MATHEMATICS UNIT M1B	75	61	53	45	37	30
MM2A	GCE MATHEMATICS UNIT M2A	100	81	71	61	51	41
MM2B	GCE MATHEMATICS UNIT M2B	75	62	54	46	38	31
MPC1	GCE MATHEMATICS UNIT PC1	75	61	53	45	38	31
MPC2	GCE MATHEMATICS UNIT PC2	75	60	53	46	39	33
MPC3	GCE MATHEMATICS UNIT PC3	75	60	53	46	39	32
MPC4	GCE MATHEMATICS UNIT PC4	75	61	54	47	40	33
MS03	GCE MATHEMATICS UNIT S03	75	61	53	45	38	31
MS04	GCE MATHEMATICS UNIT S04	75	61	53	45	38	31
MS1A	GCE MATHEMATICS UNIT S1A	100	80	70	60	50	41
MS1B	GCE MATHEMATICS UNIT S1B	75	60	52	44	37	30
MS2A	GCE MATHEMATICS UNIT S2A	100	78	68	58	49	40
MS2B	GCE MATHEMATICS UNIT S2B	75	60	52	44	37	30