Collision - part 1:

Impulse - Momentum

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Momentum	$I = mv - mu$	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$
	$I = Ft$	$\mathbf{I} = \mathbf{F}t$

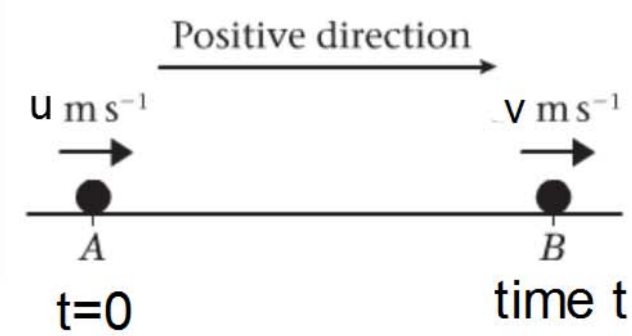
Specifications:

Collisions in one dimension

Momentum.	
Impulse as change of momentum.	Knowledge and use of the equation $I = mv - mu$.
Impulse as Force × Time.	$I = Ft$
Impulse as $\int F\,\mathrm{d}t$	Applied to explosions as well as collisions.

Definitions and formulae

Consider a particule of mass m moving in a straight line with an initial speed "u".



- The momentum of the particule, at any time t, is **p**=m**v**

According to Newton's law,

$$\mathbf{F}=m\mathbf{a}$$

$$\mathbf{F}=m\frac{d\mathbf{v}}{dt}=\frac{d(m\mathbf{v})}{dt}$$

$$\mathbf{F}=\frac{d\mathbf{p}}{dt}$$

This means that if a force is applied on a particule, there is a change of momentum, and if there is a change of momentum, a force has been applied on the particule.

- The IMPULSE of a force is equal to the change of momentum produced.

Case 1: The force applied is **constant** with respect to time

We have established that $\mathbf{F} = \frac{d\mathbf{p}}{dt}$ (*as vectors*).

In this chapter, we work on an horizontal line, the vectors have only one component so

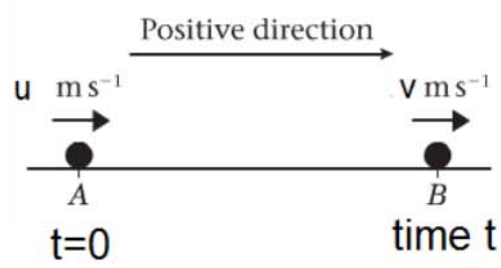
$$F = \frac{dp}{dt} \quad (x - components)$$

By integration between $t = 0$ *and* t, *we have*

$\int_0^t F dt = \int_0^t dp$ and because F does not depends on the time t:

$Ft = p_t - p_0$ $\qquad$ $Ft = mv - mu$

F is constant



Example

A body of mass 2 kg is initially at rest on a smooth horizontal plane. A horizontal force of magnitude 4.5 N acts on the body for 6 s. Find

a the magnitude of the impulse given to the body by the force,

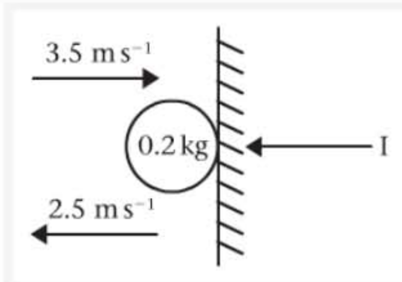
b the final speed of the body.

$v = 13.5 \text{ m s}^{-1}$

27 N s

Example

A ball of mass 0.2 kg hits a fixed vertical wall at right angles with speed 3.5 m s^{-1}. The ball rebounds with speed 2.5 m s^{-1}. Find the magnitude of the impulse exerted on the wall by the ball.



1.2 N s.

Exercises:

1 A ball of mass 0.5 kg is at rest when it is struck by a bat and receives an impulse of 15 N s. Find its speed immediately after it is struck.

2 A ball of mass 0.3 kg moving along a horizontal surface hits a fixed vertical wall at right angles with speed 3.5 m s^{-1}. The ball rebounds at right angles to the wall. Given that the magnitude of the impulse exerted on the ball by the wall is 1.8 N s, find the speed of the ball just after it has hit the wall.

3 A ball of mass 0.2 kg is dropped from a height of 2.5 m above horizontal ground. After hitting the ground it rises to a height of 1.8 m above the ground. Find the magnitude of the impulse received by the ball from the ground.

4 A ball of mass 0.2 kg, moving along a horizontal surface, hits a fixed vertical wall at right angles. The ball rebounds at right angles to the wall with speed 3.5 m s^{-1}. Given that the magnitude of the impulse exerted on the ball by the wall is 2 N s, find the speed of the ball just before it hit the wall.

5 A toy car of mass 0.2 kg is pushed from rest along a smooth horizontal floor by a horizontal force of magnitude 0.4 N for 1.5 s. Find its speed at the end of the 1.5 s.

1 30 m s^{-1}
2 2.5 m s^{-1}
3 2.59 N s
4 6.5m s^{-1}
5 3 m s^{-1}

Case 2: The force applied depends on the time

Going back to $F = \dfrac{dp}{dt}$

By integration between $t = 0$ *and* t, *we have*

$$\int_0^t F dt = \int_0^t dp$$

$$\int_0^t F dt = p_t - p_0$$

$$\boxed{\int_0^t F dt = mv - mu}$$

This also applied between any two times t_1 and t_2:

$$\int_{t_1}^{t_2} F dt = mv_2 - mv_1$$

Example

A particle P of mass 0.5 kg is moving along the x-axis. At time t seconds the force acting on P has magnitude $(5t^2 + e^{0.2t})$ N and acts in the direction OP. When $t = 0$, P is at rest at O. Calculate

a the speed of P when $t = 2$,

When $t = 2$ the speed of P is $30.6\ \text{m s}^{-1}$ (3 s.f.).

Example

A pebble of mass 0.2 kg is moving on a smooth horizontal sheet of ice. At time t seconds (where $t \geqslant 0$) a horizontal force of magnitude $2t^2$ N and constant direction acts on the pebble. When $t = 0$ the pebble is moving in the same direction as the force and has speed $6\ \text{m s}^{-1}$. When $t = T$ the pebble has speed $36\ \text{m s}^{-1}$. Calculate the value of T.

$T = 2.08$ (3 s.f.)

Exercises:

1 A particle P of mass 0.2 kg is moving on the x-axis. At time t seconds P is x metres from the origin O. The force acting on P has magnitude $2\cos t$ N and acts in the direction OP. When $t = 0$, P is at rest at O. Calculate

a the speed of P when $t = 2$,

b the speed of P when $t = 3$,

c the time when P first comes to instantaneous rest,

8 A particle P of mass 1.5 kg is moving in a straight line. The particle is initially at rest at a point O on the line. At time t seconds (where $t \geqslant 0$) the force acting on P has magnitude $(3t + 8)$ N and acts in the direction OP. When $t = T$, P has speed 75 m s^{-1}. Calculate

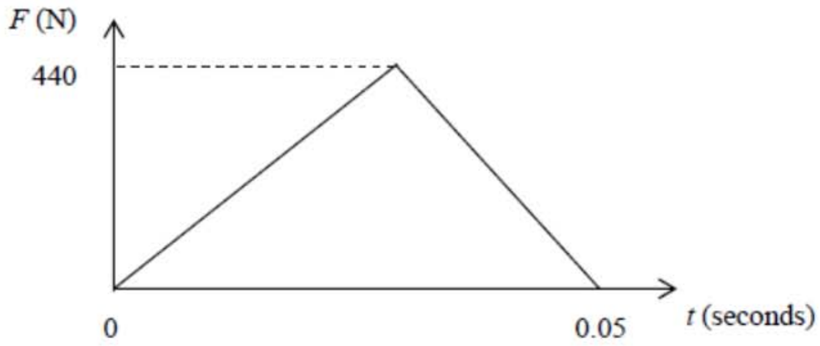
a the magnitude of the impulse exerted by the force between the times $t = 1$ and $t = 4$,

b the speed of P when $t = 3$,

c the value of T.

AQA exam question

A particle has mass 2 kg and moves in a straight line on a smooth horizontal surface. The particle strikes a vertical barrier that is perpendicular to its path and rebounds. The graph below shows how the magnitude of the force on the particle varies while it is in contact with the barrier.



(a) Calculate the magnitude of the impulse on the ball. *(2 marks)*

(b) The ball rebounds at a speed of 3 m s^{-1}. Find the speed of the ball when it hit the barrier. *(3 marks)*

1 **a** 9.09 m s^{-1} (3 s.f.) **b** 1.41 m s^{-1} (3 s.f.)
c P first comes to rest when $t = \pi$

8 **a** 46.5 N s **b** 25 m s^{-1}
c $T = 6.39$ (3 s.f.)

(a) $I = \frac{1}{2} \times 0.05 \times 440 = 11\text{ N s}$

(b) $11 = 2 \times 3 - 2(-u)$

$u = \frac{5}{2}\text{ m s}^{-1}$