General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MFP4

Unit Further Pure 4

Thursday 22 May 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.6 & -0.8 \\ 0 & 0.8 & -0.6 \end{bmatrix}$$

represents a rotation.

(a) State the axis of rotation.

[1 mark]

(b) Find the angle of rotation, giving your answer to the nearest degree.

[2 marks]

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2 (a) Factorise the determinant

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

as a product of three linear factors.

[4 marks]

(b) The matrices $\mathbf{A}$ and $\mathbf{B}$ are such that

$$\mathbf{AB} = \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix} \quad \text{and} \quad \det \mathbf{A} = z^2 - y^2$$

Given that $\det \mathbf{AB} \neq 0$, find and simplify an expression for $\det \mathbf{B}^{-1}$.

[3 marks]

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3 The matrix $\mathbf{M} = \begin{bmatrix} 1 & 4 & 2 \\ 3 & k & 3 \\ 2 & k & 1 \end{bmatrix}$, where k is a constant.

(a) Show that $\mathbf{M}$ is non-singular for all values of k.

[3 marks]

(b) Obtain $\mathbf{M}^{-1}$ in terms of k.

[5 marks]

(c) Use $\mathbf{M}^{-1}$ to solve the equations

$$\begin{aligned} x + 4y + 2z &= 25 \\ 3x + ky + 3z &= 3 \\ 2x + ky + z &= 2 \end{aligned}$$

giving your solution in terms of k.

[4 marks]

4 Three vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are such that $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, where $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{v} \neq \mathbf{w}$.

Show that $\mathbf{v} - \mathbf{w} = \lambda \mathbf{u}$, where λ is a scalar.

[3 marks]

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5 The points A, B, C and D have coordinates $(1, 3, p)$, $(4, 5, 2)$, $(2, 1, -1)$ and $(6, 5, 0)$ respectively, where p is a constant.

(a) Write down the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ in terms of p.

[2 marks]

(b) Show that $(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD}$ is of the form $m(5 - 2p)$, where m is an integer to be found.

[5 marks]

(c) In the case where $p = 2.5$, describe the configuration of the points A, B, C and D. Justify your answer.

[2 marks]

(d) In the case where the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ define the edges of a parallelepiped of volume 60, find the possible values of p.

[3 marks]

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6 The plane transformation S is a shear and is represented by the matrix $\begin{bmatrix} a & b \\ c & -2 \end{bmatrix}$, where a, b and c are constants.

(a) Show that $2a + bc = -1$.

[2 marks]

(b) Given further that $(2, 2)$ is an invariant point of S, find the values of a, b and c.

[4 marks]

(c) Show that all lines of the form $y = x + k$, where k is a constant, are invariant lines of S.

[3 marks]

Turn over ▶

7 The line l_1 has Cartesian equations

$$\frac{x-1}{4}=\frac{y-2}{-2}=\frac{z+1}{3}$$

and the line l_2 has vector equation

$$\mathbf{r}=\begin{bmatrix}4\\3\\c\end{bmatrix}+\lambda\begin{bmatrix}5\\-1\\0\end{bmatrix}$$

where c is a constant.

The plane Π_1 contains the lines l_1 and l_2.

(a) Show that an equation for the plane Π_1 is $x+5y+2z=d$, where d is an integer to be found.

[4 marks]

(b) Find the value of c.

[1 mark]

(c) The plane Π_2 has equation $2x-y+2z=4$.

(i) Find the acute angle between the planes Π_1 and Π_2, giving your answer to the nearest $0.1°$.

[4 marks]

(ii) Find an equation of the line of intersection of Π_1 and Π_2, giving your answer in the form

$$(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$$

[5 marks]

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8 The matrix $\mathbf{M}$ is given by

$$\mathbf{M}=\begin{bmatrix}p & q\\ q & p\end{bmatrix}$$ where p and q are constants and $q\neq 0$.

(a) Find the eigenvalues of $\mathbf{M}$ in terms of p and q.

[4 marks]

(b) Find corresponding eigenvectors of $\mathbf{M}$.

[3 marks]

(c) Write down a matrix $\mathbf{U}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M}=\mathbf{U}\,\mathbf{D}\,\mathbf{U}^{-1}$.

[2 marks]

(d) Show that $\mathbf{M}^n=\mathbf{U}\,\mathbf{D}^n\,\mathbf{U}^{-1}$.

[2 marks]

(e) Given that $p=0.6$ and $q=0.4$ and $\mathbf{M}^n\to\mathbf{L}$ as $n\to\infty$, find the matrix $\mathbf{L}$.

[4 marks]

Q	Solution	Mark	Total	Comment
1 **a)**	x axis	**B1**	**1**	
b)	$\cos\theta = -0.6$ and $\sin\theta = 0.8$	**M1**		Values of sine **and** cosine correctly identified and use of inverse trig to find angle
	$\theta = 127^0$	**A1**	**2**	**SC1 –B1** for **NMS** or only $\cos\theta = -0.6$ seen. Accept -233^0 **NB** 53^0 scores **M0 A0** **CAO -** Must be to the nearest degree
	Total		**3**	

Q	Solution	Mark	Total	Comment
2 **a)**	Row 2 →row 2 – row 1 Row 3 →row 3 – row 1 $= \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$			
	$= (y-x)(z-x)\begin{vmatrix} 1 & y+x \\ 1 & z+x \end{vmatrix}$	**M1** **A1**		Finding one factor correctly Two factors correct – correct working
	$= (y-x)(z-x)(z-y)$	**m1** **A1**	**4**	Complete method to find third factor All correct – any equivalent form
b)	$(y-x)(z-x)(z-y) = (z^2-y^2) \times \det \mathbf{B}$	**M1**		Use of det **AB** = det **A** × det **B –** alternatives are det **A** = det **AB** × det $\mathbf{B}^{-1}$ or det $(\mathbf{AB})^{-1}$ = det $\mathbf{B}^{-1}$ × det $\mathbf{A}^{-1}$
	Hence det **B** = $\dfrac{(y-x)(z-x)(z-y)}{(z-y)(z+y)}$			
	det $\mathbf{B}^{-1} = \dfrac{(z-y)(z+y)}{(y-x)(z-x)(z-y)}$	**M1**		Correct use of det $\mathbf{B}^{-1} = \frac{1}{\det \mathbf{B}}$ to obtain their expression for det $\mathbf{B}^{-1}$. Numerator does not need to be factorised.
	det $\mathbf{B}^{-1} = \dfrac{(z+y)}{(y-x)(z-x)}$	**A1**	**3**	**CSO -** Fully correct with factor cancelled
	Total		**7**	

Q	Solution	Mark	Total	Comment
3a)	det M = $\begin{vmatrix} k & 3 \\ k & 1 \end{vmatrix} - 3\begin{vmatrix} 4 & 2 \\ k & 1 \end{vmatrix} + 2\begin{vmatrix} 4 & 2 \\ k & 3 \end{vmatrix}$	M1		Correct expansion by row or column
	$= (k - 3k) - 3(4 - 2k) + 2(12 - 2k)$			
	= 12	A1		**CAO**
	(Constant/Independent of k and) therefore can never equal zero – hence non singular	E1	3	Explanation – must refer to non-zero answer and **M1A1** must have been scored
b)	$\begin{bmatrix} -2k & 3 & k \\ 2k-4 & -3 & 8-k \\ 12-2k & 3 & k-12 \end{bmatrix}$	M1 A(2,1)		**M1** Cofactor matrix - one full row or column correct. **A1**at least six entries correct **A2** all entries correct
	$\mathbf{M}^{-1} = \frac{1}{12}\begin{bmatrix} -2k & 2k-4 & 12-2k \\ 3 & -3 & 3 \\ k & 8-k & k-12 \end{bmatrix}$	m1 A1F	5	**m1** Divide by determinant and transpose their matrix Follow through their determinant answer in part a) – must be non-zero
c)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 25 \\ 3 \\ 2 \end{pmatrix}$			
	$= \frac{1}{12}\begin{pmatrix} -50k + 6k - 12 + 24 - 4k \\ 75 - 9 + 6 \\ 25k + 24 - 3k + 2k - 24 \end{pmatrix}$	M1A1F		**M1 -** Attempt at $\mathbf{M}^{-1}$ **v –** One of their components correct – can be unsimplified **A1** Two of their components correct – can be unsimplified. Follow through their $\mathbf{M}^{-1}$
	$= \frac{1}{12}\begin{pmatrix} 12 - 48k \\ 72 \\ 24k \end{pmatrix}$	A1		Three components correct – terms collected
	$= \begin{pmatrix} 1 - 4k \\ 6 \\ 2k \end{pmatrix}$			
	Hence $x = 1 - 4k$ $y = 6$ $z = 2k$	A1	4	Fully correct and simplified – **CSO** Any method with does not use $\mathbf{M}^{-1}$ **v** scores zero marks
	Total		**12**	

Q	Solution	Mark	Total	Comment
4	**u** x **v** = **u** x **w** **u** x **v** - **u** x **w** = **0** **u** x (**v** – **w**) = **0**	**M1**		Collect together on one side and factorise – must include x sign and ether **v – w** or **w – v**
	(Either **u = 0 or v – w = 0** or) the angle between **u** and **v – w** is 0^0 or 180^0	**E1**		Correct deduction about the angle between the vectors **u** and **v – w.** Condone reference to parallel vectors.
	Given **u** ≠ **0** and **v** ≠ **w** hence **v – w** = λ **u**	**A1**	**3**	Fully correct proof
	Total		**3**	

Q	Solution	Mark	Total	Comment
5a)	$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \\ 2-p \end{pmatrix}$	M1		Any one vector correct
	$\overrightarrow{AC} = \begin{pmatrix} 1 \\ -2 \\ -1-p \end{pmatrix}$			
	$\overrightarrow{AD} = \begin{pmatrix} 5 \\ 2 \\ -p \end{pmatrix}$	A1	2	All three vectors fully correct and consistent with labels or clearly listed in the order stated.
b)	$(\overrightarrow{AB} \times \overrightarrow{AC}).\overrightarrow{AD}$ $= \begin{vmatrix} 5 & 3 & 1 \\ 2 & 2 & -2 \\ -p & 2-p & -1-p \end{vmatrix}$			
	$5\begin{vmatrix} 2 & -2 \\ 2-p & -1-p \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 2-p & -1-p \end{vmatrix}$ $-p\begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix}$	M1		**M1 -** Correct expansion of appropriate determinant by row or column
	$= 5(2-4p) - 2(-5-2p) - p(-6-2)$	m1A1		**m1** Expansion of 2 by 2 determinants with two correct. **A1** all three correct.
	$= 20 - 8p$	A1		Correct linear expression in p
	$= 4(5-2p)$, hence $m = 4$	A1	5	Correct value of m stated or implied by factorisation
	ALTERNATIVE for b) $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & 3 & 1 \\ j & 2 & -2 \\ k & 2-p & -1-p \end{vmatrix}$			
	$= \begin{pmatrix} 2-4p \\ 5+2p \\ -8 \end{pmatrix}$	(M1A1)		**M1** Use of vector product – two components correct. **A1** – all components correct
	$(\overrightarrow{AB} \times \overrightarrow{AC}).\overrightarrow{AD} = \begin{pmatrix} 2-4p \\ 5+2p \\ -8 \end{pmatrix}.\begin{pmatrix} 5 \\ 2 \\ -p \end{pmatrix}$			
	$= 20 - 8p$	(m1A1)		**m1** Scalar product – must obtain linear expression in p. **A1** Correct expression
	$= 4(5-2p)$, hence $m = 4$	(A1)	(5)	Correct value of m stated or implied by factorisation
c)	when $p = 2.5$ $(\overrightarrow{AB} \times \overrightarrow{AC}).\overrightarrow{AD} = 0$ and as this represents the volume of a parallelepiped	E1		Reference to volume being zero or triple scalar product being zero
	the points lie in a single plane (coplanar)	E1	2	Deduction about coplanar points
d)	$20 - 8p = \pm 60$			
	$p = -5$ or $p = 10$	M1 A1, A1	3	**M1** Attempt to solve **both** equations **A1** each answer **SC1–B1** for one solution $p = -5$ or $p = 10$

	Total		**12**	

Q	Solution	Mark	Total	Comments
6a)	Determinant = $-2a - bc$	**M1**		Correct evaluation of the determinant
	Determinant of **shear** = 1/**shear** leaves area unchanged hence $-2a - bc = 1$ Giving $2a + bc = -1$	**A1**	**2**	Set determinant equal to 1 with **justification** and manipulate correctly to obtain result
b)	Fixed point $\begin{pmatrix} a & b \\ c & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$			
	Gives $2a + 2b = 2$ and $2c - 4 = 2$ hence	**M1**		Correct use of fixed point to set up two equations
	$c = 3$	**A1**		
	$b = -3$	**A1**		**A1** each correct value
	$a = 4$	**A1**		
			4	
c)	$\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ x+k \end{pmatrix} = \begin{pmatrix} 4x - 3x - 3k \\ 3x - 2x - 2k \end{pmatrix}$	**M1**		Correct substitution and multiplication of their values form part b) – can be unsimplified
	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x - 3k \\ x - 2k \end{pmatrix}$			
	$x' + k = x - 3k + k$ $= x - 2k$ $= y'$	**m1**		Both components correctly simplified and attempt to show that $y' = x' + k$ works
	Hence $y' = x' + k$	**A1**	**3**	Fully shown – **CSO**
	ALTERNATIVE for c)			
	$\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ mx+k \end{pmatrix} = \begin{pmatrix} 4x - 3mx - 3k \\ 3x - 2mx - 2k \end{pmatrix}$	**(M1)**		Correct substitution and multiplication of their values form part b) – can be unsimplified
	Using $y' = mx' + k$ $3x - 2mx - 2k = m(4x - 3mx - 3k) + k$	**(m1)**		Substitution into $y' = mx' + k$ to obtain a quadratic in m – can be unsimplified
	Giving $3(m-1)^2x + 3(m-1)k = 0$			
	Hence $m = 1$ and k can take any value So $y' = x' + k$ is invariant	**(A1)**	**3**	Fully shown - **CSO**
	Total		**9**	

Q	Solution	Mark	Total	Comment
7a)	$\begin{pmatrix}4\\-2\\3\end{pmatrix} \times \begin{pmatrix}5\\-1\\0\end{pmatrix} = \begin{pmatrix}3\\15\\6\end{pmatrix}$	M1		Use of vector product – two components correct
	Hence **n** = $\begin{pmatrix}1\\5\\2\end{pmatrix}$	A1		Correct **n –** accept equivalents
	Hence plane is $x + 5y + 2z = d$ (1, 2, -1) on plane gives 1 + 10 – 2 = d Hence d = 9	m1 A1	4	Substitution of valid point on the plane to find d. Dependant on finding vector product first. Correct value of d - **CAO**
b)	Substitute x = 4 , y = 3 and z = c to get 4 + 15 + 2c = 9, hence c = - 5	B1	1	Substitute point to find correct value of c
c)i)	**a**= $\begin{pmatrix}1\\5\\2\end{pmatrix}$ and **b** = $\begin{pmatrix}2\\-1\\2\end{pmatrix}$			
	∴ **a.b** = 1	B1		Correct value of **a.b**
	$3\sqrt{30}\cos\theta = 1$	M1		Form appropriate scalar product equation using two normals to find cos θ
	$\cos\theta = \dfrac{1}{3\sqrt{30}}$	A1		cos θ correct
	$\theta = 86.5^0$	A1	4	Using $\cos^{-1}$ to find angle - **CAO**
	ALTERNATIVE to c)i)			
	∴ $\lvert \mathbf{a}\mathrm{x}\mathbf{b} \rvert = \sqrt{269}$	(B1)		Correct value of |**a**x**b** |
	$3\sqrt{30}\sin\theta = \sqrt{269}$	(M1)		Form appropriate vector product equation using two normals to find sin θ
	$\sin\theta = \dfrac{\sqrt{269}}{3\sqrt{30}}$	(A1)		sin θ correct
	$\theta = 86.5^0$	(A1)	(4)	Using $\sin^{-1}$ to find angle - **CAO**

Q	Solution	Mark	Total	Comment
7c)ii)	$\begin{pmatrix}1\\5\\2\end{pmatrix}$ X $\begin{pmatrix}2\\-1\\2\end{pmatrix} = \begin{pmatrix}12\\2\\-11\end{pmatrix}$	**M1** **A1**		**M1** Use of vector product – two components correct. **A1** all correct
	Common point $y = 0$ so $x + 2z = 9$ $2x + 2z = 4$	**M1**		Attempt to find common point set one variable = 0 (or other value)
	Gives (-5 , 0 , 7)	**A1**		Other common possibilities are $\left(0, \frac{5}{6}, \frac{29}{12}\right)$, $\left(\frac{29}{11}, \frac{14}{11}, 0\right)$ and $\left(1, 1, \frac{3}{2}\right)$
	$\left(\boldsymbol{r} - \begin{pmatrix}-5\\0\\7\end{pmatrix}\right) \times \begin{pmatrix}12\\2\\-11\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$	**A1F**	**5**	Correct format –their **a** and **b** placed in correct positions – must have scored **both M1**s above
	ALTERNATIVE for 7c)ii) Eqn 1 $x + 5y + 2z = 9$ Eqn 2 $2x - y + 2z = 4$ 2 x Eqn 1 – Eqn 2 gives $11y + 2z = 14$			
	Let $z = t$	**(M1)**		Set one variable to a parameter and attempt to find other variables
	Hence $y = \frac{14-2t}{11}$	**(A1)**		Correct expression for x
	and $x = \frac{29-12t}{11}$	**(A1)**		Correct expression for y
	$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}29/11\\14/11\\0\end{pmatrix} + t\begin{pmatrix}-12/11\\-2/11\\1\end{pmatrix}$	**(M1)**		Rewriting to identify point and direction – can be implied
	Hence $\left(\boldsymbol{r} - \begin{pmatrix}29/11\\14/11\\0\end{pmatrix}\right) \times \begin{pmatrix}-12\\-2\\11\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$	**(A1F)**	**(5)**	Correct format –their **a** and **b** placed in correct positions – must have scored **both M1**s above
	Total		**14**	

Q	Solution	Mark	Total	Comment
8a)	Det ($\mathbf{M} - \lambda\mathbf{I}$) = 0 $\therefore (p-\lambda)^2 - q^2 = 0$ $\therefore p - \lambda = \pm q$ Hence eigenvalues are $p+q$, $p-q$	M1 m1 A1,A1	4	Correct characteristic equation obtained Correct method used to solve equation to obtain **two distinct** solutions **A1** each correct eigenvalue expression
b)	$\lambda = p+q$ $\therefore \begin{pmatrix} -q & q \\ q & -q \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	M1		**Correct** equation seen or implied by correct matrix equation. Either $-qx + qy = 0$ or $qx + qy = 0$
	Hence $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector	A1		One eigenvector correct
	$\lambda = p - q$ $\therefore \begin{pmatrix} q & q \\ q & q \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$			
	Hence $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is an eigenvector	A1	3	Second eigenvector correct
c)	$\mathbf{D} = \begin{pmatrix} p+q & 0 \\ 0 & p-q \end{pmatrix}$	B1F		Use of their eigenvalues– must have first **M1** in part a)
	$\mathbf{U} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	B1F	2	Use of their numerical eigenvectors – must have first **M1**from part a) **and** part b**).** Columns must correspond to **D**
d)	$\mathbf{M} = \mathbf{UDU}^{-1}$ Combines $\mathbf{M}^n = \mathbf{UDU}^{-1}\,\mathbf{UDU}^{-1}\ldots\mathbf{UDU}^{-1}$ and $\mathbf{U}^{-1}\mathbf{U} = \mathbf{I}$ to simplify	M1		**M1 -** Idea of repeated triples and attempt at simplification using $\mathbf{U}^{-1}\mathbf{U} = \mathbf{I}$. Allow **M1** if shown for a particular value of n.
	Extends the process fully to give $\mathbf{M}^n = \mathbf{U\,DDD}\ldots\mathbf{U}^{-1} = \mathbf{UD}^n\mathbf{U}^{-1}$	A1	2	**A1** Repeated use of $\mathbf{U}^{-1}\mathbf{U} = \mathbf{I}$ and hence result
e)	$\mathbf{U}^{-1} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$	B1		Correct $\mathbf{U}^{-1}$ seen and used in part d) or e)
	Use of $p + q = 1$ and $p - q = 0.2$			
	$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.2^n \end{pmatrix}\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$	M1		**M1** Multiplying three appropriate matrices ($\mathbf{UD}^n\mathbf{U}^{-1}$) together
	$= \frac{1}{2}\begin{pmatrix} 1+0.2^n & 1-0.2^n \\ 1-0.2^n & 1+0.2^n \end{pmatrix}$	A1		Obtains the correct single 2 by 2 matrix
	$\mathbf{L} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$	A1	4	Correct **L** obtained by letting n approach infinity **- CSO**

	ALTERNATIVE for e) $\mathbf{U}^{-1} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$ Use of $p + q = 1$ and $p - q = 0.2$ $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix}$ $\mathbf{L} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$	**(B1)** **(M1)** **(A1)** **(A1)**	**(4)**	Correct $\mathbf{U}^{-1}$ seen to be used in part d) or e) **M1** Multiplying three appropriate numerical matrices ($\mathbf{U}\mathbf{D}^n\mathbf{U}^{-1}$) to get a single 2 by 2 matrix **A1** $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ identified as $\mathbf{D}^n$ Correct **L - CSO**
	NB – ALTERNATIVES for c) $\mathbf{D} = \begin{pmatrix} p-q & 0 \\ 0 & p+q \end{pmatrix}$ $\mathbf{U} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$	**(B1F)** **(B1F)**	**(2)**	Use of their eigenvalues– must have first **M1** in part a) Use of their numerical eigenvectors – must have first **M1**from part a) **and** part b**).** Columns must correspond to **D**
	Correspondingly in e) $\mathbf{U}^{-1} = \begin{pmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{pmatrix}$ Limiting $\mathbf{D}^n = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ Method as before leading to answer $\mathbf{L} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$	**(B1)** **(M1)** **(A1)** **(A1)**	**(4)**	Correct $\mathbf{U}^{-1}$ seen to be used in part d) or e) Marks allocated as above dependant on method chosen
	Total		**15**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2014 exams

A-level

Code	Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points A*	A	B	C	D	E
LAW01	LAW UNIT 1	96	-	77	69	62	55	48
LAW02	LAW UNIT 2	94	-	72	63	54	45	37
LAW03	LAW UNIT 3	80	69	63	58	53	48	43
LAW04	LAW UNIT 4	85	71	64	58	53	48	43
MD01	MATHEMATICS UNIT MD01	75	-	61	55	50	45	40
MD02	MATHEMATICS UNIT MD02	75	69	63	57	52	47	42
MFP1	MATHEMATICS UNIT MFP1	75	-	55	48	41	35	29
MFP2	MATHEMATICS UNIT MFP2	75	63	56	49	43	37	31
MFP3	MATHEMATICS UNIT MFP3	75	65	60	55	50	45	41
MFP4	MATHEMATICS UNIT MFP4	75	70	66	59	53	47	41
MM03	MATHEMATICS UNIT MM03	75	71	68	61	54	47	40
MM04	MATHEMATICS UNIT MM04	75	68	61	53	45	38	31
MM05	MATHEMATICS UNIT MM05	75	67	59	51	43	35	27
MM1B	MATHEMATICS UNIT MM1B	75	-	53	46	40	34	28
MM2B	MATHEMATICS UNIT MM2B	75	68	62	55	48	41	35
MPC1	MATHEMATICS UNIT MPC1	75	-	62	56	50	44	38
MPC2	MATHEMATICS UNIT MPC2	75	-	55	49	43	37	32
MPC3	MATHEMATICS UNIT MPC3	75	65	59	53	47	41	36
MPC4	MATHEMATICS UNIT MPC4	75	59	54	49	44	39	34
MS03	MATHEMATICS UNIT MS03	75	68	62	55	48	41	35
MS04	MATHEMATICS UNIT MS04	75	67	60	52	44	37	30
MS1A	MATHEMATICS UNIT MS1A	100	-	85	75	66	57	48
MS1A/W	*MATHEMATICS UNIT MS1A - WRITTEN*	*75*		*65*				*38*
MS1A/C	*MATHEMATICS UNIT MS1A - COURSEWORK*	*25*		*20*				*10*