General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MFP4

Unit Further Pure 4

Tuesday 18 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The points A, B, C and D have position vectors **a**, **b**, **c** and **d** respectively relative to the origin O, where

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$$

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. *(3 marks)*

(b) The points A, B and C lie in the plane Π. Find a Cartesian equation for Π. *(2 marks)*

(c) Find the volume of the parallelepiped defined by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$. *(3 marks)*

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2 The system of equations

$$\begin{aligned} 2x - y - z &= 3 \\ x + 2y - 3z &= 4 \\ 2x + y + az &= b \end{aligned}$$

does not have a unique solution.

(a) Show that $a = -3$. *(3 marks)*

(b) Given further that the equations are inconsistent, find the possible values of b. *(2 marks)*

(c) State, with a reason, whether the vectors $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ -3 \\ -3 \end{bmatrix}$ are linearly dependent or linearly independent. *(1 mark)*

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3 The determinant Δ is given by

$$\Delta = \begin{vmatrix} x^2 - x & y^2 - y & z^2 - z \\ x & y & z \\ x^2 + y^2 + z^2 & x^2 + y^2 + z^2 & x^2 + y^2 + z^2 \end{vmatrix}$$

where x, y and z are distinct real numbers.

(a) Express Δ as a product of one quadratic factor and three linear factors. *(6 marks)*

(b) Deduce that $\Delta \neq 0$. *(2 marks)*

4 Two planes have equations

$$2x - 2y + z = 24$$

and $$x + 3y + 4z = 8$$

They meet in a line L.

(a) Find Cartesian equations for the line L. *(5 marks)*

(b) The direction cosines of the line L are given by $\cos\alpha$, $\cos\beta$ and $\cos\gamma$.

(i) Find the exact value of each of the direction cosines. *(2 marks)*

(ii) Show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$. *(3 marks)*

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5 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$$

(a) Show that $\lambda = 2$ is an eigenvalue for $\mathbf{M}$, and find the other two eigenvalues. *(5 marks)*

(b) Find an eigenvector that corresponds to $\lambda = 2$. *(3 marks)*

(c) The matrix $\mathbf{N}$ is given by

$$\mathbf{N} = \begin{bmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{bmatrix}$$

(i) Show that $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector for $\mathbf{N}$, and find the corresponding eigenvalue. *(2 marks)*

(ii) Hence state one eigenvector for the matrix $\mathbf{MN}$, and find the corresponding eigenvalue. *(3 marks)*

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6 The plane transformation T is defined by

$$\mathrm{T} : \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) A shape has an area of 3 square units. Find the area of the shape after being transformed by T. *(2 marks)*

Turn over ▶

(b) (i) Find the equations of all the invariant lines of T. *(5 marks)*

(ii) State the equation of the line of invariant points of T. *(1 mark)*

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7 The 3×3 matrices $\mathbf{A}$ and $\mathbf{B}$ satisfy

$$\mathbf{AB} = \begin{bmatrix} k & 8 & 1 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix}, \text{ where } \mathbf{A} = \begin{bmatrix} k & 6 & 8 \\ 0 & 1 & 2 \\ -3 & 4 & 8 \end{bmatrix}$$

and k is a constant.

(a) Show that $\mathbf{AB}$ is non-singular. *(1 mark)*

(b) Find $(\mathbf{AB})^{-1}$ in terms of k. *(5 marks)*

(c) Find $\mathbf{B}^{-1}$. *(4 marks)*

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8 A line and a plane have equations

$$\frac{x-3}{p} = \frac{y-q}{3} = \frac{z-1}{-1}$$

and $$\mathbf{r} \, . \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 10$$

respectively, where p and q are constants.

(a) Show that the line is **not** perpendicular to the plane. *(1 mark)*

(b) In the case where the line lies in the plane, find the values of p and q. *(4 marks)*

(c) In the case where the angle, θ, between the line and the plane satisfies $\sin\theta = \frac{1}{\sqrt{6}}$, and the line intersects the plane at $z = 2$:

(i) find the value of p; *(5 marks)*

(ii) find the value of q. *(2 marks)*

Q	Solution	Marks	Total	Comments
1(a)	$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & 2 & -2 \\ \mathbf{j} & 2 & -2 \\ \mathbf{k} & 3 & 5 \end{vmatrix} = \begin{pmatrix} 16 \\ -16 \\ 0 \end{pmatrix}$	B1		$\overrightarrow{AB}$ or $\overrightarrow{AC}$ correct
		M1		Attempt at cross product – at least one component correct
		A1	3	All correct components – no further working seen or attempted
(b)	$16:-16:\ 0 = 1:-1:\ 0$ Cartesian equation is $x-y=$ constant	M1		Correct structure using their perpendicular from a) : $ax+by+cz=d$
	Use $\mathbf{c} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} \Rightarrow$ constant $= -1$	A1	2	Finding correct value of d - CAO
	$\therefore\ x-y=-1$			
	Alternative:			
	$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}$			
	$x = 1 + 2\mu - 2\lambda$ ① $y = 2 + 2\mu - 2\lambda$ ②	(M1)		Correct parametric structure with an attempt to eliminate variables
	$z = -1 + 3\mu + 5\lambda$ ③			
	① – ② $\Rightarrow\ x-y=-1$	(A1)	(2)	
(c)	$\overrightarrow{AD} = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$	B1		Obtaining $\overrightarrow{AD}$ or $\overrightarrow{DA}$
	$\overrightarrow{AB}\times\overrightarrow{AC}\,.\,\overrightarrow{AD} = \begin{pmatrix} 16 \\ -16 \\ 0 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$	M1		Scalar product with their $\overrightarrow{AD}$ and answer from part (a) or determinant seen $\overrightarrow{AB}\times\overrightarrow{AC}\,.\,\overrightarrow{AD} = \begin{vmatrix} 2 & -2 & 3 \\ 2 & -2 & -1 \\ 3 & 5 & -1 \end{vmatrix}$
	$= 64$ (cubic units)	A1	3	CAO
	Total		**8**	

Q	Solution	Marks	Total	Comments
2(a)	$\left[\begin{array}{ccc\|c} 2 & -1 & -1 & 3 \\ 1 & 2 & -3 & 4 \\ 2 & 1 & a & b \end{array}\right]$			
	$r_3 \to r_3 - r_1$, $r_2 \to 2r_2 - r_1$ $\left[\begin{array}{ccc\|c} 2 & -1 & -1 & 3 \\ 0 & 5 & -5 & 5 \\ 0 & 2 & a+1 & b-3 \end{array}\right]$	M1		Correct row operations used to create two zeros in first column – coefficients must be correct
	$r_3 \to 5r_3 - 2r_2$ $\left[\begin{array}{ccc\|c} 2 & -1 & -1 & 3 \\ 0 & 5 & -5 & 5 \\ 0 & 0 & 5a+15 & 5b-25 \end{array}\right]$	M1		Use of row operations to create third zero in second column or compare ratios of coefficients in rows 2 and 3
	No unique solution: $5a + 15 = 0$ $a = -3$	A1	3	Solves equation with coefficient of $z = 0$ or equation formed from comparison of ratios (eg $a + 1 = -2$) $a = -3$ is a printed answer
	Alternative 1:			
	$2x - y - z = 3$ ① $x + 2y - 3z = 4$ ② $2x + y + az = b$ ③			
	① + ③ $\Rightarrow$ $4x + (a-1)z = b + 3$ ④ 2③ – ② $\Rightarrow$ $3x + (2a+3)z = 2b - 4$ ⑤	(M1)		Correct elimination of 1 variable. Coefficients must be correct.
	4⑤ –3④ $\Rightarrow (5a + 15)z = 5b - 25$	(M1)		Correctly reduce to one equation with a, b
	$5a + 15 = 0$ $a = -3$	(A1)	(3)	Solves equation with coefficient of $z = 0$
	Alternative 2:			
	Solve $\begin{vmatrix} 2 & -1 & -1 \\ 1 & 2 & -3 \\ 2 & 1 & a \end{vmatrix} = 0$			
	$2\begin{vmatrix} 2 & -3 \\ 1 & a \end{vmatrix} - 1\begin{vmatrix} -1 & -1 \\ 1 & a \end{vmatrix} + 2\begin{vmatrix} -1 & -1 \\ 2 & -3 \end{vmatrix} = 0$	(M1)		Correct expansion by row or column
	$2(2a+3) - (-a+1) + 2(5) = 0$	(M1)		Correct expansion of 2 by 2 determinants
	$5a + 15 = 0$ $a = -3$	(A1)	(3)	Solves equation with determinant = 0

Q	Solution	Marks	Total	Comments
2(b)	Either $5b-25\neq 0$ or $5b-25=0$ Inconsistent $b\neq 5$	M1 A1	2	Sets their constant $\neq 0$ (or 0) CSO (accept $b>5$, $b<5$)
(c)	Linearly dependent since determinant/triple scalar product $=0$ or $\begin{pmatrix}2\\1\\2\end{pmatrix}+\begin{pmatrix}-1\\2\\1\end{pmatrix}+\begin{pmatrix}-1\\-3\\-3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$	E1	1	Correct deduction with appropriate reason given
	Total		**6**	
3(a)	First factor (quadratic) $=x^2+y^2+z^2$ $\left(x^2+y^2+z^2\right)\begin{vmatrix}x^2-x & y^2-y & z^2-z\\ x & y & z\\ 1 & 1 & 1\end{vmatrix}$	B1		Quadratic factor identified anywhere
	$C_2\to C_2-C_1$ $C_3\to C_3-C_1$ $\left(x^2+y^2+z^2\right)$ $\begin{vmatrix}x^2-x & y^2-x^2-(y-x) & z^2-x^2-(z-x)\\ x & y-x & z-x\\ 1 & 0 & 0\end{vmatrix}$	M1		Correct use of a column/row operation to obtain first linear factor – no more than one slip
	Two linear factors $(y-x)$ and $(z-x)$ $\left(x^2+y^2+z^2\right)(y-x)(z-x)\begin{vmatrix}x^2-x & y+x-1 & z+x-1\\ x & 1 & 1\\ 1 & 0 & 0\end{vmatrix}$	A1,A1		Two correct linear factors found $(y-x)$ $(z-x)$ or equivalent
	Expand to get $\left(x^2+y^2+z^2\right)(y-x)(z-x)(y-z)$	M1 A1	6	Finding the final linear factor $(y-z)$ or equivalent All correct - CSO
(b)	$x,\ y,\ z$ distinct $\Rightarrow$ $x\neq y\neq z$ $\Rightarrow$ $(y-x)(z-x)(y-z)\neq 0$	E1		Explaining why linear factors are $\neq 0$
	$x, y,\ z$ distinct, real $\Rightarrow$ $x^2+y^2+z^2\neq 0$ $\Rightarrow$ $\Delta\neq 0$	E1	2	Explaining why the quadratic factor is $\neq 0$
	Total		**8**	

Q	Solution	Marks	Total	Comments
4(a)	direction vector $=\begin{vmatrix} \mathbf{i} & 2 & 1 \\ \mathbf{j} & -2 & 3 \\ \mathbf{k} & 1 & 4 \end{vmatrix} = \begin{pmatrix} -11 \\ -7 \\ 8 \end{pmatrix}$	M1A1		Finding direction of line M1 for attempt at vector product – one component correct
	common point, let $z = 0$ $\left.\begin{array}{l} x-y=12 \\ x+3y=8 \end{array}\right\}$ $\begin{array}{l} y=-1 \\ x=11 \end{array}$	M1A1		Finding common point - M1 for letting $z = 0$ and attempt to solve or equivalent (x = 0 gives y = -8, z = 8 and y = 0 gives x = $\frac{88}{7}$ and z = $-\frac{8}{7}$)
	So line is $\dfrac{x-11}{-11} = \dfrac{y+1}{-7} = \dfrac{z}{8}$	A1	5	CAO – any correct equivalent form
	Alternative 1 :			
	Let $z = \lambda$	(M1)		Let $z = \lambda$ and attempt to solve for x, y
	Then $y = -1 - \dfrac{7\lambda}{8}$	(A1)		For y correct
	and $x = 11 - \dfrac{11\lambda}{8}$	(A1)		For x correct
	Gives point $(11, -1, 0)$ and direction $\begin{pmatrix} -11 \\ -7 \\ 8 \end{pmatrix}$	(M1)		Elimination of parameter
	$\Rightarrow \dfrac{x-11}{-11} = \dfrac{y+1}{-7} = \dfrac{z}{8}$	(A1)	(5)	CAO – any correct equivalent form
	Alternative 2 :			
	Let $z = \lambda$	(M1)		Let $z = \lambda$ and attempt to express λ in terms of x, y
	Then $\lambda = \dfrac{8y+8}{-7}$	(A1)		Correct expression in terms of y only
	and $\lambda = \dfrac{8x-88}{-11}$	(A1)		Correct expression in terms of x only
	Hence $\dfrac{8x-88}{-11} = \dfrac{8y+8}{-7} = z$	(M1) (A1)	(5)	Elimination of parameter CAO – any equivalent form
(b)(i)	$\sqrt{11^2 + 7^2 + 8^2} = \sqrt{234}$	M1		Modulus of their direction vector seen and correct structure used for direction cosines
	$\cos\alpha = \dfrac{-11}{\sqrt{234}}$, $\cos\beta = \dfrac{-7}{\sqrt{234}}$, $\cos\gamma = \dfrac{8}{\sqrt{234}}$	A1F	2	ft error in their direction vector

Q	Solution	Marks	Total	Comments
4(b)(ii)	$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$	B1		Seen / stated
	$\Rightarrow 1-\sin^2\alpha+1-\sin^2\beta+1-\sin^2\gamma=1$	M1		Trig identity $\cos^2\theta=1-\sin^2\theta$ used
	$\Rightarrow 3-\sin^2\alpha-\sin^2\beta-\sin^2\gamma=1$			
	$\Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=3-1=2$	A1	3	All correct
	Alternative:			
	$\sin^2\alpha=\frac{113}{234}$, $\sin^2\beta=\frac{185}{234}$, $\sin^2\gamma=\frac{170}{234}$	(M1) (A1F)		Attempt to get all of $\sin^2\alpha, \sin^2\beta, \sin^2\gamma$ All correct – ft their direction vector
	$\Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=\frac{113+185+170}{234}$			
	$=\frac{468}{234}=2$	(B1)	(3)	Correct verification (CSO) – must see explicit calculation to arrive at 2
	Total		**10**	
5(a)	$\begin{vmatrix}1-\lambda & 1 & 2\\ 0 & 2-\lambda & 2\\ -1 & 1 & 3-\lambda\end{vmatrix}=0$			
	$(1-\lambda)\begin{vmatrix}2-\lambda & 2\\ 1 & 3-\lambda\end{vmatrix}-\begin{vmatrix}1 & 2\\ 2-\lambda & 2\end{vmatrix}=0$	M1		Correct row/column expansion of $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$
	$(1-\lambda)(\lambda-4)(\lambda-1)-2(\lambda-1)=0$ $(1-\lambda)[(2-\lambda)(3-\lambda)-2]-[2-2(2-\lambda)]=0$	m1		Correct expansion of 2 by 2 determinants – dependent on first M1
	$[\lambda-1][-\lambda^2+5\lambda-6]=0$	A1		or $-\lambda^3+6\lambda^2-11\lambda+6=0$
	$-(\lambda-1)(\lambda-2)(\lambda-3)=0$	M1		Attempt to show $\lambda=2$ is an eigenvalue
	$\lambda=1,\ 2$ or 3	A1	5	Fully correct eigenvalues - CAO
(b)	$\begin{bmatrix}-1 & 1 & 2\\ 0 & 0 & 2\\ -1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$	M1		Attempt to solve $(\mathbf{M}-2\mathbf{I})\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$
	$\Rightarrow 2z=0$, $-x+y+z=0$ $\Rightarrow z=0$, $x=y$	A1		Both relationships obtained (can be unsimplified)
	$\Rightarrow \begin{pmatrix}1\\ 1\\ 0\end{pmatrix}$ is an eigenvector	A1	3	Stated clearly; accept multiples

Q	Solution	Marks	Total	Comments
5(c)(i)	$\begin{pmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	M1		Attempt at $\mathbf{N}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$
	$\Rightarrow \lambda = 4$ is an eigenvalue	A1	2	
(ii)	Eigenvector = $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	B1		Accept multiples – part b) must be correct
	Eigenvalue $= 2 \times 4 = 8$	M1A1	3	M1 for product of relevant eigenvalues
	Alternative for (c)(ii)			
	Eigenvector = $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	(B1)		Accept multiples – part b) must be correct
	$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 10 & 6 \\ -2 & 10 & 12 \\ -9 & 9 & 15 \end{pmatrix}$			
	$\begin{pmatrix} -2 & 10 & 6 \\ -2 & 10 & 12 \\ -9 & 9 & 15 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ 0 \end{pmatrix} = 8\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	(M1)		Multiplies to get MN and attempts to find eigenvalue
	So eigenvalue is 8	(A1)	(3)	Correct eigenvalue
	Total		**13**	

Q	Solution	Marks	Total	Comments
6(a)	Determinant of matrix $= -8+9=1$	M1		Finding determinant and multiplying by area
	Area $= 3\times 1 = 3$ (square units)	A1	2	CAO – must show multiplication or refer to scale factor/invariant area or equivalent
(b)(i)	$\begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} x \\ mx+c \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $\Rightarrow$ $(x') = 4x + 3(mx+c)$ $(y') = -3x - 2(mx+c)$	M1		x', y' in terms of x, y, m, c
	Invariant lines $\Rightarrow y' = mx' + c$ $\Rightarrow -3x - 2mx - 2c = 4mx + 3m^2x + 3mc + c$	A1		Use of $y' = mx' + c$
	$\Rightarrow\ 0 = \left(3m^2 + 6m + 3\right)x + 3mc + 3c$ $\Rightarrow\ 3m^2 + 6m + 3 = 0 \quad 3mc + 3c = 0$	M1		Attempt at solving equations where coefficients = 0 or compares coefficients
	$3(m+1)^2 = 0 \qquad 3c(m+1) = 0$ $\Rightarrow\ m = -1 \qquad c$ can be any value	A1		Finding the correct value of m
	$\Rightarrow$ lines are $y = -x + c$	A1	5	Fully correct line – no restriction on c
(ii)	When $c = 0$, $y = -x$ is a line of invariant points	B1	1	Any equivalent form
	SPECIAL CASES – (b)(i) $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ -x+c \end{pmatrix} = \begin{pmatrix} x+3c \\ -x-2c \end{pmatrix}$			SC1 – Correct multiplication as shown
	$x' = x + 3c$ $y' = -x - 2c$ Consider $-x' + c$ $= -(x+3c) + c$ $= -x - 3c + c$ $= -x - 2c$ $= y'$ Hence $y = -x + c$ is an invariant line			SC2 – correct multiplication as shown above and full algebraic solution using $y' = -x' + c$
	Total		**8**	

Q	Solution	Marks	Total	Comments
7(a)	$\text{Det}(AB) = (4)(1) - (1)(1) = 3 \neq 0$	B1	1	Must state non-zero or $\neq 0$
(b)	Matrix of cofactors $\begin{bmatrix} 0 & 0 & 3 \\ 4 & -1 & 8-4k \\ -1 & 1 & k-8 \end{bmatrix}$	M1 A1		Attempt at matrix of cofactors – at least five correct entries Fully correct matrix of cofactors
	$(AB)^{-1} = \frac{1}{3}\begin{bmatrix} 0 & 4 & -1 \\ 0 & -1 & 1 \\ 3 & 8-4k & k-8 \end{bmatrix}$	M1 A2,1	 5	Their cofactor matrix transposed correctly At least five correct = A1 (exclude effect of determinant) All entries fully correct = A2
(c)	$(AB)^{-1} = B^{-1}A^{-1}$ $\Rightarrow B^{-1} = (AB)^{-1}A$			
	$B^{-1} = \frac{1}{3}\begin{bmatrix} 0 & 4 & -1 \\ 0 & -1 & 1 \\ 3 & 8-4k & k-8 \end{bmatrix}\begin{bmatrix} k & 6 & 8 \\ 0 & 1 & 2 \\ -3 & 4 & 8 \end{bmatrix}$	M1 A1		Use of $(AB)^{-1}$ and A multiplied Correct order of multiplication
	$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ -3 & 3 & 6 \\ 24 & -6 & -24 \end{bmatrix}$			
	$= \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ 8 & -2 & -8 \end{bmatrix}$	A2,1	4	All correct = A2 5+ correct = A1 (exclude effect of determinant) NB – if an attempt is made to find B by setting up a system of simultaneous equations then M1 – 9 correct equations used A1 for $B = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} & -\frac{1}{4} \end{pmatrix}$ Final A2 as above
	Total		**10**	

Q	Solution	Marks	Total	Comments
8(a)	direction ratios of line $= p : 3 : -1$ normal to plane $= 1 : 1 : 2$ } not equal	B1	1	Accept not parallel or showing vector product is non zero
(b)	$x = 3 + pt$ $y = q + 3t$ $z = 1 - t$	M1		Parametric form seen
	Meets plane $\Rightarrow 3 + pt + q + 3t + 2(1 - t) = 10$	A1		Correct substitution in plane
	$\Rightarrow (5 + q) + t(p + 1) = 10$ Within plane $\Rightarrow q = 5,\ p = -1$	M1A1	4	M1 - Finding one correct value A1 - Both values correct
	Alternative Point (3, q, 1) is common to line and plane Hence $\begin{pmatrix}3\\q\\1\end{pmatrix}.\begin{pmatrix}1\\1\\2\end{pmatrix}=10$ which gives q = 5	(M1A1)		Uses common point to find q
	Another point common to both is (3 + p, 8, 0) Hence $\begin{pmatrix}3+p\\8\\0\end{pmatrix}.\begin{pmatrix}1\\1\\2\end{pmatrix}=10$ which gives p = -1	(M1A1)	(4)	Use of second point and value of q to find p or consideration of scalar product $\begin{pmatrix}p\\3\\-1\end{pmatrix}.\begin{pmatrix}1\\1\\2\end{pmatrix}=0$
(c)(i)	**n** = $\begin{pmatrix}1\\1\\2\end{pmatrix}$ **d** = $\begin{pmatrix}p\\3\\-1\end{pmatrix}$ Let α be angle between normal (plane) and direction ratios (line)			
	$\mathbf{n}.\mathbf{d} = p + 1$	M1		**n . d** correct
	$\sin\theta = \frac{1}{\sqrt{6}} \Rightarrow \cos\alpha = \frac{\pm 1}{\sqrt{6}}$	B1		Correct cos α stated or implied
	$\Rightarrow \frac{p+1}{\sqrt{6}\sqrt{p^2+10}} = \frac{\pm 1}{\sqrt{6}}$	m1 A1		Forming equation connecting all relevant parts **and** attempting to solve for p (condone missing $\pm$) Dependent on first M1 – fully correct for A1
	$\Rightarrow (p+1)^2 = p^2 + 10$ $\Rightarrow p^2 + 2p + 1 = p^2 + 10$ $\Rightarrow 2p = 9$ giving $p = 4.5$	A1	5	CAO

Q	Solution	Marks	Total	Comments
8(c)(ii)	$z = 2 \Rightarrow t = -1 \Rightarrow x = -1.5$ $p = 4.5 \quad y = q - 3$ $\Rightarrow -1.5 + q - 3 + 4 = 10$	M1		Attempt to form an equation for *q* using $t = -1$
	$q = 10.5$	A1	2	CAO
	Alternative for (c)(i)			
	$\lvert\mathbf{n}\times\mathbf{d}\rvert = \sqrt{49 + (1+2p)^2 + (3-p)^2}$	(M1)		$\lvert\mathbf{n}\times\mathbf{d}\rvert$ correct
	$\sin\theta = \frac{1}{\sqrt{6}} \Rightarrow \sin\alpha = \frac{\sqrt{5}}{\sqrt{6}}$	(B1)		Correct $\sin\alpha$ stated or implied
	$\frac{\sqrt{49 + (1+2p)^2 + (3-p)^2}}{\sqrt{6}\sqrt{p^2+10}} = \frac{\sqrt{5}}{\sqrt{6}}$	(m1A1)		Forming equation connecting all relevant parts **and** attempting to solve for *p*. Dependent on first M1 – fully correct for A1
	Leading to $p = 4.5$	(A1)	(5)	CAO
	Total		**12**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2013 exams

A-level

Code	Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points A*	A	B	C	D	E
LAW02	LAW UNIT 2	94	-	77	69	61	53	45
LAW03	LAW UNIT 3	80	69	63	57	52	47	42
LAW04	LAW UNIT 4	85	73	67	61	56	51	46
MD01	MATHEMATICS UNIT MD01	75	-	64	59	54	50	46
MD02	MATHEMATICS UNIT MD02	75	69	64	56	48	41	34
MFP1	MATHEMATICS UNIT MFP1	75	-	55	49	43	37	32
MFP2	MATHEMATICS UNIT MFP2	75	65	61	54	47	40	34
MFP3	MATHEMATICS UNIT MFP3	75	67	64	56	49	42	35
MFP4	MATHEMATICS UNIT MFP4	75	64	60	52	44	36	28
MM1B	MATHEMATICS UNIT MM1B	75	-	56	49	42	35	29
MM2B	MATHEMATICS UNIT MM2B	75	67	61	55	49	43	37
MM03	MATHEMATICS UNIT MM03	75	70	65	58	51	44	37
MM04	MATHEMATICS UNIT MM04	75	67	59	51	44	37	30
MM05	MATHEMATICS UNIT MM05	75	68	61	52	44	36	28
MPC1	MATHEMATICS UNIT MPC1	75	-	59	53	47	41	36
MPC2	MATHEMATICS UNIT MPC2	75	-	61	55	49	43	37
MPC3	MATHEMATICS UNIT MPC3	75	66	60	54	49	44	39
MPC4	MATHEMATICS UNIT MPC4	75	60	55	50	45	40	35
MS1A	MATHEMATICS UNIT MS1A	100	-	76	67	59	51	43
MS/SS1A/W	*MATHEMATICS UNIT S1A - WRITTEN*	*75*		*56*				*33*
MS/SS1A/C	*MATHEMATICS UNIT S1A - COURSEWORK*	*25*		*20*				*10*
MS1B	MATHEMATICS UNIT MS1B	75	-	56	50	44	39	34
MS2B	MATHEMATICS UNIT MS2B	75	71	67	60	53	46	39