General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MFP4

Unit Further Pure 4

Wednesday 30 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Two planes have equations

$$x + 2y + 2z = 5 \quad \text{and} \quad px + 3y = 10$$

where p is a non-zero constant.

Given that the acute angle, θ, between the planes is such that $\cos\theta = \frac{2}{3}$, find the value of p. *(5 marks)*

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2 It is given that $\mathbf{A}$ and $\mathbf{B}$ are 3×3 matrices such that

$$\det(\mathbf{AB}) = 24 \quad \text{and} \quad \det(\mathbf{A}^{-1}) = -3$$

(a) State the value of $\det \mathbf{A}$. *(1 mark)*

(b) A three-dimensional shape S, with volume $20\,\text{cm}^3$, is transformed using matrix $\mathbf{B}$.

Find the volume of the image of S. *(3 marks)*

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3 (a) Expand and simplify, as far as possible,

$$(\mathbf{a} - 4\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b})$$

where $\mathbf{a}$ and $\mathbf{b}$ are vectors. *(3 marks)*

(b) Given that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, deduce that

$$|(\mathbf{a} - 4\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b})| = \lambda|\mathbf{a}|\,|\mathbf{b}|$$

where λ is an integer. *(2 marks)*

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4 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(a) Given that $\mathbf{A}^2 = \begin{bmatrix} p & -2 & -4 \\ 5 & 6 & 4 \\ 10 & q & 9 \end{bmatrix}$, find the value of p and the value of q. *(2 marks)*

(b) Given that $\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I} = \mathbf{0}$, prove that

$$\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I})$$ *(2 marks)*

(c) Given that $\mathbf{A}^{-1} = \frac{1}{6}\begin{bmatrix} r & -2 & 2 \\ -1 & 5 & -2 \\ -2 & s & 2 \end{bmatrix}$, find the value of r and the value of s.

(2 marks)

(d) Hence, or otherwise, find the solution of the system of equations

$$\begin{aligned} x \quad\;\; - z &= k \\ x + 2y + z &= 5 \\ 2x + 2y + 3z &= 7 \end{aligned}$$

giving your answers in terms of k. *(3 marks)*

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5 (a) By direct expansion, or otherwise, show that the value of $\begin{vmatrix} -2 & 1 & 2k \\ -1 & 1 & k+1 \\ 2 & k-1 & 1 \end{vmatrix}$ is independent of k. *(4 marks)*

(b) State, with a reason, whether the vectors

$$\begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

are linearly dependent or linearly independent. *(2 marks)*

(c) (i) State, with a reason, whether the equations

$$\begin{aligned} -2x + y + 6z &= 1 \\ -x + y + 4z &= 0 \\ 2x + 2y + z &= -1 \end{aligned}$$

are consistent or inconsistent. *(2 marks)*

(ii) The three equations given in part **(c)(i)** are the Cartesian equations of three planes.

State the geometrical configuration of these three planes. *(1 mark)*

6 The linear transformations T_1 and T_2 are represented by the matrices

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } \mathbf{M}_2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

respectively.

(a) Give a full geometrical description of the transformations:

(i) T_1 ; *(2 marks)*

(ii) T_2 . *(3 marks)*

(b) Find the matrix which represents the transformation T_1 followed by T_2 . *(2 marks)*

(c) The linear transformation T_3 is represented by the matrix

$$\mathbf{M}_3 = \begin{bmatrix} k & 2 & -1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

where k is a constant.

For one particular value of k, T_3 has a line L of invariant points.

(i) Find k.

(ii) Find the Cartesian equations of L in the form $\frac{x}{p} = \frac{y}{q} = \frac{z}{r}$. *(7 marks)*

7 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} -a & 0 & a \\ 0 & 6 & 0 \\ a & 0 & 2 \end{bmatrix}$$

where a is a real number. The distinct eigenvalues of $\mathbf{M}$ are λ_1, λ_2 and λ_3 with corresponding eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

(a) Given that $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, find λ_1. *(2 marks)*

(b) Given that $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, find the value of a. *(3 marks)*

(c) Given that $\lambda_3 = -6$, find a possible eigenvector $\mathbf{v}_3$. *(3 marks)*

(d) The matrix $\mathbf{M}$ can be expressed as $\mathbf{UDU}^{-1}$, where $\mathbf{D}$ is a diagonal matrix.

Write down possible matrices $\mathbf{D}$ and $\mathbf{U}$. *(3 marks)*

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8 The four vertices of a parallelogram $ABCD$ have coordinates

$$A(1,\ 0,\ 2),\ B(3,\ -1,\ 5),\ C(7,\ 2,\ 4) \text{ and } D(5,\ 3,\ 1)$$

(a) (i) Find $\overrightarrow{AB} \times \overrightarrow{AD}$. *(3 marks)*

(ii) Show that the area of the parallelogram is $p\sqrt{10}$, where p is an integer to be found. *(2 marks)*

(b) The diagonals AC and BD of the parallelogram meet at the point M. The line L passes through M and is perpendicular to the plane $ABCD$.

Find an equation for the line L, giving your answer in the form $(\mathbf{r} - \mathbf{u}) \times \mathbf{v} = \mathbf{0}$. *(4 marks)*

(c) The plane Π is parallel to the plane $ABCD$ and passes through the point $Q(6,\ 5,\ 17)$.

(i) Find the coordinates of the point of intersection of the line L with the plane Π. *(6 marks)*

(ii) One face of a parallelepiped is $ABCD$ and the opposite face lies in the plane Π.

Find the volume of the parallelepiped. *(3 marks)*

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$\mathbf{n}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad \mathbf{n}_2 = \begin{bmatrix} p \\ 3 \\ 0 \end{bmatrix}$			
	$\mathbf{n}_1 \cdot \mathbf{n}_2 = p+6$	B1		$\mathbf{n}_1 \cdot \mathbf{n}_2$ correct
	$\lvert \mathbf{n}_1 \rvert = \sqrt{1^2+2^2+2^2} = 3$			
	$\lvert \mathbf{n}_2 \rvert = \sqrt{p^2+9}$			
	Using $\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$:			
	$'their'\ p+6 = (3)\left(\sqrt{p^2+9}\right)\left(\frac{2}{3}\right)$	M1A1		forming an equation using scalar product
	$\Rightarrow (p+6)^2 = 4p^2+36$			
	$p^2+12p+36 = 4p^2+36$ $0 = 3p(p-4)$	m1		correctly forming and attempting to solve their quadratic equation
	$p \neq 0 \Rightarrow p = 4$	A1	5	$p = 4$ stated clearly (must reject p=0)
	Total		**5**	
2(a)	$\det \mathbf{A}^{-1} = -3 \Rightarrow \det \mathbf{A} = -\frac{1}{3}$	B1	1	
(b)	$\det(\mathbf{AB}) = 24 \Rightarrow \det \mathbf{B} = \frac{24}{\det \mathbf{A}} = -72$	M1 A1F		M1 for use of det(**AB**) = det **A** × det **B** A1F ft their det**A**
	Volume = $20 \times 72 = 1440\,\text{cm}^3$	A1cso	3	Must be positive
	Total		**4**	
3(a)	$(\mathbf{a}-4\mathbf{b})\times(\mathbf{a}+3\mathbf{b}) = \mathbf{a}\times\mathbf{a} - 4\mathbf{b}\times\mathbf{a} + 3\mathbf{a}\times\mathbf{b} - 12\mathbf{b}\times\mathbf{b}$	M1		Three terms correct
	$= -4\mathbf{b}\times\mathbf{a} + 3\mathbf{a}\times\mathbf{b}$	A1		$\mathbf{b}\times\mathbf{b} = \mathbf{a}\times\mathbf{a} = \mathbf{0}$ — correct use
	$= 7\mathbf{a}\times\mathbf{b}$	A1cso	3	$7\,\mathbf{a}\times\mathbf{b}$ or $-7\,\mathbf{b}\times\mathbf{a}$
(b)	$\mathbf{a}\perp\mathbf{b} \Rightarrow \sin\theta = 1$			
	$\Rightarrow \lvert\mathbf{a}\times\mathbf{b}\rvert = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert$	M1		Use of sin θ = 1 to simplify
	$\Rightarrow \lvert(\mathbf{a}-4\mathbf{b})\times(\mathbf{a}+3\mathbf{b})\rvert = 7\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert$			
	$\lambda = 7$	A1F	2	Should match 'their' 7
	Total		**5**	

Q	Solution	Marks	Total	Comments
4(a)	$\mathbf{A}^2 = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$			
	$= \begin{bmatrix} -1 & -2 & -4 \\ 5 & 6 & 4 \\ 10 & 10 & 9 \end{bmatrix}$ $p = -1$	B1		p-value
	$q = 10$	B1	2	q-value
(b)	$\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I} = \mathbf{0}$ multiply by $\mathbf{A}^{-1}$	M1		Multiplication by $\mathbf{A}^{-1}$
	$\left(\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I}\right)\mathbf{A}^{-1} = (\mathbf{0})\mathbf{A}^{-1}$			
	$\mathbf{A}^3\mathbf{A}^{-1} - 6\mathbf{A}^2\mathbf{A}^{-1} + 11\mathbf{A}\mathbf{A}^{-1} - 6\mathbf{I}\mathbf{A}^{-1} = \mathbf{0}$			
	$\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I} - 6\mathbf{A}^{-1} = \mathbf{0}$			
	$6\mathbf{A}^{-1} = \mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I}$			
	$\mathbf{A}^{-1} = \frac{1}{6}\left(\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I}\right)$	A1	2	**AG**
(c)	$\mathbf{A}^{-1} = \frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$ $r = 4$	B1		r-value
	$s = -2$	B1	2	s-value
(d)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}\begin{bmatrix} k \\ 5 \\ 7 \end{bmatrix}$			
	$= \frac{1}{6}\begin{bmatrix} 4k - 10 + 14 \\ -k + 25 - 14 \\ -2k - 10 + 14 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 4k + 4 \\ 11 - k \\ 4 - 2k \end{bmatrix}$	M1		use of $\mathbf{A}^{-1}\mathbf{v}$ – one row correct
	$x = \frac{2k+2}{3},\ y = \frac{11-k}{6},\ z = \frac{2-k}{3}$	A1		correct solution for one variable
		A1	3	all correct CAO
	Total		**9**	
(d)	**alternative** If solving equations by elimination, M1 A1 for correct solution for one variable, A1 all correct			

Q	Solution	Marks	Total	Comments
5(a)	$\begin{vmatrix} -2 & 1 & 2k \\ -1 & 1 & k+1 \\ 2 & k-1 & 1 \end{vmatrix} =$			
	$= -2\begin{vmatrix} 1 & k+1 \\ k-1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2k \\ k-1 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 2k \\ 1 & k+1 \end{vmatrix}$	M1		correctly expanding by any row or column
	$= -2\left[1-(k+1)(k-1)\right]+\left[1-2k(k-1)\right]+2\left[k+1-2k\right]$	A1		correct unsimplified expansion of 2 × 2 determinants
	$= -2\left[1-k^2+1\right]+\left[1-2k^2+2k\right]+2\left[1-k\right]$ $= -4+\cancel{2k^2}+1-\cancel{2k^2}+\cancel{2k}+2-\cancel{2k}$ $= -1$	A1cso		–1 obtained
	either all k's cancel *or* independent of k *etc*	E1	4	comment required (must score previous 3 marks
(b)	Identifying that $k = 2$ Value of determinant $\neq 0$ (or $= -1$ etc) therefore vectors are linearly independent	B1 E1F	2	$k = 2$ ft answer (a) if $0 \Rightarrow$ lin dep
(c)(i)	Identifying that $k = 3$ Value of determinant $\neq 0$ (or $= -1$ etc) therefore equations are consistent	B1 E1F	2	$k = 3$ ft answer (a) if $0 \Rightarrow$ inconsistent
(ii)	3 planes intersect in a unique point	B1	1	
	Total		**9**	
	Alternative for (a):			
	$\begin{vmatrix} -2 & 1 & 2k \\ -1 & 1 & k+1 \\ 2 & k-1 & 1 \end{vmatrix}$ $\begin{matrix} r_1 \to r_1 - 2r_2 \\ r_3 \to r_3 + 2r_2 \end{matrix}$			
	$= \begin{vmatrix} 0 & -1 & -2 \\ -1 & 1 & k+1 \\ 0 & k+1 & 2k+3 \end{vmatrix}$ $r_3 \to r_3 + (k+1)r_1$	(M1)		correctly expanding by any row or column after row operations
	$= \begin{vmatrix} 0 & -1 & -2 \\ -1 & 1 & k+1 \\ 0 & 0 & 1 \end{vmatrix} = -1$	(A1) (A1)		correct expansion unsimplified -1 obtained
	either all k's cancel *or* independent of k *etc*	(E1)	(4)	comment required

Q	Solution	Marks	Total	Comments
6(a)(i)	Reflection In (the plane) $z=0$ (or in the x–y plane)	M1 A1	2	Reflection stated for M1 Either version for A1
(ii)	Rotation About the y-axis through $\frac{\pi}{3}$ radians	M1 A1 B1	3	Rotation stated. y-axis (or 60°)
(b)	$T_2\ T_1 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{-\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{-\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix}$	M1A1	2	M1 correct order of matrices A1 fully correct [N.B $\begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix}$ scores M0A0]
(c)(i)	For line of invariants points $\begin{bmatrix} k & 2 & -1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	M1		Set up equations – uses $\mathbf{M}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
	$\Rightarrow kx+2y-z=x \Rightarrow (k-1)x+2y-z=0$ ①			
	$x+y+z=y \Rightarrow x+z=0$ ②	A1		Two equations correct
	$3x+4y+z=z \Rightarrow 3x+4y=0$ ③	A1		All three equations correct
	From ② $z=-x$ From ③ $y=\frac{-3x}{4}$	M1		Defines variables in terms of one letter *or* 2 components in $\begin{bmatrix} 4 \\ -3 \\ -4 \end{bmatrix}$ correct
	Substitute in ① $(k-1)x-\frac{3}{2}x+x=0$ $x\left[k-1-\frac{3}{2}+1\right]=0$ $x\left[k-\frac{3}{2}\right]=0$	A1		Substitution into other equations *or* $\begin{bmatrix} 4 \\ -3 \\ -4 \end{bmatrix}$ correct
	$x \neq 0 \Rightarrow k=\frac{3}{2}$	A1		k-value obtained.
(ii)	Line $x=\frac{-4}{3}y=-z$ $\frac{x}{4}=\frac{y}{-3}=\frac{z}{-4}$	B1cao	7	Or equivalent
	Total		**14**	

Q	Solution	Marks	Total	Comments
	Alternative to 6(c):			
	$\begin{bmatrix} k-1 & 2 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix}$	(M1)		Substitute and set up
	$r_1 \to r_1 + r_2$	(A1)		Row operation
	$\begin{bmatrix} k & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix}$			
	$r_1 \to r_1 - \frac{1}{2} r_3$	(A1)		Row operation
	$\begin{bmatrix} k-\frac{3}{2} & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix} \Rightarrow k = \frac{3}{2}$	(A1)		k-value obtained
	$v = \lambda \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix}$	(M1A1)		M1 obtains v in terms of single vector. A1 correct
	$\frac{x}{4} = \frac{y}{-3} = \frac{z}{-4}$	(B1cao)	(7)	Correct form

Q	Solution	Marks	Total	Comments
7(a)	$\begin{bmatrix} -a & 0 & a \\ 0 & 6 & 0 \\ a & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$	M1		
	$\Rightarrow \lambda_1 = 6$	A1	2	
(b)	$\begin{bmatrix} -a & 0 & a \\ 0 & 6 & 0 \\ a & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ a+4 \end{bmatrix}$	M1		
	$\begin{bmatrix} a \\ 0 \\ a+4 \end{bmatrix} = \begin{bmatrix} \lambda_2 \\ 0 \\ 2\lambda_2 \end{bmatrix}$ **i** component $\Rightarrow a = \lambda_2 \;\otimes$ **k** component $\Rightarrow a + 4 = 2\lambda_2$ using $\otimes$, $a + 4 = 2a$	m1		Eliminating λ_2
	$4 = a$	A1	3	Value of a obtained
(c)	Let $\mathbf{v_3} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$			
	$\begin{bmatrix} -4 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ x \end{bmatrix} = -6\begin{bmatrix} x \\ y \\ z \end{bmatrix}$	M1		Substitute 'their value of a' and attempt to get a system of equations.
	$\Rightarrow -4x + 4z = -6x \Rightarrow x + 2z = 0$ $6y = -6y \quad \Rightarrow y = 0$ $\left[4x + 2z = -6z \;\Rightarrow\; x + 2z = 0\right]$	A1F		Both equations "correct" FT their a
	$\mathbf{v_3} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ (or equivalent)	A1cao	3	
(d)	$\mathbf{D} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -6 \end{bmatrix}$	B1F		Diagonal matrix using –6 and "their 4" and "their 6"
	$\mathbf{U} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix}$	M1		FT their non-zero $\mathbf{v_3}$ in **U**
		A1cao	3	**U** correct and corresponding to **D**
	Total		**11**	

Q	Solution	Marks	Total	Comments
	Alternative to 7(c) $\begin{bmatrix} -4 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -6\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 2 & 0 & 4 \\ 0 & 12 & 0 \\ 4 & 0 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$			
	$r_3 \to r_3 - 2r_1 \quad \begin{bmatrix} 2 & 0 & 4 \\ 0 & 12 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	(M1)		Row operations
	$\Rightarrow y = 0 \quad x = -2z$	(A1F)		"correct" FT their a
	$\mathbf{v_3} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ (or equivalent)	(A1cao)		

Q	Solution	Marks	Total	Comments
8(a)(i)	$\overrightarrow{AB}=\begin{bmatrix}2\\-1\\3\end{bmatrix}$ $\quad \overrightarrow{AD}=\begin{bmatrix}4\\3\\-1\end{bmatrix}$	B1		either $\overrightarrow{AB}$ or $\overrightarrow{AD}$
	$\begin{vmatrix}\mathbf{i} & 2 & 4\\ \mathbf{j} & -1 & 3\\ \mathbf{k} & 3 & -1\end{vmatrix}=\begin{pmatrix}-8\\14\\10\end{pmatrix}$	M1 A1cao	3	one component of $\overrightarrow{AB}\times\overrightarrow{AD}$ correct all correct
(ii)	Area $ABCD=\left\lvert\overrightarrow{AB}\times\overrightarrow{AD}\right\rvert$ $=\sqrt{8^2+14^2+10^2}$ $=\sqrt{64+196+100}$ $=\sqrt{360}$ $=\sqrt{36}\sqrt{10}$ $=6\sqrt{10}$	M1 A1cso	 2	FT their $\left\lvert\overrightarrow{AB}\times\overrightarrow{AD}\right\rvert$ or $p=6$
(b)	$\overrightarrow{AB}\times\overrightarrow{AD}$ is perpendicular to plane $ABCD$ Hence direction ratios of line $=-8:14:10$ $=-4:7:5$	M1 A1		used for $\mathbf{v}$ in vector line equation or any multiple of $\begin{bmatrix}-4\\7\\5\end{bmatrix}$
	M is mid-point of either diagonal $=\left(\frac{1+7}{2},\frac{0+2}{2},\frac{2+4}{2}\right)$ $=(4,1,3)$	B1		mid-point calculation
	Hence line is $\left(\mathbf{r}-\begin{bmatrix}4\\1\\3\end{bmatrix}\right)\times\begin{bmatrix}-4\\7\\5\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$	A1cso	4	All correct or equivalent multiple of $\begin{bmatrix}-4\\7\\5\end{bmatrix}$

Q	Solution	Marks	Total	Comments
8(c)(i)	Perpendicular vector to $\Pi = \begin{bmatrix} -4 \\ 7 \\ 5 \end{bmatrix}$			
	$\Rightarrow \Pi$ has equation $-4x+7y+5z=c$	M1		ft their perpendicular vector
	Through (6, 5, 17) $\Rightarrow c = -4(6)+7(5)+5(17)$	m1		using (6, 5, 17)
	$= -24+35+85$			
	$= 96$	A1		
	Equation is $-4x+7y+5z=96$			ACF
	$x=4-4t;\quad y=1+7t;\quad z=3+5t$	B1F		parametric form of line
	Line meets plane when $-4(4-4t)+7(1+7t)+5(3+5t)=96$	M1		substitution of parametric form
	$-16+16t+7+49t+15+25t=96$			and attempt to solve for t
	$90t = 90$			
	$t = 1$			
	$\Rightarrow$ point of intersection = (0, 8, 8)	A1cao	6	correct point of intersection
(ii)	Volume $= \left(\overrightarrow{AB} \times \overrightarrow{AD}\right) \cdot \overrightarrow{AQ}$	M1		Attempt to use formula
	$\overrightarrow{AQ} = \begin{bmatrix} 5 \\ 5 \\ 15 \end{bmatrix}$			
	$\Rightarrow \begin{bmatrix} -8 \\ 14 \\ 10 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 5 \\ 15 \end{bmatrix} = -40+70+150$	A1F		Follow through $\overrightarrow{AB} \times \overrightarrow{AD}$ from (a)(i). May use $\overrightarrow{BQ}$ etc instead of $\overrightarrow{AQ}$
	$= 180$ (cubic units)	A1cso	3	
	Alternative			
	Vol = Area of base × perp dist	(M1)		Volume formula used
	Perp distance			
	$= \sqrt{(0-4)^2+(8-1)^2+(8-3)^2}$			
	$= \sqrt{16+49+25}$			
	$= \sqrt{90}$	(A1F)		Perp distance calculated FT their points or the equation of their plane *or* $\dfrac{\lvert -4\times1+7\times0+5\times2-96\rvert}{\sqrt{(-4)^2+7^2+5^2}} = \sqrt{90}$ etc
	Volume $= 6\sqrt{10}\times\sqrt{90}$			
	$= 6 \times 30$			
	$= 180$ (cubic units)	(A1cso)	(3)	
	Total		**18**	
	TOTAL		**75**	

Q	Solution	Marks	Total	Comments
	Alternative to 8(c)(i)			
	$\mathbf{r}=\begin{bmatrix}6\\5\\17\end{bmatrix}+s\begin{bmatrix}2\\-1\\3\end{bmatrix}+t\begin{bmatrix}4\\3\\-1\end{bmatrix}$	(M1)		$\mathbf{r}=\mathbf{a}+s\mathbf{d}_1+t\mathbf{d}_2$ fully correct
	$\begin{vmatrix}i & 2+2s+4t & -4\\ j & 4-s+3t & 7\\ k & 14+3s-t & 5\end{vmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$	(m1) (B1F)		Substitute in $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$ Use of $\mathbf{r}-\mathbf{a}$ in parametric form-simplified
	$-26s+22t=78$ $22s+16t=-66$ $10s+40t=-30$	(A1)		Three correct equations obtained from vector product-terms collected
	$s=-3$, $t=0$	(M1)		Correctly solving equations to get both parameters
	$\begin{bmatrix}0\\8\\8\end{bmatrix}$	(A1)		Correct point of intersection
	Alternative 2 to 8(c)(i)			
	$\mathbf{r}=\begin{bmatrix}6\\5\\17\end{bmatrix}+s\begin{bmatrix}2\\-1\\3\end{bmatrix}+t\begin{bmatrix}4\\3\\-1\end{bmatrix}$	(M1)		$\mathbf{r}=\mathbf{a}+s\mathbf{d}_1+t\mathbf{d}_2$ fully correct
	$\mathbf{r}=\begin{pmatrix}4-4p\\1+7p\\3+5p\end{pmatrix}$	(B1F)		Parametric form of line
	$2s+4t+4p=-2$ $-s+3t-7p=-4$ $3s-t-5p=-14$	(m1)		Equating components, simplifying and attempting to solve-must at least reduce to 2 equations in two unknowns
	$p=2$ $t=0$ and $s=-3$	(M1) (A1)		Solving equations-one parameter correct All values correct
	$\begin{bmatrix}0\\8\\8\end{bmatrix}$	(A1)		Correct point of intersection

Scaled mark unit grade boundaries - January 2013 exams

A-level

Code	Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points A*	A	B	C	D	E
LAW03	LAW UNIT 3	80	66	60	54	48	43	38
MD01	MATHEMATICS UNIT MD01	75	-	63	57	52	47	42
MD02	MATHEMATICS UNIT MD02	75	68	62	55	49	43	37
MFP1	MATHEMATICS UNIT MFP1	75	-	69	61	54	47	40
MFP2	MATHEMATICS UNIT MFP2	75	67	60	53	47	41	35
MFP3	MATHEMATICS UNIT MFP3	75	68	62	55	48	41	34
MFP4	MATHEMATICS UNIT MFP4	75	68	61	53	45	37	30
MM1B	MATHEMATICS UNIT MM1B	75	-	58	52	46	40	35
MM2B	MATHEMATICS UNIT MM2B	75	66	59	52	46	40	34
MPC1	MATHEMATICS UNIT MPC1	75	-	64	58	52	46	40
MPC2	MATHEMATICS UNIT MPC2	75	-	62	55	48	41	35
MPC3	MATHEMATICS UNIT MPC3	75	69	63	56	49	42	36
MPC4	MATHEMATICS UNIT MPC4	75	58	53	48	43	38	34
MS1A	MATHEMATICS UNIT MS1A	100	-	78	69	60	52	44
MS/SS1A/W	*MATHEMATICS UNIT S1A - WRITTEN*	*75*		*58*				*34*
MS/SS1A/C	*MATHEMATICS UNIT S1A - COURSEWORK*	*25*		*20*				*10*
MS1B	MATHEMATICS UNIT MS1B	75	-	60	54	48	42	36
MS2B	MATHEMATICS UNIT MS2B	75	70	66	58	50	42	35
MEST1	MEDIA STUDIES UNIT 1	80	-	54	47	40	33	26
MEST2	MEDIA STUDIES UNIT 2	80	-	63	54	45	36	28
MEST3	MEDIA STUDIES UNIT 3	80	68	58	48	38	28	18
MEST4	MEDIA STUDIES UNIT 4	80	74	68	56	45	34	23