# Simultaneous equations

## SET OF LINEAR EQUATIONS

Consider the line $L_1 : ax + by = c$

and the line $L_2 : dx + ey = f$

To work out the coordinates of the point of INTERSECTION, solve the equations SIMULTANEOUSLY.

Solving by *combination* / elimination:

$$\begin{cases} ax + by = c & (\times d) \\ dx + ey = f & (\times -a) \end{cases} \qquad \begin{cases} adx + bdy = cd \\ -adx - aey = -af \end{cases}$$

Then add the equations to find the value of $y$. Use any other equation to find the value of $x$.

Solving by *identification*:

Make $y$ the subject in both equations and identify the values of $y$:

$L_1 : y = m_1 x + c_1$

$L_2 : y = m_2 x + c_2$ *this gives* $(y =)\, m_1 x + c_1 = m_2 x + c_2$ *and solve.*

Solving by *substitution*:

*M*ake $y$ the subject in one of the equation then substitute $y$ by this expression in the second equation:

$L_1 : y = mx + c$

$L_2 : dx + ey = f$ *this gives* $dx + e(mx + c) = f$ *then solve.*

## SET OF QUADRATIC AND LINEAR EQUATIONS

A parabola C has equation $y = ax^2 + bx + c$ ,

a line L has equation $y = dx + e$ (*make* $y$ *the subject if it is an implict equation*)

To work out the coordinates of the points of intersection of the parabola and the line, solve these equations simultaneoulsy

Solving by identification:

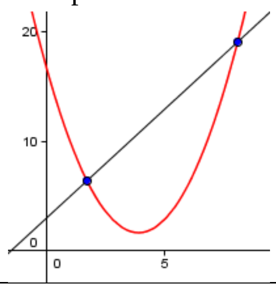
$$(y =)\ ax^2 + bx + c = dx + e \ \text{ then re-arrange into}$$

$$ax^2 + (b - d)x + c - e = 0 \ \textit{and solve.}$$

*Let's re-write as* $Ax^2 + Bx + C = 0$

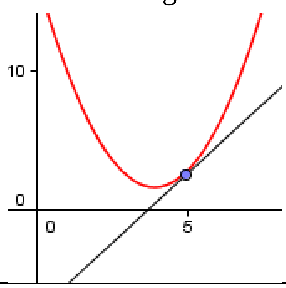
*if* $B^2 - 4AC > 0$

Two points of intersection




*if* $B^2 - 4AC = 0$

The line is tangent to the parabola




*if* $B^2 - 4AC < 0$

There is no intersection