

Quadratic functions

VOCABULARY



- A quadratic function can be written $f(x) = ax^2 + bx + c$, where a, b , and c are 3 real numbers.
- The graph of a quadratic function is called a **PARABOLA**
 - if $a > 0$, the parabola is \cup -shaped
 - if $a < 0$, the parabola is \cap -shaped
- The maximum or minimum point of the parabola is called the **VERTEX**.
- The parabola is **symmetrical** around the 'vertical' line going through the vertex.

COMPLETED SQUARE FORM



- $ax^2 + bx + c$ can be re-arrange into $a(x + p)^2 + q$.
This is the completed square form
- The **vertex** of the parabola is $V(-p, q)$.
- The axis of **symmetry** of the parabola has equation $x = -p$.
- Transformation: $y = x^2$ is mapped onto $y = (x + p)^2 + q$ by a **translation** with vector $\begin{bmatrix} -p \\ q \end{bmatrix}$

QUADRATIC EQUATIONS



A quadratic equation can be written $ax^2 + bx + c = 0$

- The **discriminant** is the value of the expression $b^2 - 4ac$.

if $b^2 - 4ac < 0$, there is no solution.

if $b^2 - 4ac = 0$, there is a repeated/double root.

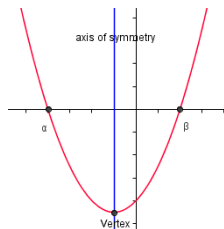
if $b^2 - 4ac > 0$, there are two solutions/roots: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- The roots are the values of x , where the parabola crosses the x -axis.

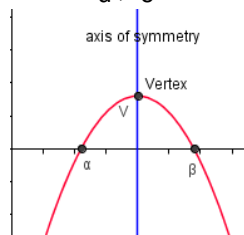
$$b^2 - 4ac > 0$$

$$f(x) = ax^2 + bx + c$$

$$f(x) = a(x - \alpha)(x - \beta)$$



$$a > 0$$



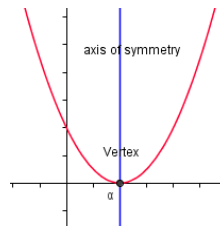
$$a < 0$$

$$b^2 - 4ac = 0$$

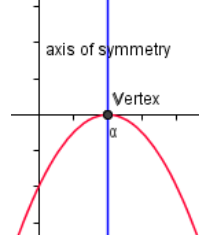
$$f(x) = ax^2 + bx + c$$

$$f(x) = a(x - \alpha)^2$$

The parabola is **TANGENT** to the x -axis



$$a > 0$$



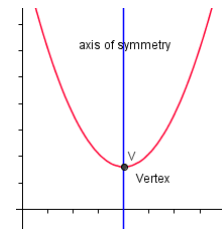
$$a < 0$$

$$b^2 - 4ac < 0$$

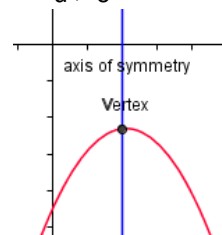
$$f(x) = ax^2 + bx + c$$

$f(x)$ can't be factorised

The parabola does not cross the x -axis.



$$a > 0$$



$$a < 0$$

