Integration

Indefinite integrals

Integration is the "opposite" of differentiation.

If $y = f(x)$ is a given function, to integrate f means finding a function $F(x)$

so that $\dfrac{dF}{dx} = f$

F is called an INTEGRAL of f and it is noted $\int f(x)dx$

Note: An integral is not unique. If $F(x)$ is an integral, then $F(x) + c$ is also one.

$$\int f(x)dx = F(x) + c \qquad \textit{where } c \textit{ is a constant}$$

Integrating x^n

The formula tells you how to integrate *powers of* x.

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \qquad \textit{for all } n \neq -1$$

Rules of integrations

$f(x)$ *and* $g(x)$ are two functions , a is a constant

$$\int a \times f(x)dx = a \times \int f(x)dx$$

$$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$$

Examples: $\int x^3 dx = \frac{1}{4}x^4 + c$, $\int (3x^2 - 3x)\,dx = 3 \times \frac{1}{3}x^3 - 3 \times \frac{1}{2}x^2 + c = x^3 - \frac{3}{2}x^2 + c$

Integrating to find the equation of a curve

A curve $y = f(x)$ is going through the point $A(x_A, y_A)$ and $\dfrac{dy}{dx} = f'(x)$ *is given.*

To find the equation of the curve,

- integrate $f'(x)$: $\int f'(x)dx = F(x) + c$
- find the value of the constant c using the coordinates of A.

Example: The curve $y = f(x)$ goes through A(2,9) and $\dfrac{dy}{dx} = 3x^2$.

Find the eqaution of the curve.

- $\int 3x^2 dx = x^3 + c \qquad so \quad y = x^3 + c$
- $A(2,9)$ belongs to the curve so $9 = (2)^3 + c \qquad c = 1$
- the eqaution of the curve is $y = x^3 + 1$