

# Transformations of graphs

## Translations of graphs

A curve  $C_f$  has equation  $y = f(x)$ .

" $a$ " is a positive number.

- The curve with equation  $y = f(x) - b$  is the translation of  $C_f$  by vector  $\begin{pmatrix} 0 \\ -b \end{pmatrix}$
- The curve with equation  $y = f(x + a)$  is the translation of  $C_f$  by vector  $\begin{pmatrix} -a \\ 0 \end{pmatrix}$



## Combined translations

- The curve with equation  $y + b = f(x + a)$  is the translation of  $C_f$  by vector  $\begin{pmatrix} -a \\ -b \end{pmatrix}$

**Examples:** The curve with equation  $y = (x - 3)^2 + 2$  is the translation of

the curve  $y = x^2$  by vector  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ .

The circle  $(x - 3)^2 + (y + 1)^2 = 9$  is the translation of the circle  $x^2 + y^2 = 9$

by the vector  $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ .

## Parabolas

All parabolas of the form  $y = x^2 + bx + c$  are the image of the parabola  $y = x^2$

To work out the vector of this translation, use the completed square form:

$$y = x^2 + bx + c = (x + p)^2 + q$$

The vector of the translation is  $\begin{pmatrix} -p \\ q \end{pmatrix}$ .

**Note:** This vector is the vector  $\overrightarrow{OV}$ , where  $V(-p, q)$  is the vertex of the parabola.

