Integration and area

Definite integrals

Definite integrals have numbers, a *and* b, next to the integral sign.

They indicate the range of x-values to integrate the function between.

a is the lower limit, b is the upper limit $\quad a < b$

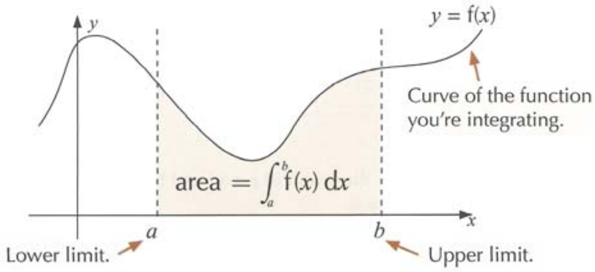
$$\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a)$$ where F is an integral of f.

Example:

$$\int_1^2 x^2 dx = \left[\frac{1}{3}x^3\right]_1^2 = \left(\frac{1}{3}\times 2^3\right) - \left(\frac{1}{3}\times 1^3\right) = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

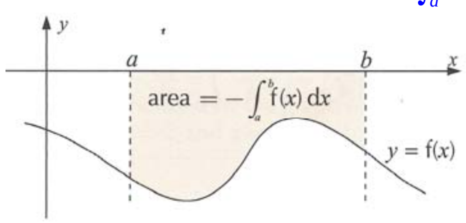
Area under a curve

The value of a definite integral represents the area between the curve of the function, the x-axis and the line $x = a$ and $x = b$.



Be careful: if the curve is below the x-axis, i.e if $f(x) < 0$, the integral will give a negative value.

In this case, $\quad Area = -\int_a^b f(x)dx$



Area between two curves

$f(x)$ *and* $g(x)$ are two functions and a *and* b are two numbers.

when $a < x < b$, $f(x) > g(x)$.

The area between the two curves and the lines $x = a$ and $x = b$ is

$$\int_a^b f(x)dx - \int_a^b g(x)dx \quad or \quad \int_a^b (f(x) - g(x))dx$$

$$Area = \int_1^2 -x^2 + 3x - \left((x-1)^2 + 1\right)dx = \int_1^2 -2x^2 + 5x - 2dx$$

$$= \left[-\frac{2}{3}x^3 + \frac{5}{2}x^2 - 2x\right]_1^2$$

$$Area = \left(-\frac{16}{3} + 10 - 4\right) - \left(-\frac{2}{3} + \frac{5}{2} - 2\right) = \frac{5}{6}$$