General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MPC1

Unit Pure Core 1

Monday 13 May 2013 1.30 pm to 3.00 pm

For this paper you must have: • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.	

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The line AB has equation $3x - 4y + 5 = 0$.

(a) The point with coordinates $(p, p + 2)$ lies on the line AB. Find the value of the constant p. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The point A has coordinates $(1, 2)$. The point $C(-5, k)$ is such that AC is perpendicular to AB. Find the value of k. *(3 marks)*

(d) The line AB intersects the line with equation $2x - 5y = 6$ at the point D. Find the coordinates of D. *(3 marks)*

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2 (a) (i) Express $\sqrt{48}$ in the form $n\sqrt{3}$, where n is an integer. *(1 mark)*

(ii) Solve the equation

$$x\sqrt{12} = 7\sqrt{3} - \sqrt{48}$$

giving your answer in its simplest form. *(3 marks)*

(b) Express $\dfrac{11\sqrt{3} + 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}}$ in the form $m - \sqrt{15}$, where m is an integer. *(4 marks)*

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3 A circle C has equation

$$x^2 + y^2 - 10x + 14y + 25 = 0$$

(a) Write the equation of C in the form

$$(x - a)^2 + (y - b)^2 = k$$

where a, b and k are integers. *(3 marks)*

(b) Hence, for the circle C, write down:

(i) the coordinates of its centre; *(1 mark)*

(ii) its radius. *(1 mark)*

(c) (i) Sketch the circle C. *(2 marks)*

(ii) Write down the coordinates of the point on C that is furthest away from the x-axis. *(2 marks)*

(d) Given that k has the same value as in part **(a)**, describe geometrically the transformation which maps the circle with equation $(x + 1)^2 + y^2 = k$ onto the circle C. *(3 marks)*

4 (a) The polynomial $f(x)$ is given by $f(x) = x^3 - 4x + 15$.

(i) Use the Factor Theorem to show that $x + 3$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x + 3)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(b) A curve has equation $y = x^4 - 8x^2 + 60x + 7$.

(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Show that the x-coordinates of any stationary points of the curve satisfy the equation

$$x^3 - 4x + 15 = 0$$ *(1 mark)*

(iii) Use the results above to show that the only stationary point of the curve occurs when $x = -3$. *(2 marks)*

(iv) Find the value of $\frac{d^2y}{dx^2}$ when $x = -3$. *(3 marks)*

(v) Hence determine, with a reason, whether the curve has a maximum point or a minimum point when $x = -3$. *(1 mark)*

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5 (a) (i) Express $2x^2 + 6x + 5$ in the form $2(x + p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(ii) Hence write down the minimum value of $2x^2 + 6x + 5$. *(1 mark)*

(b) The point A has coordinates $(-3, 5)$ and the point B has coordinates $(x, 3x + 9)$.

(i) Show that $AB^2 = 5(2x^2 + 6x + 5)$. *(3 marks)*

(ii) Use your result from part **(a)(ii)** to find the minimum value of the length AB as x varies, giving your answer in the form $\frac{1}{2}\sqrt{n}$, where n is an integer. *(2 marks)*

6 A curve has equation $y = x^5 - 2x^2 + 9$. The point P with coordinates $(-1, 6)$ lies on the curve.

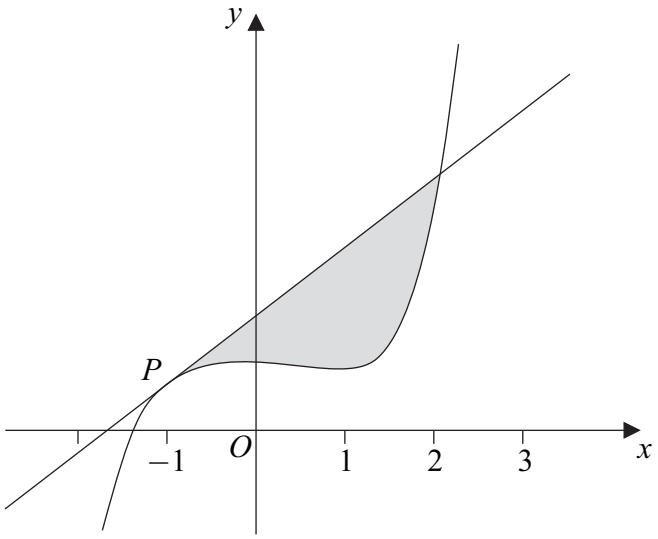
(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(5 marks)*

(b) The point Q with coordinates $(2, k)$ lies on the curve.

(i) Find the value of k. *(1 mark)*

(ii) Verify that Q also lies on the tangent to the curve at the point P. *(1 mark)*

(c) The curve and the tangent to the curve at P are sketched below.



(i) Find $\int_{-1}^{2} (x^5 - 2x^2 + 9)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the tangent to the curve at P. *(3 marks)*

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7 The quadratic equation

$$(2k - 7)x^2 - (k - 2)x + (k - 3) = 0$$

has real roots.

(a) Show that $7k^2 - 48k + 80 \leqslant 0$. *(4 marks)*

(b) Find the possible values of k. *(4 marks)*

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

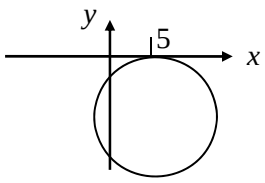
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

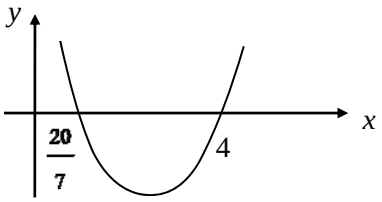
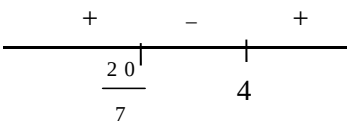
Q	Solution	Marks	Total	Comments
1(a)	$3p-4(p+2)+5=0$	M1		condone omission of brackets or one sign error
	$(\Rightarrow p=)\ -3$	A1	2	
(b)	$y=\frac{3}{4}x+\frac{5}{4}$	M1		rearranging into form $y=\pm\frac{3}{4}x+c$
	(gradient AB =) $\frac{3}{4}$	A1	2	condone slips in rearranging if gradient is correct .
(c)	(gradient AC =) $\frac{k-2}{-5-1}$	M1		*or* $\frac{2-k}{1--5}$ (condone one sign error)
	"their" $\frac{(k-2)}{-6}\times\frac{3}{4}=-1$ *OE*	m1		product of grads = –1 in terms of k
	$(\Rightarrow k=)\ 10$	A1	3	
(d)	$3x-4y+5=0$ and $2x-5y=6$ $\Rightarrow P\,x=Q$ or $R\,y=S$	M1		must use "correct" pair of equations **and** attempt to eliminate y (or x) (generous)
	$x=-7$	A1		
	$y=-4$	A1	3	(–7, – 4)
	Total		**10**	

Q	Solution	Marks	Total	Comments
2(a)(i)	$\left(\sqrt{48}\ =\right)4\sqrt{3}$	B1	1	condone $n = 4$. No ISW .
(ii)	$\sqrt{12}\ = 2\sqrt{3}$ and $\sqrt{48}\ = 4\sqrt{3}$	M1		(FT 'their'n) $2x\sqrt{3} = 7\sqrt{3} - 4\sqrt{3}$
	$\left(x =\right)\dfrac{7\sqrt{3} - 4\sqrt{3}}{2\sqrt{3}}$	A1		correct quotient unsimplified or correct equation in integers eg $6x = 21 - 12$
	$= \dfrac{3}{2}$	A1cso	3	accept 1.5 but not $\dfrac{9}{6}$ etc
				alternative 1 $x = \dfrac{7\sqrt{3} - \sqrt{48}}{\sqrt{12}} \times \dfrac{\sqrt{12}}{\sqrt{12}}$ M1 integer terms $= \dfrac{42 - 24}{12}$ A1 $= \dfrac{3}{2}$ A1
(b)	$\dfrac{11\sqrt{3} + 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}} \times \dfrac{2\sqrt{3} - \sqrt{5}}{2\sqrt{3} - \sqrt{5}}$	M1		
	(numerator =) $22 \times 3 + 4\sqrt{15} - 11\sqrt{15} - 2 \times 5$	A1		correct unsimplified but must simplify $\left(\sqrt{3}\right)^2$, $\left(\sqrt{5}\right)^2$ and $\sqrt{3} \times \sqrt{5}$ correctly
	(denominator = 12 – 5 =) 7	B1		must be seen or identified as denominator giving $\dfrac{56 - 7\sqrt{15}}{7}$
	(Answer =) $8 - \sqrt{15}$	A1cso	4	$m = 8$
	Total		**8**	

Q	Solution	Marks	Total	Comments
3(a)	$(x-5)^2+(y+7)^2$	M1		one term correct
		A1		both terms correct and added
	$(x-5)^2+(y+7)^2=49$	A1cao	3	must see 49 not just 7^2 condone $(x-5)^2+(y--7)^2=49$
(b)(i)	(Centre is) (5, –7)	B1√	1	correct or FT their a and b
(ii)	Radius = 7	B1√	1	condone $\sqrt{49}$ but **no**t ± 7 or $\pm\sqrt{49}$ correct or FT their $\sqrt{k}$ provided $k > 0$
(c)(i)		M1		freehand circle with centre in correct quadrant or FT from their **(b)(i)** must have both axes shown clearly
		A1	2	correct position cutting negative y-axis twice and touching x-axis at $x = 5$ 5 must be marked on x-axis or centre clearly marked as (5, –7) must have correct centre and radius in **(b)**
(ii)	$x=5$	B1		
	$y=-14$	B1	2	(5, –14)
(d)	Translation	E1		and no other transformation
	through $\begin{bmatrix}6\\*\end{bmatrix}$	M1		
	$\begin{bmatrix}6\\-7\end{bmatrix}$	A1cso	3	both components correct for A1; may describe in words or use a column vector
	Total		**12**	

Q	Solution	Marks	Total	Comments
4(a)	$f(-3) = (-3)^3 - 4\times(-3) + 15$ $f(-3) = -27 + 12 + 15$ $= 0 \Rightarrow x + 3$ is a factor	M1 A1	 2	f(–3) attempted **not** long division shown = 0 plus statement
(ii)	Quadratic factor $(x^2 - 3x + 5)$ $(f(x) =) \quad (x+3)(x^2 - 3x + 5)$	M1 A1	 2	$-3x$ or + 5 term by inspection *or* full long division attempt must see correct product
(b) (i)	$\left(\frac{dy}{dx} =\right) 4x^3 - 16x + 60$	M1 A1 A1	 3	one of these terms correct another term correct all correct (no +*c* etc)
(ii)	$4x^3 - 16x + 60 = 0$ $\Rightarrow x^3 - 4x + 15 = 0$	 B1	 1	must see this line OE **AG**
(iii)	Discriminant of quadratic = $(-3)^2 - 4\times 5$ $b^2 - 4ac = -11$ (*or* $b^2 - 4ac < 0$) therefore quadratic has no (real)roots Hence only stationary point is when $x = -3$	M1 A1	 2	discriminant of " their" quadratic or correct use of quad eqn "formula" **correct discriminant** evaluated correctly (or shown to be < 0) with appropriate conclusion **plus** final statement
(iv)	$\left(\frac{d^2y}{dx^2} =\right) 12x^2 - 16$ $= 12(-3)^2 - 16$ (*or* $12\times 9 - 16$ etc) $= 92$	B1√ M1 A1	 3	 sub $x = -3$ into "their" $\frac{d^2y}{dx^2}$
(v)	Minimum **since** $\frac{d^2y}{dx^2} > 0$ (*or* 92 > 0 etc)	E1√	1	FT appropriate conclusion from their value from **(iv) plus** reason treat parts **(iv) & (v)** holistically
	Total		**14**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$2(x+1.5)^2$	M1		OE
	$2(x+1.5)^2+0.5$	A1	2	$2\left(x+\frac{3}{2}\right)^2+\frac{1}{2}$ OE
(ii)	(Minimum value is) 0.5	B1√	1	ft their q
(b)(i)	$(AB^2=)\quad (x+3)^2+(3x+9-5)^2$	M1		condone one sign error inside one bracket
	$(3x+4)^2=9x^2+24x+16$	B1		OE
	$AB^2=x^2+6x+9+9x^2+24x+16=10x^2+30x+25$ $\Rightarrow AB^2=5(2x^2+6x+5)$	A1cso	3	**AG**
(ii)	*Either* $\sqrt{5\times 'their'}$ **(a)(ii)** *or* $5\times 'their'$ **(a)(ii)**	M1		using their minimum value from **(a)(ii)** and 5 provided "their" **(a)(ii)** > 0
	(Minimum length of AB =) $\frac{1}{2}\sqrt{10}$	A1cso	2	
	Total		**8**	
6(a)	$\frac{dy}{dx}=5x^4-4x$	M1 A1		one of these terms correct all correct (no +c etc)
	$\left(=5(-1)^4-4(-1)\right)\qquad =9$	A1		
	Tangent has equation $y='their\ 9'x+c$ **and** $6='their\ 9'(-1)+c\quad \Rightarrow c=...$	m1		tangent using 'their' gradient, and attempt to find c using $x=-1$ and $y=6$
	$\Rightarrow y=9x+15$	A1	5	equation must be seen in this form
(b)(i)	When $x=2$, $y=2^5-2\times 2^2+9=32-8+9=33$ $(k=)$ 33	B1	1	be convinced that they are using **curve** equation **NMS** $k=33$ scores B0
(ii)	When $x=2$, $y=9\times 2+15=33$ so lies on tangent	B1	1	be convinced that they are using **tangent** equation **and** have statement

Q	Solution	Marks	Total	Comments
6(c)(i)	$\frac{x^6}{6}-\frac{2x^3}{3}+9x$	M1 A1 A1		one of these terms correct another term correct all correct (may have +c)
	$\left[\frac{2^6}{6}-\frac{2\times2^3}{3}+9\times2\right]-\left[\frac{(-1)^6}{6}-\frac{2\times(-1)^3}{3}+9\times(-1)\right]$	m1		F(2) – F(–1) unsimplified FT "their terms" from integration
	$\left[\frac{64}{6}-\frac{16}{3}+18\right]-\left[\frac{1}{6}+\frac{2}{3}-9\right]$			$=\frac{70}{3}-\left(-\frac{49}{6}\right)$
	= 31.5			
	$\left(or\ \frac{189}{6}\ etc\right)$	A1	5	condone single fraction
(ii)	Area of trapezium = $\frac{1}{2}\times3\times(6+'their'k)$	B1√		= 58.5 when k = 33
	Shaded area = **Trapezium** – 'their' **(c)(i)** value	M1		
	= 27	A1	3	OE $eg\ \frac{162}{6}$
	Total		**15**	
7(a)	$(k-2)^2-4\times(2k-7)(k-3)$	M1		discriminant – condone one slip –condone omission of brackets
	$k^2-4k+4-4(2k^2-6k-7k+21)$	A1		
	"their" $-7k^2+48k-80\geqslant0$	B1		real roots condition ; f(k) $\geqslant$ 0 must appear before final line
	$7k^2-48k+80\leqslant0$	A1cso	4	**AG** (all working correct with no missing brackets etc)
(b)	$7k^2-48k+80=(7k-20)(k-4)$	M1		correct factors
				(or roots unsimplified) $\frac{48\pm\sqrt{64}}{14}$
	critical values are 4 and $\frac{20}{7}$	A1		accept $\frac{56}{14}$, $\frac{40}{14}$ etc here
		M1		sketch or sign diagram including values
				
	$\frac{20}{7}\leqslant k\leqslant4$	A1cao	4	fractions must be simplified here
	Take their final line as their answer			
	Total		**8**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2013 exams

A-level

Code	Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points					
			A*	A	B	C	D	E
LAW02	LAW UNIT 2	94	-	77	69	61	53	45
LAW03	LAW UNIT 3	80	69	63	57	52	47	42
LAW04	LAW UNIT 4	85	73	67	61	56	51	46
MD01	MATHEMATICS UNIT MD01	75	-	64	59	54	50	46
MD02	MATHEMATICS UNIT MD02	75	69	64	56	48	41	34
MFP1	MATHEMATICS UNIT MFP1	75	-	55	49	43	37	32
MFP2	MATHEMATICS UNIT MFP2	75	65	61	54	47	40	34
MFP3	MATHEMATICS UNIT MFP3	75	67	64	56	49	42	35
MFP4	MATHEMATICS UNIT MFP4	75	64	60	52	44	36	28
MM1B	MATHEMATICS UNIT MM1B	75	-	56	49	42	35	29
MM2B	MATHEMATICS UNIT MM2B	75	67	61	55	49	43	37
MM03	MATHEMATICS UNIT MM03	75	70	65	58	51	44	37
MM04	MATHEMATICS UNIT MM04	75	67	59	51	44	37	30
MM05	MATHEMATICS UNIT MM05	75	68	61	52	44	36	28
MPC1	MATHEMATICS UNIT MPC1	75	-	59	53	47	41	36
MPC2	MATHEMATICS UNIT MPC2	75	-	61	55	49	43	37
MPC3	MATHEMATICS UNIT MPC3	75	66	60	54	49	44	39
MPC4	MATHEMATICS UNIT MPC4	75	60	55	50	45	40	35
MS1A	MATHEMATICS UNIT MS1A	100	-	76	67	59	51	43
MS/SS1A/W	*MATHEMATICS UNIT S1A - WRITTEN*	*75*		*56*				*33*
MS/SS1A/C	*MATHEMATICS UNIT S1A - COURSEWORK*	*25*		*20*				*10*
MS1B	MATHEMATICS UNIT MS1B	75	-	56	50	44	39	34
MS2B	MATHEMATICS UNIT MS2B	75	71	67	60	53	46	39