General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MPC1

Unit Pure Core 1

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Express $\dfrac{5\sqrt{3}-6}{2\sqrt{3}+3}$ in the form $m + n\sqrt{3}$, where m and n are integers. *(4 marks)*

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2 The line AB has equation $4x - 3y = 7$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Find an equation of the straight line that is parallel to AB and which passes through the point $C(3, -5)$, giving your answer in the form $px + qy = r$, where p, q and r are integers. *(3 marks)*

(b) The line AB intersects the line with equation $3x - 2y = 4$ at the point D. Find the coordinates of D. *(3 marks)*

(c) The point E with coordinates $(k - 2, 2k - 3)$ lies on the line AB. Find the value of the constant k. *(2 marks)*

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3 The polynomial $\text{p}(x)$ is given by

$$\text{p}(x) = x^3 + 2x^2 - 5x - 6$$

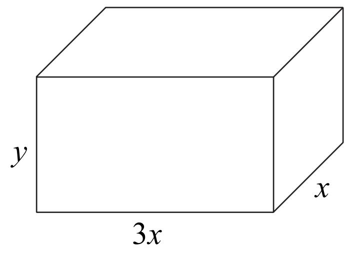
(a) (i) Use the Factor Theorem to show that $x + 1$ is a factor of $\text{p}(x)$. *(2 marks)*

(ii) Express $\text{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Verify that $\text{p}(0) > \text{p}(1)$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 + 2x^2 - 5x - 6$, indicating the values where the curve crosses the x-axis. *(3 marks)*

4 The diagram shows a solid cuboid with sides of lengths x cm, $3x$ cm and y cm.



The total surface area of the cuboid is 32 cm^2.

(a) (i) Show that $3x^2 + 4xy = 16$. *(2 marks)*

(ii) Hence show that the volume, $V\text{ cm}^3$, of the cuboid is given by

$$V = 12x - \frac{9x^3}{4}$$ *(2 marks)*

(b) Find $\frac{dV}{dx}$. *(2 marks)*

(c) (i) Verify that a stationary value of V occurs when $x = \frac{4}{3}$. *(2 marks)*

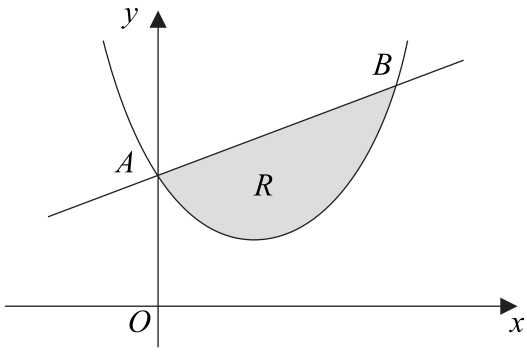
(ii) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = \frac{4}{3}$. *(2 marks)*

Turn over ▶

5 (a) (i) Express $x^2 - 3x + 5$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$. *(1 mark)*

(b) The curve C with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point B, as shown in the diagram below.

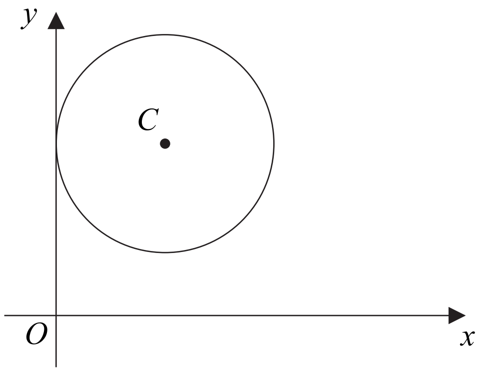


(i) Find the coordinates of the point B. *(3 marks)*

(ii) Find $\int (x^2 - 3x + 5)\,dx$. *(3 marks)*

(iii) Find the area of the shaded region R bounded by the curve C and the line segment AB. *(4 marks)*

6 The circle with centre $C(5, 8)$ touches the y-axis, as shown in the diagram.



(a) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

(b) (i) Verify that the point $A(2, 12)$ lies on the circle. *(1 mark)*

(ii) Find an equation of the tangent to the circle at the point A, giving your answer in the form $sx + ty + u = 0$, where s, t and u are integers. *(5 marks)*

(c) The points P and Q lie on the circle, and the mid-point of PQ is $M(7, 12)$.

(i) Show that the length of CM is $n\sqrt{5}$, where n is an integer. *(2 marks)*

(ii) Hence find the area of triangle PCQ. *(3 marks)*

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7 The gradient, $\frac{dy}{dx}$, of a curve C at the point (x, y) is given by

$$\frac{dy}{dx} = 20x - 6x^2 - 16$$

(a) (i) Show that y is increasing when $3x^2 - 10x + 8 < 0$. *(2 marks)*

(ii) Solve the inequality $3x^2 - 10x + 8 < 0$. *(4 marks)*

(b) The curve C passes through the point $P(2, 3)$.

(i) Verify that the tangent to the curve at P is parallel to the x-axis. *(2 marks)*

(ii) The point $Q(3, -1)$ also lies on the curve. The normal to the curve at Q and the tangent to the curve at P intersect at the point R. Find the coordinates of R.
(7 marks)

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

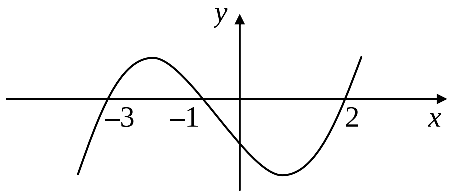
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1	$\frac{5\sqrt{3}-6}{2\sqrt{3}+3} \times \frac{2\sqrt{3}-3}{2\sqrt{3}-3}$	M1		
	(Numerator =) $30-15\sqrt{3}-12\sqrt{3}+18$	m1		correct $\left(=48-27\sqrt{3}\right)$
	(Denominator = $12-9=$) 3	B1		must be seen as denominator
	$\left(\frac{48-27\sqrt{3}}{3}\right)=16-9\sqrt{3}$	A1	4	CSO; accept $16+-9\sqrt{3}$
	Total		**4**	
2(a)(i)	$y=\frac{4}{3}x-\frac{7}{3}$	M1		$y=\pm\frac{4}{3}x+k$ or $\frac{\Delta y}{\Delta x}$ with 2 **correct** points
	$\Rightarrow$ grad $AB=\frac{4}{3}$	A1	2	condone slip in rearranging if gradient is correct; condone 1.33 or better
(ii)	$y=$ 'their grad' $x+c$ **and** attempt to use $x=3,\ y=-5$	M1		***or*** $y--5=$ 'their grad AB' $(x-3)$ ***or*** $4x-3y=k$ and attempt to find k using $x=3$ and $y=-5$
	$y+5=\frac{4}{3}(x-3)$ *or* $y=\frac{4}{3}x-\frac{27}{3}$	A1		correct equation in any form but must simplify $--$ to +
	$4x-3y=27$	A1	3	integer coefficients in required form eg $-8x+6y=-54$
(b)	$4x-3y=7$ and $3x-2y=4$ $\Rightarrow 8x-9x=14-12$ etc	M1		must use **correct pair** of equations and **attempt** to eliminate x or y (generous)
	$x=-2$	A1		
	$y=-5$	A1	3	or $D\ (-2,-5)$
(c)	$4(k-2)-3(2k-3)=7$ $4k-8-6k+9=7$	M1		sub $x=k-2,\ y=2k-3$ into $4x-3y=7$ and attempt to multiply out with all k terms on one side (condone one slip)
	$\Rightarrow k=-3$	A1	2	
	Total		**10**	

MPC1

Q	Solution	Marks	Total	Comments
3(a)(i)	$\mathrm{p}(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$	M1		$\mathrm{p}(-1)$ attempted **not** long division
	$\mathrm{p}(-1) = -1 + 2 + 5 - 6 = 0 \Rightarrow x + 1$ is a factor	A1	2	CSO; correctly shown = 0 plus statement
(ii)	Quad factor in this form: $(x^2 + bx + c)$	M1		long division as far as constant term ***or*** comparing coefficients, ***or*** $b = 1$ ***or*** $c = -6$ by inspection
	$x^2 + x - 6$	A1		correct quadratic factor
	$[\mathrm{p}(x) =]\ (x+1)(x+3)(x-2)$	A1	3	**must** see correct product
(b)	$\mathrm{p}(0) = -6$; $\mathrm{p}(1) = -8$	M1		**both** $\mathrm{p}(0)$ and $\mathrm{p}(1)$ attempted **and** at least one value correct
	$\Rightarrow \mathrm{p}(0) > \mathrm{p}(1)$	A1	2	**AG** both values correct plus correct statement involving p(0) and p(1)
(c)		M1 A1 A1	3	cubic with one max and one min  with $-3, -1, 2$ marked correct with minimum to right of y-axis AND going beyond –3 and 2
	Total		**10**	

MPC1

Q	Solution	Marks	Total	Comments
4(a)(i)	$3x^2+3x^2+xy+xy+3xy+3xy$	M1		correct expression for surface area
	$6x^2+8xy=32$			$2(3x^2+xy+3xy)=32$ etc
	$\Rightarrow 3x^2+4xy=16$	A1	2	**AG** be convinced
(ii)	$(V=)3x^2y$ OE	M1		correct volume in terms of x and y
	$=3x\left(\frac{16-3x^2}{4}\right)$ **or** $=3x^2\left(\frac{16-3x^2}{4x}\right)$			OE
	$=12x-\frac{9x^3}{4}$	A1	2	CSO **AG** be convinced that all working is correct
(b)	$\left(\frac{dV}{dx}=\right)12-\frac{27}{4}x^2$	M1 A1	2	one of these terms correct all correct with 9 × 3 evaluated (no + c etc)
(c)(i)	$x=\frac{4}{3}\Rightarrow\frac{dV}{dx}=12-\frac{27}{4}\times\left(\frac{4}{3}\right)^2$	M1		attempt to sub $x=\frac{4}{3}$ into '*their*' $\frac{dV}{dx}$
	$\frac{dV}{dx}=12-\frac{27}{4}\times\frac{16}{9}=12-12$			or $12-\frac{432}{36}=12-12$ or $12-\frac{48}{4}=0$ etc
	$\frac{dV}{dx}=0\Rightarrow$ stationary value	A1	2	CSO; shown **=** 0 plus statement
(ii)	$\frac{d^2V}{dx^2}=-\frac{27x}{2}$ OE	B1√		FT for '*their*' $\frac{dV}{dx}=a+bx^2$
	when $x=\frac{4}{3}$, $\frac{d^2V}{dx^2}<0\Rightarrow$ maximum	E1√	2	**or** sub of $x=\frac{4}{3}$ into '*their*' $\frac{d^2V}{dx^2}$ $\Rightarrow$ maximum
	(FT *"minimum"* if their $\frac{d^2V}{dx^2}>0$)			E0 if numerical error seen
	Total		**10**	

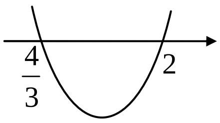
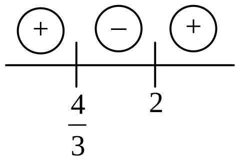
MPC1

Q	Solution	Marks	Total	Comments
5(a)(i)	$\left(x-\frac{3}{2}\right)^2$	M1		or $p = 1.5$ stated
	$\left(x-\frac{3}{2}\right)^2 + \frac{11}{4}$	A1	2	$(x-1.5)^2 + 2.75$
	Mark their final line as their answer			
(ii)	$x=\frac{3}{2}$	B1√	1	correct or FT their " $x = p$ "
(b)(i)	$x^2-3x+5=x+5 \Rightarrow x^2=4x$	M1		eliminating x or y and collecting like terms (condone **one** slip) **or** $(y-5)^2-3(y-5)+5=y$ $\Rightarrow y^2-14y+45=0$
	$(x \neq 0) \quad \Rightarrow x=4$	A1		
	$y=9$	A1	3	
(ii)	$\frac{x^3}{3} - \frac{3x^2}{2} + 5x\ (+c)$	M1 A1 A1	3	one of these terms correct another term correct all correct (need not have $+c$)
(iii)	$[\ ]_0^4 = \frac{4^3}{3} - 3\times\frac{4^2}{2} + 5\times 4$	M1		must have earned M1 in part(b)(ii) $\mathrm{F}(\text{their } x_B)\{-\mathrm{F}(0)\}$ "correctly sub'd"
	$=17\frac{1}{3}$	A1		$\left(\frac{64}{3} - 24 + 20 =\right) \frac{52}{3}$ or $\frac{104}{6}$ etc condone 17.3 but not $16\frac{4}{3}$ etc
	Area trapezium $=\frac{1}{2}(x_B)(5+y_B)$	B1√		FT their numerical values of x_B, y_B Area = $\frac{1}{2}\times 4\times 14$ (= 28)
	Area of shaded region $=28-17\frac{1}{3}$			
	$=10\frac{2}{3}$	A1	4	CSO; $\frac{32}{3}$, accept 10.7 or better
	Total		**13**	

MPC1

Q	Solution	Marks	Total	Comments
6(a)	$(x-5)^2+(y-8)^2$	B1		
	$= 25$	B1	2	condone 5^2
(b)(i)	$(2-5)^2+(12-8)^2$ $= 9+16 \quad = 25$ $\Rightarrow A$ lies on circle (must have concluding statement and circle equation correct if using equation)	B1	1	or $AC^2 = 3^2+4^2$ hence $AC = 5$; (also radius = 5) CSO $(\Rightarrow \text{ radius} = AC) \Rightarrow A$ lies on circle (must have concluding statement & RHS of circle equation correct or r =5 stated if Pythagoras is used)
(ii)	grad $AC = -\frac{4}{3}$	B1		
	Gradient of tangent is $\frac{3}{4}$	B1√		FT their –1/ grad AC
	$y-12 = $ '*their tangent grad*' $(x-2)$	M1		**or** y = '*their tangent grad*' $x + c$ & attempt to find c using $x = 2$, $y = 12$
	$y-12=\frac{3}{4}(x-2)$ ***or*** $y=\frac{3}{4}x+\frac{21}{2}$ etc	A1		correct equation in any form
	$3x-4y+42=0$	A1	5	CSO; must have integer coefficients with all terms on one side of equation accept $0 = 8y - 6x - 84$ etc
(c)(i)	$(CM^2=) \quad (7-5)^2+(12-8)^2$	M1		**or** $(CM^2=) \quad 20$
	$(\Rightarrow CM = \sqrt{20}) \quad \Rightarrow (CM =) \quad 2\sqrt{5}$	A1	2	
(ii)	$PM^2 = PC^2 - CM^2 \quad = 25-20$	M1		Pythagoras used correctly eg $d^2+(2\sqrt{5})^2=5^2$
	$\Rightarrow PM = \sqrt{5}$	A1		
	Area $\Delta PCQ = \sqrt{5}\times 2\sqrt{5}$ $= 10$	A1	3	CSO
	Total		**13**	

MPC1

Q	Solution	Marks	Total	Comments
7(a)(i)	$\left.\begin{array}{l}\left(\text{Increasing} \Rightarrow\right) \dfrac{\mathrm{d}y}{\mathrm{d}x} > 0 \\ 20x - 6x^2 - 16 > 0\end{array}\right\}$ either	M1		correct interpretation of y increasing
	$\Rightarrow 6x^2 - 20x + 16 < 0$ *or* (2) $(10x - 3x^2 - 8) > 0$			must see at least one of these steps before final answer for A1
	$\Rightarrow 3x^2 - 10x + 8 < 0$	A1	2	CSO **AG** no errors in working
(ii)	$(3x-4)(x-2)$	M1		correct factors or correct use of quadratic equation formula as far as $\dfrac{10 \pm \sqrt{4}}{6}$
	CVs are $\dfrac{4}{3}$ and 2	A1		condone $\dfrac{8}{6}$ and $\dfrac{12}{6}$ here but not in final line
	 	M1		sketch or sign diagram
	$\dfrac{4}{3} < x < 2$ ***Mark their final line as their answer***	A1	4	or $2 > x > \dfrac{4}{3}$ accept $x < 2$ **AND** $x > \dfrac{4}{3}$ but **not** $x < 2$ **OR** $x > \dfrac{4}{3}$ **nor** $x < 2$, $x > \dfrac{4}{3}$

MPC1

Q	Solution	Marks	Total	Comments
7(b)(i)	$x = 2$; $\left(\frac{dy}{dx} =\right) 40 - 24 - 16$	M1		sub $x = 2$ into $\frac{dy}{dx}$ and simplify terms
	$\frac{dy}{dx} = 0 \Rightarrow$ tangent at P is parallel to the x-axis	A1	2	must be all correct working plus statement
(ii)	$x = 3$; $\frac{dy}{dx} = 20\times3 - 6\times3^2 - 16$	M1		must attempt to sub $x = 3$ into $\frac{dy}{dx}$
	(= 60–54–16) $= -10$	A1		
	Gradient of normal $= \frac{1}{10}$	A1√		$\frac{-1}{\text{"}their\ -10\text{"}}$
	Normal: $(y - -1) = \text{'their grad'}(x - 3)$	m1		normal attempted with correct coordinates used and gradient obtained from their $\frac{dy}{dx}$ value
	$y + 1 = \frac{1}{10}(x - 3)$	A1		any correct form, eg $10y = x - 13$ but must simplify – – to +
	(Equation of tangent at P is) $y = 3$	B1		
	$x = 43$	A1	7	CSO; $\Rightarrow R\,(43, 3)$
	Total		**15**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2012 exams

A-level

Code	Title	Max. Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points A*	A	B	C	D	E
MD01	MATHEMATICS UNIT MD01	75	-	60	55	50	45	40
MD02	MATHEMATICS UNIT MD02	75	68	61	53	45	38	31
MFP1	MATHEMATICS UNIT MFP1	75	-	61	54	47	41	35
MFP2	MATHEMATICS UNIT MFP2	75	68	63	56	49	42	35
MFP3	MATHEMATICS UNIT MFP3	75	70	65	57	49	41	33
MFP4	MATHEMATICS UNIT MFP4	75	61	55	48	41	34	28
MM1A	MATHEMATICS UNIT MM1A	100	-	79	69	59	49	39
MM1A/W	*MATHEMATICS UNIT MM1A - WRITTEN*	*75*		*59*				*29*
MM1A/C	*MATHEMATICS UNIT MM1A - COURSEWORK*	*25*		*20*				*10*
MM1B	MATHEMATICS UNIT MM1B	75	-	57	49	41	33	26
MM2B	MATHEMATICS UNIT MM2B	75	69	63	55	48	41	34
MM03	MATHEMATICS UNIT MM03	75	62	55	48	41	34	27
MM04	MATHEMATICS UNIT MM04	75	67	60	52	44	37	30
MM05	MATHEMATICS UNIT MM05	75	67	60	52	44	37	30
MPC1	MATHEMATICS UNIT MPC1	75	-	58	51	44	37	30
MPC2	MATHEMATICS UNIT MPC2	75	-	51	46	41	36	31
MPC3	MATHEMATICS UNIT MPC3	75	67	61	55	49	43	38
MPC4	MATHEMATICS UNIT MPC4	75	59	53	47	41	36	31
MS1A	MATHEMATICS UNIT MS1A	100	-	76	67	59	51	43