General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MPC1

Unit Pure Core 1

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have: • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.	

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The line AB has equation $7x + 3y = 13$.

(a) Find the gradient of AB. *(2 marks)*

(b) The point C has coordinates $(-1, 3)$.

(i) Find an equation of the line which passes through the point C and which is parallel to AB. *(2 marks)*

(ii) The point $(1\frac{1}{2}, -1)$ is the mid-point of AC. Find the coordinates of the point A. *(2 marks)*

(c) The line AB intersects the line with equation $3x + 2y = 12$ at the point B. Find the coordinates of B. *(3 marks)*

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2 (a) (i) Express $\sqrt{48}$ in the form $k\sqrt{3}$, where k is an integer. *(1 mark)*

(ii) Simplify $\dfrac{\sqrt{48} + 2\sqrt{27}}{\sqrt{12}}$, giving your answer as an integer. *(3 marks)*

(b) Express $\dfrac{1 - 5\sqrt{5}}{3 + \sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers. *(4 marks)*

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3 The volume, $V\,\text{m}^3$, of water in a tank after time t seconds is given by

$$V = \frac{t^3}{4} - 3t + 5$$

(a) Find $\dfrac{\text{d}V}{\text{d}t}$. *(2 marks)*

(b) (i) Find the rate of change of volume, in $\text{m}^3\,\text{s}^{-1}$, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the volume is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the positive value of t for which V has a stationary value. *(3 marks)*

(ii) Find $\dfrac{\text{d}^2V}{\text{d}t^2}$, and hence determine whether this stationary value is a maximum value or a minimum value. *(3 marks)*

4 (a) Express $x^2 + 5x + 7$ in the form $(x + p)^2 + q$, where p and q are rational numbers. *(3 marks)*

(b) A curve has equation $y = x^2 + 5x + 7$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) State the equation of the line of symmetry of the curve. *(1 mark)*

(iii) Sketch the curve, stating the value of the intercept on the y-axis. *(3 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$. *(3 marks)*

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5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 2x^2 + 3$.

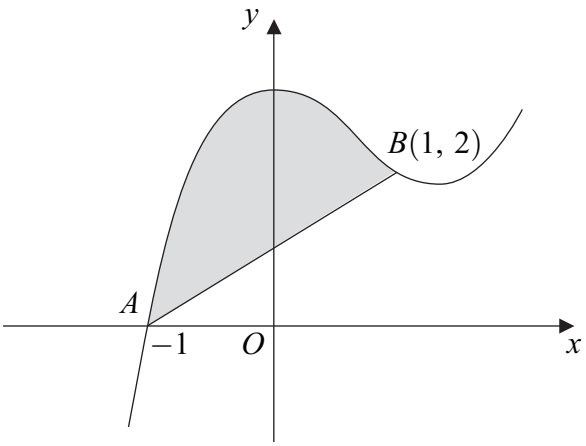
(a) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(b) Use the Factor Theorem to show that $x + 1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(c) (i) Express $\mathrm{p}(x) = x^3 - 2x^2 + 3$ in the form $(x + 1)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(ii) Hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root. *(2 marks)*

6 The curve with equation $y = x^3 - 2x^2 + 3$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and passes through the point $B(1, 2)$.

(a) Find $\int_{-1}^{1} (x^3 - 2x^2 + 3)\,dx$. *(5 marks)*

(b) Hence find the area of the shaded region bounded by the curve $y = x^3 - 2x^2 + 3$ and the line AB. *(3 marks)*

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7 Solve each of the following inequalities:

(a) $2(4 - 3x) > 5 - 4(x + 2)$; *(2 marks)*

(b) $2x^2 + 5x \geqslant 12$. *(4 marks)*

8 A circle has centre $C(3, -8)$ and radius 10.

(a) Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k$$ *(2 marks)*

(b) Find the x-coordinates of the points where the circle crosses the x-axis. *(3 marks)*

(c) The tangent to the circle at the point A has gradient $\frac{5}{2}$. Find an equation of the line CA, giving your answer in the form $rx + sy + t = 0$, where r, s and t are integers. *(3 marks)*

(d) The line with equation $y = 2x + 1$ intersects the circle.

(i) Show that the x-coordinates of the points of intersection satisfy the equation

$$x^2 + 6x - 2 = 0$$ *(3 marks)*

(ii) Hence show that the x-coordinates of the points of intersection are of the form $m \pm \sqrt{n}$, where m and n are integers. *(2 marks)*

END OF QUESTIONS

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1(a)	$y = \frac{13}{3} - \frac{7}{3}x$	M1		attempt at $y = a + bx$ or $\frac{\Delta y}{\Delta x}$ with 2 correct points
	(gradient =) $-\frac{7}{3}$	A1	2	condone slip in rearranging if gradient is correct
(b)(i)	$y - 3 =$ 'their grad'$(x - -1)$	M1		*or* $7x + 3y = k$ and attempt at k using $x = -1$ and $y = 3$ *or* $y =$ (their m)$x + c$ and attempt at c using $x = -1$ and $y = 3$
	$y - 3 = -\frac{7}{3}(x+1)$ or $7x + 3y = 2$ or $y = -\frac{7}{3}x + c, \quad c = \frac{2}{3}$	A1cso	2	correct equation in any form and replacing $- -$ with + sign
(ii)	$(4, -5)$	B1,B1	2	$x = 4$, $y = -5$ withhold if clearly from incorrect working
(c)	$7x + 3y = 13$ and $3x + 2y = 12$ $\Rightarrow$ equation in x or y only	M1		must use correct pair of equations and attempt to eliminate y (or x)
	$x = -2$	A1		
	$y = 9$	A1	3	
	Total		**9**	

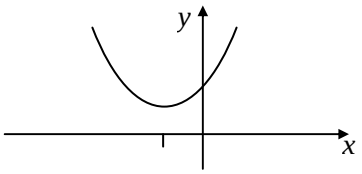
MPC1 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$\sqrt{48} = 4\sqrt{3}$	B1	1	condone $k = 4$ stated
(ii)	$\dfrac{4\sqrt{3}+6\sqrt{3}}{2\sqrt{3}}$	M1		attempt to write each term in form $k\sqrt{3}$ with at least 2 terms correctly obtained
		A1		correct unsimplified in terms of $\sqrt{3}$ only
	$= 5$	A1cso	3	must simplify fraction to 5
				Alternative 1 $\times\dfrac{\sqrt{12}}{\sqrt{12}}$ $\left(or \times \dfrac{\sqrt{3}}{\sqrt{3}}\right)$ M1
				correct with integer terms $= \dfrac{24+36}{12}$ A1
				$= 5$ A1cso
				Alternative 2 $\dfrac{\sqrt{48}+\sqrt{108}}{\sqrt{12}}$ M1
				$= \sqrt{4}+\sqrt{9}$ A1
				$= 5$ A1cso
				Alternative 3 $\sqrt{\dfrac{48}{12}}+2\sqrt{\dfrac{27}{12}}$ M1
				$= 2+2\sqrt{\dfrac{9}{4}}$ A1
				$= 5$ A1cso
				if hybrid of methods used, award M1 and most appropriate first A1
				NMS (answer =) 5 scores full marks
(b)	$\dfrac{1-5\sqrt{5}}{3+\sqrt{5}} \times \dfrac{3-\sqrt{5}}{3-\sqrt{5}}$	M1		
	(numerator =) $3-\sqrt{5}-15\sqrt{5}+25$	m1		correct unsimplified but must write $5\sqrt{5}\sqrt{5} = 25$ PI by 28 seen later
	(denominator = 9 – 5 =) 4	B1		must be seen as denominator
	giving $\dfrac{28-16\sqrt{5}}{4}$			
	(answer =) $7-4\sqrt{5}$	A1	4	$m = 7$, $n = -4$
	Total		**8**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\left(\frac{dV}{dt} =\right) \frac{3t^2}{4} - 3$	M1 A1	2	one of these terms correct all correct (no + *c* etc)
(b)(i)	$t = 1 \Rightarrow \frac{dV}{dt} = \frac{3}{4} - 3$	M1		substituting $t = 1$ into their $\frac{dV}{dt}$
	$= -2\frac{1}{4}$	A1cso	2	(–2.25 OE) *BUT* must have $\frac{dV}{dt}$ correct
(ii)	Volume is decreasing when $t = 1$			must have used $\frac{dV}{dt}$ in (b)(i) or starts again
	because $\frac{dV}{dt} < 0$	E1√	1	must state that $\frac{dV}{dt} < 0$ (or $-2\frac{1}{4} < 0$ etc) ft increasing plus explanation if their $\frac{dV}{dt} > 0$
(c)(i)	$\left(\frac{dV}{dt} = 0 \Rightarrow\right) \frac{3t^2}{4} - 3 = 0$	M1		PI by "correct" equation being solved
	$\Rightarrow t^2 = 4$	A1√		obtaining $t^n = k$ correctly from their $\frac{dV}{dt}$
	$t = 2$	A1cso	3	withhold if answer left as $t = \pm 2$
(ii)	$\left(\frac{d^2V}{dt^2} =\right) \frac{3t}{2}$	B1√		(condone unsimplified) ft their $\frac{dV}{dt}$
	When $t = 2$, $\frac{d^2V}{dt^2} = 3$ or $\frac{d^2V}{dt^2} > 0$	M1		ft their $\frac{d^2V}{dt^2}$ and value of *t* from (c)(i)
	$\Rightarrow$ minimum	A1cso	3	
	Total		**11**	

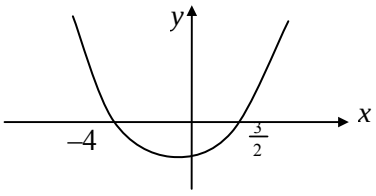
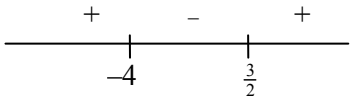
MPC1 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$(x+2.5)^2$	B1		$p = \frac{5}{2}$
	$q = 7 - \text{'their'}\, p^2$	M1		unsimplified attempt at $q = 7 - \text{'their'}\, p^2$
				$q = 7 - \frac{25}{4} = \frac{3}{4}$
	$(x+2.5)^2 + 0.75$	A1	3	
	mark their final line as their answer			
(b)(i)	$x =$ – 'their' p or $y =$ 'their' q	M1		or $x = -\frac{5}{2}$ cao found using calculus
	$\left(-\frac{5}{2},\ \frac{3}{4}\right)$	A1cao	2	condone correct coordinates stated $x = -2.5$, $y = 0.75$
(ii)	$x = -\frac{5}{2}$	B1√	1	correct *or* ft "$x =$ – 'their' p"
(iii)		B1		y intercept = 7 stated or seen in table as $y = 7$ when $x = 0$ or 7 marked as intercept on y-axis (any graph)
		M1		∪ shape
		A1	3	vertex above x-axis in correct quadrant and parabola extending beyond y-axis into first quadrant
(c)	Translation	E1		and no other transformation
	through $\begin{bmatrix} -\frac{5}{2} \\ \frac{3}{4} \end{bmatrix}$	M1		ft either 'their' $-p$ or 'their' q or one component correct for M1
		A1cao	3	both components correct for A1; may describe in words or use a vector
	Total		**12**	

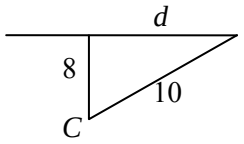
MPC1 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$p(3) = 3^3 - 2\times 3^2 + 3\,(= 27 - 18 + 3)$ $= 12$	M1 A1	2	p(3) attempted; not long division
(b)	$p(-1) = (-1)^3 - 2(-1)^2 + 3$	M1		p(–1) attempted; not long division
	$p(-1) = -1 - 2 + 3 = 0 \Rightarrow x+1$ is a factor	A1cso	2	correctly shown = 0 plus statement
(c)(i)	Quadratic factor $(x^2 - 3x + 3)$	M1		$b = -3$ or $c = 3$ by inspection or full long division attempt or comparing coefficients
	$(p(x) =)\quad (x+1)(x^2 - 3x + 3)$	A1	2	must see correct product
(ii)	Discriminant of quadratic $b^2 - 4ac = (-3)^2 - 4\times 3$	M1		'their' discriminant considered possibly within quadratic equation formula
	$b^2 - 4ac < 0 \Rightarrow$ no real roots from quadratic $\Rightarrow$ only one real root	A1cso	2	
	Total		**8**	
6(a)	$\int_{-1}^{1} (x^3 - 2x^2 + 3)\,dx$			
	$= \left[\frac{x^4}{4} - \frac{2x^3}{3} + 3x\right]_{-1}^{1}$	M1 A1 A1		one term correct another term correct all correct (condone + *c*)
	$= \left(\frac{1}{4} - \frac{2}{3} + 3\right) - \left(\frac{1}{4} + \frac{2}{3} - 3\right)$	B1√		'their' F(1) – F(–1) with $(-1)^3$ etc evaluated correctly but must have earned M1
	$= 4\frac{2}{3}$	A1cso	5	$\frac{14}{3}$, $\frac{56}{12}$ etc but combined as single fraction
(b)	Area of $\Delta\ \left(= \frac{1}{2}\times 2\times 2\right)$ $= 2$	B1		PI
	Shaded region has area $4\frac{2}{3} - 2$	M1		± their (a) ± their Δ area
	$= 2\frac{2}{3}$	A1cso	3	$\frac{8}{3}$, $\frac{32}{12}$ etc but combined as single fraction
	Total		**8**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$8-6x > 5-4x-8$ $11 > 2x$	M1		multiplying out correctly and $>$ sign used
	$x < 5\frac{1}{2}$ $\left(\text{or } x < \frac{11}{2} \right)$	A1cso	2	accept $5.5 > x$ OE
(b)	$2x^2+5x-12 \geqslant 0$			
	$(x+4)(2x-3)$	M1		correct factors (or roots unsimplified) $\frac{-5\pm\sqrt{121}}{4}$
	Critical values are -4 and $\frac{3}{2}$	A1		both CVs correct; condone $\frac{6}{4}$, $-\frac{16}{4}$ etc here but must be single fractions
		M1		sketch or sign diagram including values 
	$x \leqslant -4$, $x \geqslant \frac{3}{2}$ ***take their final line as their answer***	A1	4	fractions must be simplified condone use of **OR** but not **AND**
	Total		**6**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$(x-3)^2+(y+8)^2$	B1		accept $(y--8)^2$
	$=100$	B1	2	condone RHS $=10^2$ or $k=10^2$
(b)	$y=0 \Rightarrow$ 'their' $(x-a)^2+b^2=k$	M1		**Alternative** 
	$(x-3)^2=36$ or $x^2-6x-27\ (=0)$ (PI)	A1		
	$\Rightarrow x=-3,\ 9$	A1	3	$(d^2=)\ 10^2-8^2$ M1
				$d^2=36$ A1 or $d=6$
				$\Rightarrow x=-3,\ 9$ A1
(c)	Line CA has gradient $-\frac{2}{5}$	M1		
	CA has equation $(y+8)=-\frac{2}{5}(x-3)$	A1		any form of correct equation eg $y=-\frac{2}{5}x+c,\quad c=-\frac{34}{5}$
	$2x+5y+34=0$	A1cso	3	integer coefficients - all terms on 1 side
(d)(i)	their $(x-3)^2+(2x+1+8)^2$ *or* $x^2+(2x+1)^2-6x+16(2x+1)\quad(+73)$	M1		substituting $y=2x+1$ correctly into LHS of "their" circle equation and attempt to expand in terms of x only
	$x^2-6x+9+4x^2+36x+81=100$ *or* $x^2+4x^2+4x+1-6x+32x+16+73=100$	A1		any correct equation (with brackets expanded)
	$\Rightarrow 5x^2+30x-10=0$			must see this line or equivalent
	$\Rightarrow x^2+6x-2=0$	A1cso	3	AG; all algebra must be correct
(ii)	$(x+3)^2=11$	M1		or correct use of formula
				must get as far as $x=\frac{-6\pm\sqrt{44}}{2}$
	$x=-3\pm\sqrt{11}$	A1cso	2	exactly this
	Total		**13**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - June 2011 exams

A-level

Code	Title	Max. Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points					
			A*	A	B	C	D	E
HBIO2	GCE HUMAN BIOLOGY UNIT 2	80	-	56	51	46	41	37
HBI3T	GCE HUMAN BIOLOGY UNIT 3T	50	-	41	38	35	33	31
HBI3X	GCE HUMAN BIOLOGY UNIT 3X	50	-	34	30	26	22	19
HBIO4	GCE HUMAN BIOLOGY UNIT 4	90	61	56	51	46	41	36
HBIO5	GCE HUMAN BIOLOGY UNIT 5	90	65	60	55	50	45	41
HBI6T	GCE HUMAN BIOLOGY UNIT 6T	50	43	40	37	34	31	28
HBI6X	GCE HUMAN BIOLOGY UNIT 6X	50	43	39	35	31	27	24
INFO1	GCE INFO AND COMM TECH UNIT 1	80	-	53	48	43	38	33
INFO2	GCE INFO AND COMM TECH UNIT 2	80	-	53	47	41	35	30
INFO3	GCE INFO AND COMM TECH UNIT 3	100	72	66	60	54	48	43
INFO4	GCE INFO AND COMM TECH UNIT 4	70	63	57	50	43	36	30
LAW01	GCE LAW UNIT 1	96	-	73	66	59	52	46
LAW02	GCE LAW UNIT 2	94	-	70	61	52	43	35
LAW03	GCE LAW UNIT 3	80	69	62	55	48	42	36
LAW04	GCE LAW UNIT 4	85	73	66	59	53	47	41
XMCAS	GCE MATHEMATICS UNIT XMCAS	125	-	99	89	79	70	61
MD01	GCE MATHEMATICS UNIT D01	75	-	59	52	45	39	33
MFP1	GCE MATHEMATICS UNIT FP1	75	-	61	54	48	42	36
MM1A	GCE MATHEMATICS UNIT M1A	100	No candidates were entered for this unit					
MM1B	GCE MATHEMATICS UNIT M1B	75	-	62	55	48	41	34
MPC1	GCE MATHEMATICS UNIT PC1	75	-	62	55	48	41	34
MS1A	GCE MATHEMATICS UNIT S1A	100	-	73	63	54	45	36
MS/SS1A/W	*GCE MATHEMATICS UNIT S1A - WRITTEN*	*75*		*53*				*26*
MS/SS1A/C	*GCE MATHEMATICS UNIT S1A - COURSEWORK*	*25*		*20*				*10*