General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MPC1

Unit Pure Core 1

Monday 24 May 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

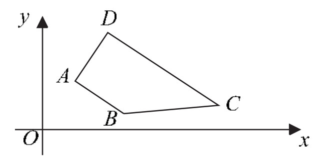
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The trapezium $ABCD$ is shown below.



The line AB has equation $2x + 3y = 14$ and DC is parallel to AB.

(a) Find the gradient of AB. *(2 marks)*

(b) The point D has coordinates $(3, 7)$.

(i) Find an equation of the line DC. *(2 marks)*

(ii) The angle BAD is a right angle. Find an equation of the line AD, giving your answer in the form $mx + ny + p = 0$, where m, n and p are integers. *(4 marks)*

(c) The line BC has equation $5y - x = 6$. Find the coordinates of B. *(3 marks)*

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2 (a) Express $\left(3 - \sqrt{5}\right)^2$ in the form $m + n\sqrt{5}$, where m and n are integers. *(2 marks)*

(b) Hence express $\dfrac{\left(3 - \sqrt{5}\right)^2}{1 + \sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers. *(4 marks)*

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3 The polynomial $p(x)$ is given by

$$p(x) = x^3 + 7x^2 + 7x - 15$$

(a) (i) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

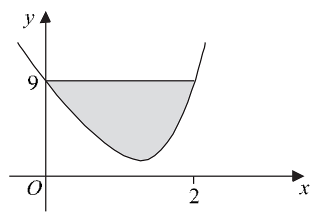
(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 2$. *(2 marks)*

(c) (i) Verify that $p(-1) < p(0)$. *(1 mark)*

(ii) Sketch the curve with equation $y = x^3 + 7x^2 + 7x - 15$, indicating the values where the curve crosses the coordinate axes. *(4 marks)*

4 The curve with equation $y = x^4 - 8x + 9$ is sketched below.



The point (2, 9) lies on the curve.

(a) (i) Find $\int_0^2 (x^4 - 8x + 9)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line $y = 9$. *(2 marks)*

(b) The point $A(1, 2)$ lies on the curve with equation $y = x^4 - 8x + 9$.

(i) Find the gradient of the curve at the point A. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

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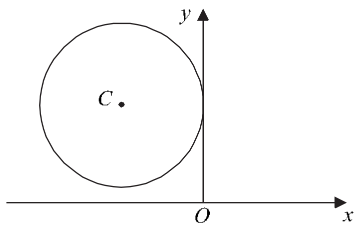
5 A circle with centre $C(-5, 6)$ touches the y-axis, as shown in the diagram.



(a) Find the equation of the circle in the form

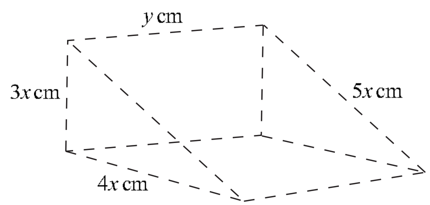
$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) (i) Verify that the point $P(-2, 2)$ lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point P. *(3 marks)*

(iii) The mid-point of PC is M. Determine whether the point P is closer to the point M or to the origin O. *(4 marks)*

6 The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths $3x$ cm, $4x$ cm and $5x$ cm, and the length of the prism is y cm, as shown in the diagram.



The total surface area of the five faces is 144 cm^2.

(a) (i) Show that $xy + x^2 = 12$. *(3 marks)*

(ii) Hence show that the volume of the block, $V\text{ cm}^3$, is given by

$$V = 72x - 6x^3$$ *(2 marks)*

(b) (i) Find $\frac{dV}{dx}$. *(2 marks)*

(ii) Show that V has a stationary value when $x = 2$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 2$. *(2 marks)*

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7 (a) (i) Express $2x^2 - 20x + 53$ in the form $2(x - p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Use your result from part **(a)(i)** to explain why the equation $2x^2 - 20x + 53 = 0$ has no real roots. *(2 marks)*

(b) The quadratic equation $(2k - 1)x^2 + (k + 1)x + k = 0$ has real roots.

(i) Show that $7k^2 - 6k - 1 \leqslant 0$. *(4 marks)*

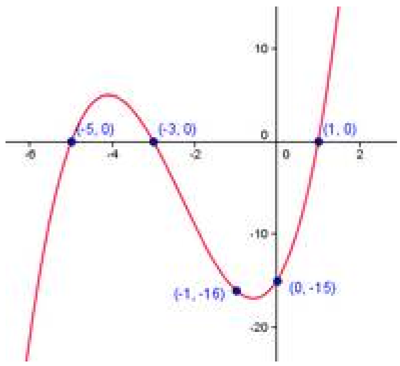
(ii) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 - Jun 2010 – Answers

Question 1:	Exam report
$AB: 2x+3y=14$ $a)\ 3y=-2x+14 \qquad y=-\frac{2}{3}x+\frac{14}{3}$ The gradient of AB is $-\frac{2}{3}$. $b)\ i)\ D(3,7)$ *and* Dc is paralle to AB so $m_{DC}=m_{AB}=-\frac{2}{3}$ The equation of DC: $y-7=-\frac{2}{3}(x-3)$ $3y-21=-2x+6$ $2x+3y=27$ $ii)$ The line AD is perpendicular to AB so $m_{AD}=-\frac{1}{m_{AB}}=\frac{3}{2}$ The eqaution of AD: $y-7=\frac{3}{2}(x-3)$ $2y-14=3x-9$ $3x-2y+5=0$ $c)$ The point B is the intersection of AB and BC. Solve simultaneously $\begin{cases}2x+3y=14\\-x+5y=6\end{cases}$ $(\times 2)$ $\begin{cases}2x+3y=14\\-2x+10y=12\end{cases}$ $13y=26\ \ so\ \ y=2$ *and* $x=5y-6=10-6=4$ The coordinates of B are $(4,2)$.	Part (a) Many candidates were unable to make *y* the subject of the equation 2*x*+3*y* =14 and, as a result, many incorrect answers for the gradient were seen. Those who tried to use two points on the line to find the gradient were rarely successful. Part (b)(i) Those candidates who obtained a value for the gradient in part (a) were usually aware that the line *DC* had the same gradient. Those using *y=mx+c* often made errors when finding the value of *c*, whereas those writing down an equation of the form $y - y_1 = m(x - x_1)$ usually scored full marks. Part (b)(ii) Most candidates realised that the product of the gradients of perpendicular lines should be –1 and credit was given for using this result together with their answer from part (a). Careless arithmetic prevented many from obtaining the final equation in the given form with integer coefficients. Part (c) Although many correct answers for the coordinates of *B* were seen, the simultaneous equations defeated a large number of candidates. No credit was given for mistakenly using their equation from part (b)(i) or part (b)(ii) instead of the correct equation for *AB*, clearly printed below the diagram. Many did not recognize the need to use the equation of *AB* at all. It was common to see *x* = 0 or *y* = 0 substituted into the equation for *BC* and then solved to obtain the other coordinate.

Question 2:	Exam report
$a)\ \left(3-\sqrt{5}\right)^2=9+5-6\sqrt{5}=14-6\sqrt{5}$ $b)\ \frac{(3-\sqrt{5})^2}{1+\sqrt{5}}=\frac{(3-\sqrt{5})^2}{1+\sqrt{5}}\times\frac{1-\sqrt{5}}{1-\sqrt{5}}=\frac{\left(14-6\sqrt{5}\right)\left(1-\sqrt{5}\right)}{1-5}$ $=\frac{14-14\sqrt{5}-6\sqrt{5}+30}{-4}=\frac{44-20\sqrt{5}}{-4}$ $=-11+5\sqrt{5}$	Part (a) Many candidates were successful with this part, although sign errors and arithmetic slips were common. Part (b) Most candidates recognised the first crucial step of multiplying the numerator and denominator by $1-\sqrt{5}$ and many obtained $\frac{44-20\sqrt{5}}{-4}$, but then inaccurate evaluation of the numerator or poor cancellation led to many failing to obtain the correct final answer.

Question 3:	Exam report
$p(x)=x^3+7x^2+7x-15$ $a)\,i)\ p(-3)=(-3)^3+7\times(-3)^2+7\times(-3)-15$ $=-27+63-21-15=63-63=0$ -3 is a root of p, so $(x+3)$ is a foacto of p $ii)\ x^3+7x^2+7x-15=(x+3)(x^2+4x-5)$ $=(x+3)(x+5)(x-1)$ $b)$ The remainder of the division by $(x-2)$ is p(2) $p(2)=2^3+7\times2^2+7\times2-15$ $p(2)=8+28+14-15=35$ $c)\,i)\ p(-1)=-1+7-7-15=-16$ $p(0)=-15$ $p(-1)<p(0)$ $ii)$ The curve crosses the $x-axis$ at $(-3,0),(-5,0),(1,0)$ The curve crosses the $y-axis$ at $(0,-15)$  	Part (a)(i) Those who used long division instead of the Factor Theorem scored no marks. Most candidates realised the need to show that p(–3) = 0. However quite a few omitted sufficient working such as p(–3) = –27 + 63 –21 –15 = 0, together with a concluding statement about x + 3 being a factor, and therefore failed to score full marks. Part (a)(ii) Many candidates have become quite skilled at writing down the correct product of a linear and quadratic factor and then writing p(x) as the product of three linear factors and these scored full marks. Others used long division or the Factor Theorem effectively but lost a mark for failing to write p(x) as a product of linear factors. Others tried methods involving comparing coefficients, but often after several lines of working were unable to find the correct values of the coefficients because of poor algebraic manipulation. Speculative attempts to write down p(x) immediately as the product of three factors were rarely successful. Part (b) Those candidates who used the Remainder Theorem were usually able to find the correct remainder, though once again arithmetic errors abounded. Those who used long division, synthetic division or other algebraic methods again scored no marks since the question specifically asked candidates to use the Remainder Theorem. Part (c)(i) This part of the question was intended to help candidates when sketching the curve. Those who found the correct values of p(–1) and p(0) usually scored the mark but in future a carefully written proof may be called for. Here again many arithmetic errors were seen. Part (c)(ii) The sketch was intended to bring various parts of the question together but even very good candidates ignored the hint from part (c)(i) and showed a minimum point on the y-axis. A few lost the final mark when their curve stopped on the x-axis. Confusion between roots and factors spoiled many sketches and several showed the y-intercept of –15 on the positive y-axis. It was disappointing that many candidates did not recognize the shape of a cubic curve at all.

Question 4:	Exam report
$y=x^4-8x+9$ $a)\,i)\ \int_0^2(x^4-8x+9)\,dx=\left[\frac{1}{5}x^5-4x^2+9x\right]_0^2$ $=\left(\frac{32}{5}-16+18\right)-(0)=8\tfrac{2}{5}$ $ii)$ The area of the shaded region is area rectangle – area beneath the curve $9\times2-8\tfrac{2}{5}=9\tfrac{3}{5}$ $b)\ A(1,2)$ lies on the curve. $i)$ The gradient of the curve at a is $\frac{dy}{dx}(x=1)$. $\frac{dy}{dx}=4x^3-8\ \ and\ \ for\ x=1, m_A=4\times1^3-8=-4$ $ii)$ The equation of the tangent at A is : $y-2=-4(x-1)$ $y=-4x+6$	Part (a)(i) Most candidates were able to integrate the expression with only the weakest candidates unable to do this basic integration. Poor notation was used with many including the integral sign after integrating. It would have been thought that this bad habit would have been corrected by the time of the examination. Many candidates did not find the actual value of the definite integral until part (a)(ii) and on this occasion full credit was given. It was alarming that many candidates who had correct fractions were unable to combine these to give a value of 8.4 or equivalent. Weaker candidates were seen substituting values into the expression for y or $\frac{dy}{dx}$ showing a complete lack of understanding. Part (a)(ii) It was necessary to consider a rectangle of area 18 and then to subtract their answer from part (a)(i) in order to obtain the area of the shaded region. Many believed that the area of the rectangle was 9 and others failed to do this basic subtraction correctly, even when their answer to part (a)(i) was correct. Part (b)(i) Many candidates did not realise the need to find $\frac{dy}{dx}$ before substituting the value x = 1 and thus failed to score some easy marks for finding the gradient of the curve. A substantial number of candidates tried to calculate the gradient of the straight line between two points on the curve and scored no marks for this. Part (b)(ii) Unfortunately many candidates tried to find the equation of the normal instead of the tangent to the curve. Otherwise, since there was a generous follow through in this part of the question, most were able to score this final mark. The only exceptions were those who insisted on using y = mx + c where poor arithmetic often prevented them from finding a value for c.

Question 5:	Exam report
a) Centre of the circle $C(-5,6)$ the circle touches the y-axis, the radius is 5. The equation of the circle is $(x+5)^2+(y-6)^2=5^2$ b) i) $P(-2,2)$ $(-2+5)^2+(2-6)^2$ $=3^2+(-4)^2=9+16=25=5^2$ P lies on the circle. ii) The normal to the circle at P is the line CP Gradient of $CP=m_{CP}=\frac{2-6}{-2+5}=\frac{-4}{3}$ The equation of the normal is : $y-2=-\frac{4}{3}(x+2)$ $3y-6=-4x-8$ $4x+3y=-2$ iii) The distance $PM=\frac{1}{2}r=2.5$ The distance $PO=\sqrt{(-2)^2+(2)^2}=\sqrt{8}=2\sqrt{2}>2.5$ $(\sqrt{2}\approx 1.4)$ The point P is closer to M.	Part (a) Most candidates obtained the correct left hand side of the circle equation but many failed to recognize that the radius was 5. Alarmingly many thought that r was equal to –5 or wrote down the right hand side of the equation as -5^2, thus displaying a fundamental misunderstanding of the idea of radius as a length. Part (b)(i) Most who had the correct circle equation were able to verify that the circle passed through the point P, although those who neglected to make a statement as a conclusion to their calculation failed to earn this mark. Part (b)(ii) The negative signs caused problems for many when finding the gradient of PC and only the better candidates obtained the correct value. Many candidates then found the negative reciprocal of this fraction instead of using the gradient of PC to find the normal to the circle at the point P. Part (b)(iii) There were basically two approaches to this question, although some candidates were merely guessing and no credit was given for a correct answer without supporting working. The most common method involved distances or squares of distances; many made errors in finding the coordinates of M and then struggled with the fractions when squaring and adding to find the length of PM; whereas others noted that the length of PM was simply half the radius. A simple comparison with the length of PO led to the correct conclusion. The second approach was essentially one using vectors or the differences of coordinates, but this method was not always explained correctly and left examiners in some doubt as to whether candidates really understood what they were doing. The best candidates wrote down the correct vectors PM and OP and reasoned that these vectors had the same y-component but different x-components and it was then easy to deduce that P was closer to the point M.

Question 6:	Exam report
a) i) The surface area is $S=\frac{1}{2}\times 3x\times 4x+\frac{1}{2}\times 3x\times 4x+5x\times y+4x\times y+3x\times y$ $S=12x^2+12xy=144cm^2 \quad (\div 12)$ $x^2+xy=12 \quad (1)$ ii) The volume $V=\left(\frac{1}{2}\times 3x\times 4x\right)\times y=6x^2y$ Making y the subject in (1): $y=\frac{12-x^2}{x}$ *Now*, substituting y in V, we have $V=6x^2y=6x^2\times\frac{12-x^2}{x}=72x-6x^3$ b) i) $\frac{dV}{dx}=72-18x^2$ ii) *When* $x=2,\ \frac{dV}{dx}=72-18\times 2^2=72-72=0$ There is a stationary point when $x=2$. c) $\frac{d^2V}{dx^2}=-36x$ *and when* $x=2,\ \frac{d^2V}{dx^2}=-72<0$ *For* $x=2$, The value of V is a MAXIMUM.	Part (a)(i) Usually after a few abortive attempts many candidates realised that they had to add together the areas of the various faces. Once they had the correct expression for the total surface area most candidates were able to obtain the printed result. There was clearly some fudging on the part of weaker candidates and they could earn little more than a single method mark. Part (a)(ii) This was surprisingly one of the biggest discriminators on the paper with only the best candidates being able to obtain the correct expression for V. Trying to make y the subject of the equation from part (a)(i) caused problems for many who took this approach; others substituted for xy in the formula V = 6x(xy) and were often more successful. Once again it was quite common to see totally incorrect expressions being miraculously transformed into the printed answer. Part (b)(i) Most candidates scored full marks for this basic differentiation although it was not always clearly identified as $\frac{dV}{dx}$ in their working. Part (b)(ii) Most substituted x = 2 into their expression for $\frac{dV}{dx}$ and found the value to be zero. It was then necessary to make a statement about the implication of there being a stationary value in order to score full marks. Part (c) Some made a sign error when finding the second derivative, but the majority of candidates scored full marks in this part. Credit was given if the correct conclusion was drawn from the sign of their second derivative, provided no further arithmetic errors occurred.

Question 7:

$$a)\,i)\;2x^2-20x+53=2\left(x^2-10x\right)+53$$
$$=2\left[(x-5)^2-25\right]+53$$
$$=2(x-5)^2-50+53$$
$$=2(x-5)^2+3$$

$$ii)\;2x^2-20x+53=0 \text{ is equivalent to}$$
$$2(x-5)^2+3=0 \text{ or } (x-5)^2=-\frac{3}{2}$$

For all x, $(x-5)^2\ge 0$ so there is no solution.

$b)\;(2k-1)x^2+(k+1)x+k=0$ has real roots,

This means that the discriminant ≥ 0

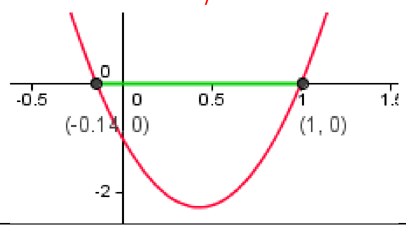
i.e $(k+1)^2-4\times(2k-1)\times(k)\ge 0$

$$k^2+2k+1-8k^2+4k\ge 0$$
$$-7k^2+6k+1\ge 0 \qquad (\times -1)$$
$$7k^2-6k-1\le 0$$

$$iii)\;7k^2-6k-1\le 0$$
$$(7k+1)(k-1)\le 0$$

critical values $-\frac{1}{7}$ and 1

$(7k+1)(k-1)\le 0$ for $-\frac{1}{7}\le k\le 1$



Exam report

Part (a)(i) Candidates did not seem well drilled in completing the square when the coefficient of x^2 is not equal to 1. It was very rare to see a correct answer here although a few did realise that p = 5. Clearly further practice is required at this type of question.

Part (a)(ii) Only the more able candidates were able to reason sufficiently well using the result from part (a)(i) as well as providing a concluding statement. No credit was given for using the discriminant to show that the equation had no real roots since the wording of the question excluded this approach.

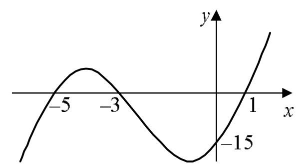
Part (b)(i) This kind of question has been set several times before and the usual errors were seen. Candidates should state the condition for real roots ($b^2 - 4ac \ge 0$) and find an expression in terms of k for the discriminant using the correct inequality throughout. The inequality is then reversed when multiplying by a negative number. Again, many candidates would benefit from practising this technique, using brackets where appropriate to avoid algebraic errors.

Part (b)(ii) The factorisation of the quadratic was usually correct, but several candidates wrote down one of the critical values as 1/7. Most found critical values and either stopped or immediately tried to write down a solution without any working. Candidates are strongly advised to use a sign diagram or a sketch graph showing their critical values when solving a quadratic inequality.

GRADE BOUNDARIES

Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	63	55	47	40	33

Q	Solution	Marks	Total	Comments
1(a)	$y=\frac{14}{3}-\frac{2}{3}x$	M1		Attempt at $y = ...$
	Gradient $AB = -\frac{2}{3}$	A1	2	Condone error in rearranging equation
(b)(i)	$y-7=$"*their grad AB*"$(x-3)$	M1		or $2x+3y=k$ and sub $x=3, y=7$ or $y=mx+c$, m – *their grad AB* and attempt to find c using $x=3$, $y=7$
	$y-7=-\frac{2}{3}(x-3)$ OE	A1	2	$2x+3y=27$, $y=-\frac{2}{3}x+9$ etc
(ii)	$m_1m_2=-1$	M1		or *negative reciprocal* (stated or used PI)
	$\Rightarrow$ grad $AD=\frac{3}{2}$	A1√		FT their grad AB
	$y-7=\frac{3}{2}(x-3)$	A1		Any correct equation unsimplified
	$\Rightarrow 3x-2y+5=0$	A1	4	Integer coefficients; all terms on one side, condone different order or multiples. eg $0=4y-6x-10$
(c)	$2x+3y=14$ and $5y-x=6$ used with x or y eliminated (generous)	M1		$2(5y-6)+3y=14$ etc
	$x=4$,	A1		
	$y=2$	A1	3	$B(4,2)$ full marks NMS
	Total		**11**	
2(a)	$\left(3-\sqrt{5}\right)^2=9-6\sqrt{5}+\left(\sqrt{5}\right)^2$	M1		Allow one slip in one of these terms M0 if middle term is omitted
	$=14-6\sqrt{5}$	A1	2	
(b)	$\frac{\left(3-\sqrt{5}\right)^2}{1+\sqrt{5}}\times\frac{1-\sqrt{5}}{1-\sqrt{5}}$	M1		or $...\times\frac{\sqrt{5}-1}{\sqrt{5}-1}$
	$14+6\sqrt{5}\sqrt{5}-6\sqrt{5}-14\sqrt{5}$ $(=44-20\sqrt{5})$	m1		Expanding ***their*** numerator (condone one error or omission)
	(Denominator) $=-4$	B1		Must be seen as denominator
	(Answer) $=-11+5\sqrt{5}$	A1	4	Accept "answer $=5\sqrt{5}-11$"
	Total		**6**	

Q	Solution	Marks	Total	Comments
3(a)(i)	$\text{p}(-3)=(-3)^3+7(-3)^2+7(-3)-15$ $=-27+63-21-15$	M1		$\text{p}(-3)$ attempted; NOT long division This line alone implies M1
	$\text{p}(-3)=0 \Rightarrow (x+3$ is) factor	A1	2	$\text{p}(-3)$ shown = 0 plus statement
(ii)	$\text{p}(x)=(x+3)(x^2+px+q)$	M1		Full long division, comparing coefficients or by inspection either $p=4$ or $q=-5$
	(Quadratic factor) (x^2+4x-5)	A1		or M1 A1 for either x–1 or x+5 ***clearly*** found using Factor Theorem
	$(\text{p}(x)=)$ $(x+3)(x-1)(x+5)$	A1	3	Must be seen as a product of 3 factors NMS full marks for correct product SC B2 for 3 correct factors listed NMS SC B1 for $(x+3)(x-1)(\ )$ or $(x+3)(x+5)(\ )$ or $(x+3)(x+1)(x-5)$
(b)	$\text{p}(2)=2^3+7\times2^2+7\times2-15$ or $(2+3)(2-1)(2+5)$ (Remainder) = 35	M1 A1cso	2	NOT long division; must be $\text{p}(2)$ May use "their" product of factors
(c)(i)	$\text{p}(-1)=-16$; $\text{p}(0)=-15$ $\Rightarrow \text{p}(-1)<\text{p}(0)$	B1	1	Values must be evaluated correctly
(ii)		B1		y- intercept –15 marked or (0,–15) stated
		M1		Cubic graph – 1 max, 1 min
		A1		∿ shape with –5, –3, 1 marked
	Cannot score M1A0A1 but can score B0M1A1A1	A1	4	Graph correct with minimum point to left of y-axis and going beyond both –5 and 1 Previous A1 must be scored
	Total		**12**	

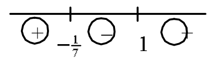
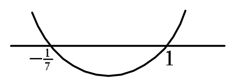
Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{x^5}{5}-\frac{8}{2}x^2+9x$	M1 A1 A1		One term correct Another term correct All correct (may have + c)
	$\frac{32}{5}-16+18$	m1		$F(2)$ attempted
	$=8\frac{2}{5}$	A1	5	$\frac{42}{5}$, 8.4
(ii)	Shaded area = 18 – '*their integral*'	M1		PI by 18 – (a)(i) NMS
	$= 9\frac{3}{5}$	A1	2	$\frac{48}{5}$, 9.6 NMS full marks
(b)(i)	$\frac{dy}{dx}=4x^3-8$	M1 A1		One term correct All correct (no + c etc)
	$x=1 \Rightarrow \frac{dy}{dx}=4-8$	m1		sub $x=1$ into *their* $\frac{dy}{dx}$
	(Gradient of curve) $= -4$	A1cso	4	No ISW
(ii)	$y-2=-4(x-1)$; $y=-4x+c$, $c=6$	B1√	1	any correct form ; FT *their* answer from (b)(i) but must use $x = 1$ and $y = 2$
	Total		**12**	

Q	Solution	Marks	Total	Comments
5(a)	$(x+5)^2+(y-6)^2=5^2$	M1 A1 B1	3	One term correct LHS LHS all correct RHS correct: condone = 25
(b)(i)	sub $x=-2$, $y=2$ into circle equation $3^2+(-4)^2=25$			Circle equation must be correct
	$\Rightarrow$ lies on circle	B1	1	Must have concluding statement
(ii)	Grad $PC = -\frac{4}{3}$	B1		Condone $\frac{4}{-3}$
	Normal to circle has equation $y-6=$ '*their gradient* PC'$(x+5)$ or $y-2=$ '*their gradient* PC'$(x+2)$	M1		M0 if tangent attempted or incorrect coordinates used
	$y-6=-\frac{4}{3}(x+5)$ or $y-2=-\frac{4}{3}(x+2)$	A1cso	3	Any correct form eg $4x+3y+2=0$ $y=-\frac{4}{3}x+c$, $c=-\frac{2}{3}$
(iii)				**Alternative 1**
	$PM=\frac{1}{2}\times$ radius	M1		Attempt at $M\left(-\frac{7}{2},4\right)$ with at least one correct coordinate **and** PM^2 attempted
	$= 2.5$	A1cso		$PM^2=\frac{9}{4}+4=\frac{25}{4}$
	$PO=\sqrt{8}$	B1		$PO^2=4+4=8$
	P is closer to the point M	E1cso	4	Statement following correct values
				Alternative 2
		(M1)		Attempt at $M\left(-\frac{7}{2},4\right)$ with at least one correct coordinate **and** attempt at vectors or difference of coordinates
		(A1cso)		$\overrightarrow{PM}=\begin{pmatrix}-1.5\\2\end{pmatrix}$ OE
		(E1cso)		P is closer to the point M
		(E1)	(4)	Components of their $\overrightarrow{PM}$ and $\overrightarrow{OP}$ considered – ***totally independent*** of M1
	Total		**11**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	S.A. $= 4xy + 5xy + 3xy + 6x^2 + 6x^2$ OE	M1		Condone one slip or omission
	$= 12xy + 12x^2$	A1		
	$144 = 12xy + 12x^2$			Must see this line
	$\Rightarrow xy + x^2 = 12$	A1cso	3	AG
(ii)	(Volume =) $\frac{1}{2} \times 3x \times 4x \times y$ OE	M1		
	$= 6x^2 \times \frac{(12 - x^2)}{x}$			Must see $(y =) \frac{(12-x^2)}{x}$ or $xy = 12 - x^2$ for A1
	$(V =) 72x - 6x^3$	A1	2	AG must be convinced not working back from answer
(b)(i)	$\frac{dV}{dx} = 72 - 18x^2$	M1 A1	2	One term correct All correct (no + c etc)
(ii)	$x = 2 \Rightarrow \frac{dV}{dx} = 72 - 18 \times 2^2$	M1		Substitute $x = 2$ into their $\frac{dV}{dx}$
	$\Rightarrow \frac{dV}{dx} = 72 - 72 = 0$			
	$\Rightarrow$ stationary (value when x = 2)	A1	2	Shown = 0 plus statement Statement may appear first
(c)	$\frac{d^2V}{dx^2} = -36x$	B1√		FT their $\frac{dV}{dx}$
	$\frac{d^2V}{dx^2} = -72$ *or* when $x = 2 \Rightarrow \frac{d^2V}{dx^2} < 0$			
	$\Rightarrow$maximum	E1√	2	FT their $\frac{d^2V}{dx^2}$ value when x = 2 with appropriate conclusion
	Total		**11**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$2(x-5)^2$	B1		$p = 5$
	$+ 3$	B1	2	$q = 3$
(ii)	Stating both $(x-5)^2 \geqslant 0$ and $3 > 0$	M1		FT their p & q, but must have $q > 0$
	$\Rightarrow 2x^2 - 20x + 53 > 0$ *or* $2(x-5)^2 + 3 > 0$			
	$\Rightarrow 2x^2 - 20x + 53 = 0$ has no real roots	A1cso	2	Must have statement and correct p & q.
(b)(i)	$b^2 - 4ac = (k+1)^2 - 4k(2k-1)$	M1		Condone one slip (including x is one slip)
	$= -7k^2 + 6k + 1$	A1		Condone recovery from missing brackets
	real roots $\Rightarrow b^2 - 4ac \geqslant 0$			Their discriminant $\geqslant 0$ (in terms of k)
	$-7k^2 + 6k + 1 \geqslant 0$	B1√		Need not be simplified & may earn earlier
	$\Rightarrow 7k^2 - 6k - 1 \leqslant 0$	A1cso	4	AG (must see sign change)
(ii)	$(7k+1)(k-1)$	M1		**Correct** factors or **correct** use of formula
	Critical values $k = 1, -\frac{1}{7}$	A1		May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working.
	Use of sign diagram or sketch  	M1		If previous A1 earned, sign diagram or sketch must be correct for M1 Otherwise M1 may be earned for an attempt at the sketch or sign diagram using ***their*** critical values.
	$-\frac{1}{7} \leqslant k \leqslant 1$	A1	4	$\left(-\frac{1}{7} < k < 1\right)$, $\left(k \geqslant -\frac{1}{7} \text{ OR } k \leqslant 1\right)$, $\left(k \geqslant -\frac{1}{7},\ k \leqslant 1\right)$ score M1A1M1A0
	Full marks for correct answer NMS Condone $-\frac{2}{14}$ throughout Condone $k \geqslant -\frac{1}{7}$ AND $k \leqslant 1$ for full marks ***Take their final line as their answer.***			*Answer only of* $k < -\frac{1}{7}$, $k < 1$ etc scores M1, A1, M0 since the critical values are evident. *Answer only of* $\frac{1}{7} \leqslant k \leqslant 1$ etc scores M0, M0 since the critical values are not both correct.
	Total		**12**	
	TOTAL		75	