

Answer all questions.

- 1** The line AB has equation $3x + 5y = 11$.
- (a) (i) Find the gradient of AB . (2 marks)
- (ii) The point A has coordinates $(2, 1)$. Find an equation of the line which passes through the point A and which is perpendicular to AB . (3 marks)
- (b) The line AB intersects the line with equation $2x + 3y = 8$ at the point C . Find the coordinates of C . (3 marks)

- 2** (a) Express $\frac{5 + \sqrt{7}}{3 - \sqrt{7}}$ in the form $m + n\sqrt{7}$, where m and n are integers. (4 marks)

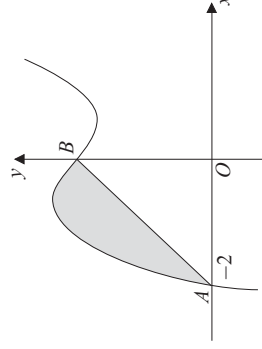
- (b) The diagram shows a right-angled triangle.



- The hypotenuse has length $2\sqrt{5}$ cm. The other two sides have lengths $3\sqrt{2}$ cm and x cm. Find the value of x . (3 marks)

- 3** The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P , where $x = -2$.
- (a) Find $\frac{dy}{dx}$. (3 marks)
- (b) Verify that the point P is a stationary point of the curve. (2 marks)
- (c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P . (3 marks)
- (ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. (1 mark)
- (d) Find an equation of the tangent to the curve at the point where $x = 1$. (4 marks)

- 4** (a) The polynomial $p(x)$ is given by $p(x) = x^3 - x + 6$.
- (i) Find the remainder when $p(x)$ is divided by $x - 3$. (2 marks)
- (ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. (2 marks)
- (iii) Express $p(x) = x^3 - x + 6$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers. (2 marks)
- (iv) The equation $p(x) = 0$ has one root equal to -2 . Show that the equation has no other real roots. (2 marks)
- (b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x -axis at the point $A(-2, 0)$ and the y -axis at the point B .

- (i) State the y -coordinate of the point B . (1 mark)
- (ii) Find $\int_{-2}^0 (x^3 - x + 6) dx$. (5 marks)
- (iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB . (3 marks)

Turn over for the next question

- 5 A circle with centre C has equation

$$(x - 5)^2 + (y + 12)^2 = 169$$

- (a) Write down:
- the coordinates of C ; (1 mark)
 - the radius of the circle. (1 mark)
- (b) (i) Verify that the circle passes through the origin O . (1 mark)
- (ii) Given that the circle also passes through the points $(10, 0)$ and $(0, p)$, sketch the circle and find the value of p . (3 marks)
- (c) The point $A(-7, -7)$ lies on the circle.
- Find the gradient of AC . (2 marks)
 - Hence find an equation of the tangent to the circle at the point A , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (3 marks)
- 6 (a) (i) Express $x^2 - 8x + 17$ in the form $(x - p)^2 + q$, where p and q are integers. (2 marks)
- (ii) Hence write down the minimum value of $x^2 - 8x + 17$. (1 mark)
- (iii) State the value of x for which the minimum value of $x^2 - 8x + 17$ occurs. (1 mark)
- (b) The point A has coordinates $(5, 4)$ and the point B has coordinates $(x, 7 - x)$.
- Expand $(x - 5)^2$. (1 mark)
 - Show that $AB^2 = 2(x^2 - 8x + 17)$. (3 marks)
 - Use your results from part (a) to find the minimum value of the distance AB as x varies. (2 marks)

- 7 The curve C has equation $y = k(x^2 + 3)$, where k is a constant.

The line L has equation $y = 2x + 2$.

- (a) Show that the x -coordinates of any points of intersection of the curve C with the line L satisfy the equation
- $$kx^2 - 2x + 3k - 2 = 0 \quad (1 \text{ mark})$$
- (b) The curve C and the line L intersect in two distinct points.
- Show that
- $$3k^2 - 2k - 1 < 0 \quad (4 \text{ marks})$$
- Hence find the possible values of k . (4 marks)

END OF QUESTIONS

AQA – Core 1 - Jun 2009 – Answers

| Question 1: | Exam report |
|---|---|
| <p>Line AB : $3x + 5y = 11$</p> <p>a) i) Make y the subject : $5y = -3x + 11$</p> $y = -\frac{3}{5}x + \frac{11}{5}$ <p>The gradient of AB = $m_{AB} = -\frac{3}{5}$</p> <p>ii) A(2,1).</p> <p>The gradient of the line perpendicular to AB is</p> $-\frac{1}{m_{AB}} = \frac{5}{3}$ <p>The equation of the line is : $y - 1 = \frac{5}{3}(x - 2)$</p> $3y - 3 = 5x - 10$ $5x - 3y = 7$ <p>b) Solve simultaneously</p> $\begin{cases} 3x + 5y = 11 & (\times 2) \\ 2x + 3y = 8 & (\times -3) \end{cases} \text{ gives}$ $\begin{cases} 6x + 10y = 22 \\ -6x - 9y = -24 \end{cases} \text{ and by adding}$ $y = -2$ <p>and $3x + 5y = 11$ $3x - 10 = 11$ $x = 7$</p> <p style="color: red;">The lines intersect at (7, -2)</p> | <p>In part (a)(i) many candidates were unable to make y the subject of the equation $3x + 5y = 11$ and, as a result, many incorrect answers for the gradient were seen. Those who tried to use two points on the line to find the gradient were rarely successful.</p> <p>In part (a)(ii) most candidates realised that the product of the gradients of perpendicular lines should be -1 and credit was given for using this result together with their answer from part(a)(i). Although many correct answers for the coordinates of C were seen in part (b)(i), the simultaneous equations defeated a large number of candidates. No credit was given for mistakenly using their equation from part (a)(ii) instead of the correct equation for AB.</p> |

| Question 2: | Exam report |
|--|--|
| <p>a) $\frac{5 + \sqrt{7}}{3 - \sqrt{7}} = \frac{5 + \sqrt{7}}{3 - \sqrt{7}} \times \frac{3 + \sqrt{7}}{3 + \sqrt{7}} = \frac{15 + 5\sqrt{7} + 3\sqrt{7} + 7}{9 - 7}$</p> $= \frac{22 + 8\sqrt{7}}{2} = 11 + 4\sqrt{7}$ <p>b) Using pythagoras' theorem, we have</p> $x^2 = (2\sqrt{5})^2 - (3\sqrt{2})^2$ $= 4 \times 5 - 9 \times 2 = 2$ $x = \sqrt{2} \text{ or } x = -\sqrt{2}$ <p>but x is a length, $x > 0$, so $x = \sqrt{2}$ is the answer.</p> | <p>In part (a) most candidates recognised the first crucial step of multiplying the numerator and denominator by $3 + \sqrt{7}$ and many obtained $\frac{22 + 8\sqrt{7}}{2}$, but then poor cancellation led to a very common incorrect answer of $11 + 8\sqrt{7}$.</p> <p>Candidates found part (b) more difficult than part (a) and revealed a lack of understanding of surds. Most candidates realised the need to use Pythagoras' Theorem but many could not square $2\sqrt{5}$ and $3\sqrt{2}$ correctly. Little credit was given for those who wrote things such as $x = \sqrt{20} - \sqrt{18} = \sqrt{2}$ and candidates need to realise that "getting the right answer" is not always rewarded with full marks. Although the equation $x^2 = 2$ has the solution $x = \pm\sqrt{2}$, it was necessary to consider the context and to give the value of x as $\sqrt{2}$.</p> |

| Question 3: | Exam report |
|---|---|
| <p> $y = x^5 + 20x^2 - 8$ $P(-2, y_P)$ a) $\frac{dy}{dx} = 5x^4 + 40x$ b) for $x = -2$, $\frac{dy}{dx} = 5 \times (-2)^4 + 40 \times (-2)$ $\frac{dy}{dx} = 5 \times 16 - 80 = 0$ P is a stationary point. c) i) $\frac{d^2y}{dx^2} = 20x^3 + 40$ and for $x = -2$ $= 20 \times (-2)^3 + 40 = -160 + 40 = -120 < 0$ ii) P is a MAXIMUM point. d) when $x = 1$, $y = 1^5 + 20 \times 1^2 - 8 = 1 + 20 - 8 = 13$ $\frac{dy}{dx}(x=1) = 5 + 40 = 45$ the equation of the tangent at $(1, 13)$ is $y - 13 = 45(x - 1)$ $y = 45x - 32$ </p> | <p> In part (a) almost everyone obtained the correct expression for $\frac{dy}{dx}$, although a few spoiled their solution by dividing each term by 5 or adding “+ c” to their answer. In part (b) most candidates substituted $x = -2$ into their expression for $\frac{dy}{dx}$, but, in order to score full marks, it was necessary to show $(-2)^4$ written as 16 or to show that $\frac{dy}{dx} = 80 - 80 = 0$ and then to write an appropriate conclusion about P being a stationary point. For part (c) many candidates simply wrote down an expression for $\frac{d^2y}{dx^2}$ in terms of x when answering part (i) and only evaluated the second derivative when determining the nature of the stationary point in part (ii). On this occasion full credit was given, but candidates need to realise what is meant by the demand to “find the value of” since this may be penalized in future examinations. In part (d) some candidates failed to find the y-coordinate of P, which was necessary in order to find the equation of the tangent. It was pleasing to see most candidates using the value of $\frac{dy}{dx}$ when $x = 1$, but unfortunately many tried to find the equation of the normal instead of the tangent to the curve. </p> |

Question 4:

$$a) p(x) = x^3 - x + 6$$

i) The remainder is $p(3)$

$$p(3) = 3^3 - 3 + 6 = 27 - 3 + 6 = 30$$

$$ii) p(-2) = (-2)^3 - (-2) + 6$$

$$= -8 + 2 + 6 = 0$$

-2 is a root of p , so $(x + 2)$ is a factor of p .

$$iii) p(x) = (x + 2)(x^2 - 2x + 3)$$

$$iv) p(x) = 0 \text{ means } (x - 2)(x^2 - 2x + 3) = 0$$

$$\text{so } x - 2 = 0 \text{ or } x^2 - 2x + 3 = 0$$

$$x = 2$$

the **discriminant**

$$= (-2)^2 - 4 \times 1 \times 3 = -8 < 0$$

no solution.

$$b) i) B(0, 6) \quad (p(0) = 0 - 0 + 6 = 6)$$

$$ii) \int_{-2}^0 (x^3 - x + 6) dx = \left[\frac{1}{4}x^4 - \frac{1}{2}x^2 + 6x \right]_{-2}^0$$

$$= (0) - (4 - 2 - 12) = 10$$

iii) The shaded area

= area beneath the curve – area of triangle ABO

$$= 10 - \frac{1}{2} \times 2 \times 6 = 4$$

Exam report

Those candidates who used the remainder theorem in part (a)(i) were usually successful in finding the correct remainder. Those who tried to use long division were usually confused by the lack of an x^2 term and were rarely successful in showing that the remainder was 30.

Those who used long division in part (a)(ii) scored no marks. Most candidates realised the need to show that $p(-2) = 0$, but quite a few omitted sufficient working such as $p(-2) = -8 + 2 + 6 = 0$ together with a concluding statement about $x + 2$ being a factor and therefore failed to score full marks.

Many candidates have become quite skilled at writing down the correct product of a linear and quadratic factor and these scored full marks in part (a)(iii). Others used long division effectively but lost a mark for failing to write $p(x)$ in the required form. Others tried methods involving comparing coefficients, but often after several lines of working were unable to find the correct values of b and c because of poor algebraic manipulation. In part (a)(iv), although many candidates tried to consider the value of the discriminant of their quadratic factor, quite a few used $a = 1$, $b = -1$ and $c = 6$ (from the cubic equation) and scored no marks for this part of the question. Others drew a correct conclusion using the quadratic equation formula, indicating that it was not possible to find the square root of -8 and others, after completing the square showed that the equation $(x - 1)^2 = -2$ has no real solutions. Some wrongly concluded that because it was not possible to factorise their quadratic then the corresponding quadratic equation had no real roots.

Most obtained the correct y -coordinate of B in part (b)(i).

In part (b)(ii) it was pleasing to see most candidates being able to integrate correctly but a large number did not answer the question set and simply found the indefinite integral in this part. Many candidates use poor techniques when finding a definite integral and it was often difficult to see the evaluation of $F(0) - F(-2)$ in their solution. Many obtained an answer of -10 which was

miraculously converted into $+10$ with some comment about an area being positive. This and similar dubious working was penalized.

In part (b)(iii) some obtained an answer of -6 for the area of the triangle by using -2 as the base. Credit was given to candidates who later realised that the area of the triangle was actually 6. Unless candidates had scored full marks in part (ii) they were not able to score full marks in this part either, even if they obtained a correct value of 4 for the shaded area.

| Question 5: | Exam report |
|--|---|
| <p>$(x-5)^2 + (y+12)^2 = 169$</p> <p>a) i) $C(5, -12)$</p> <p>ii) $r = \sqrt{169} = 13$</p> <p>b) i) $O(0,0)$</p> <p>$(0-5)^2 + (0+12)^2 = 25 + 144 = 169$</p> <p>$O$ belongs to the circle.</p> <p>ii) $(0, p) : (0-5)^2 + (p+12)^2 = 169$</p> <p>$25 + (p+12)^2 = 169$</p> <p>$(p+12)^2 = 144$</p> <p>$p+12 = \pm 12$</p> <p>$p = 0$ or $p = -24$</p> <p>$(0, -24)$ belongs to the circle.</p> <p>c) $A(-7, -7)$ lies on the circle</p> <p>i) gradient of $AC = m_{AC} = \frac{-12+7}{5+7} = -\frac{5}{12}$</p> <p>ii) The tangent is perpendicular to the radius AC</p> <p>The gradient of the tangent is $\frac{12}{5}$</p> <p>The equation of the tangent is $y+7 = \frac{12}{5}(x+7)$</p> <p>$5y+35 = 12x+94$</p> <p>$12x-5y+49 = 0$</p> | <p>In part (a)(i) most candidates realised what the correct coordinates of the centre were, although some wrote these as $(-5, 12)$ instead of $(5, -12)$.</p> <p>Some gave the radius as 169 and others evaluated $\sqrt{169}$ incorrectly in part (a)(ii). The majority of candidates obtained the correct value of the radius.</p> <p>In part (b)(i) most were able to verify that the circle passed through the origin, although some neglected to make a statement as a conclusion to their calculation and so failed to earn this mark. A surprisingly large number made no attempt at this part.</p> <p>Most sketches were correct in part (b)(ii), though some were very untidy with some making several attempts at the circle so the diagram resembled the chaotic orbit of a planet. In spite of being asked to verify that the circle passed through the origin many sketches did not do so. Credit was given for freehand circles with the centre in the correct quadrant and which passed through the origin, although it was good to see some circles drawn using compasses.</p> <p>Many used algebraic methods, putting $x = 0$, but often their poor algebra prevented them from finding the value of p. Those using the symmetry, doubling the y-coordinate, were usually more successful, although an answer of -25 (from $-12-13$) was common.</p> <p>In part (c)(i) the majority of candidates tried to find the gradient of AC but careless arithmetic meant that far fewer actually succeeded in finding its correct simplified value.</p> <p>In part (c)(ii), in order to find the tangent, it was necessary to use the negative reciprocal of the answer from part (c)(i) in order to find the gradient. Although some did, many chose to use the same gradient obtained in the previous part of the question and scored no marks at all.</p> |

| Question 6: | Exam report |
|---|---|
| <p>a) i) $x^2 - 8x + 17 = (x-4)^2 - 16 + 17 = (x-4)^2 + 1$</p> <p>ii) For all x, $(x-4)^2 \geq 0$ so $(x-4)^2 + 1 \geq 1$</p> <p>The minimum value is 1</p> <p>iii) the minimum occurs when $(x-4)^2 = 0$ i.e. $x = 4$</p> <p>b) $A(5,4)$ $B(x, 7-x)$</p> <p>i) $(x-5)^2 = x^2 + 25 - 10x$</p> <p>ii) $AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$</p> <p>$= (x-5)^2 + (3-x)^2$</p> <p>$= x^2 + 25 - 10x + 9 + x^2 - 6x$</p> <p>$= 2x^2 - 16x + 34$</p> <p>$AB^2 = 2(x^2 + 8x + 17)$</p> <p>iii) The minimum value of $x^2 + 8x + 17$ is 1</p> <p>so the minimum value of AB^2 is 2</p> <p>the minimum value of AB is $\sqrt{2}$</p> | <p>Completing the square was done well by most candidates in part (a)(i), although quite a few wrote q as 17 instead of 1.</p> <p>Part (a)(ii) of this question was answered very badly with many giving their answer as coordinates. Candidates were either "hedging their bets" or were simply presenting the coordinates of a minimum point of a curve as their answer.</p> <p>In part (a)(iii) many candidates obtained the correct value for x in this part, but there was confusion with many about how to answer parts (i) and (ii). The question was deliberately designed to test the understanding of the minimum value of a quadratic expression and when this occurred. Those who wrote "(ii) 4 and (iii) 1" scored no marks at all for these two parts of the question.</p> <p>In part (b)(i) practically everyone scored a mark for multiplying out $(x-5)^2$ correctly.</p> <p>In part (b)(ii) only the best candidates obtained a correct expression for AB^2 and then completed the resulting algebra to obtain the printed answer.</p> <p>It was good to see that many saw the link between the various parts in part (b)(iii). Many more able candidates substituted $x = 4$ into the expression and obtained $AB^2 = 2$, but they then failed to take the positive square root in order to find the minimum distance.</p> |

Question 7:**Exam report**

curve $C: y = k(x^2 + 3)$

line $L: y = 2x + 2$

a) By indentifying the y 's:

$$k(x^2 + 3) = 2x + 2$$

$$kx^2 - 2x + 3k - 2 = 0$$

b) The curve and the line have

two distinct points of intersection,

this means that the discriminant > 0 .

$$(-2)^2 - 4 \times k \times (3k - 2) > 0$$

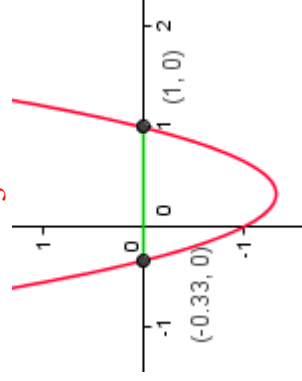
$$4 - 12k^2 + 8k > 0 \quad (\div -4)$$

$$3k^2 - 2k - 1 < 0$$

iii) $(3k + 1)(k - 1) < 0$

critical values $-\frac{1}{3}$ and 1

$(3k + 1)(k - 1) < 0$ for $-\frac{1}{3} < k < 1$



Most candidates scored the mark for the correct printed equation in part (a), but some omitted “= 0” and others made algebraic slips when taking terms from one side of their equation to the other. In part (b)(i) only the more able candidates were able to obtain the printed inequality using correct algebraic steps. Many began by stating that the discriminant was less than 0, clearly being influenced by the answer. Not all assigned the correct terms to a, b and c in the expression $b^2 - 4ac$ and others made sign errors when removing brackets.

The factorisation was usually correct in part (b)(ii), but many wrote down one of the critical values as $1/3$. Most found critical values and either stopped or immediately tried to write down a solution without any working. Candidates are strongly advised to use a sign diagram or a sketch showing their critical values when solving a quadratic inequality.

GRADE BOUNDARIES

| Component title | Max mark | A | B | C | D | E |
|-------------------|----------|----|----|----|----|----|
| Core 1 – Unit PC1 | 75 | 63 | 55 | 48 | 41 | 34 |

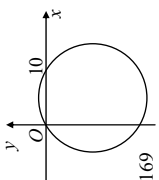
MPC1

| Q | Solution | Marks | Total | Comments |
|----------------|--|----------------------|----------|---|
| 1(a)(i) | $y = -\frac{3}{5}x + \frac{11}{5}$ Or correct expression for gradient using two correct points | M1 | | Attempt at $y = f(x)$ Or answer = $\frac{3}{5}$ or $-\frac{3}{5}x$ gets M1 But answer of $\frac{3}{5}x$ gets M0 |
| | (Gradient of AB) = $-\frac{3}{5}$ | A1 | 2 | Correct answer scores 2 marks. Condone error in rearranging formula if answer for gradient is correct. |
| (ii) | $m_1 m_2 = -1$ Gradient of perpendicular = $\frac{5}{3}$ | M1 A1 ✓ | | Used or stated ft their answer from (a)(i) or correct |
| (b) | Eliminating x or y but must use $3x + 5y = 11$ & $2x + 3y = 8$ $x = 7$ $y = -2$ | A1 A1 A1 | 3 | $5x - 3y = 7$; or $y = \frac{5}{3}x + c$, $c = -\frac{7}{3}$ etc CSO An equation in x only or y only |
| | Total | | 8 | Answer only of (7, -2) scores 3 marks |
| 2(a) | $\frac{5+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$ Numerator = $15 + 5\sqrt{7} + 3\sqrt{7} + 7$ Denominator = $9 - 7$ (= 2) (Answer =) $11 + 4\sqrt{7}$ | M1 m1 B1 A1 | | Condone one error or omission Must be seen as the denominator |
| (b) | $(2\sqrt{5})^2 = 20$ or $(3\sqrt{2})^2 = 18$ their $(2\sqrt{5})^2 - (3\sqrt{2})^2$ ($x^2 = 20 - 18$) ($\Rightarrow x = \pm\sqrt{2}$) | B1 M1 A1 | 3 | Either correct Condone missing brackets and x^2 $x^2 = 2 \Rightarrow$ B1, M1 $\pm\sqrt{2}$ scores A0 Answer only of 2 scores B0, M0 Answer only of $\sqrt{2}$ scores 3 marks |
| | Total | | 7 | |


MPC1 (cont)

| Q | Solution | Marks | Total | Comments |
|---------------|---|------------------------------|-----------|---|
| 3(a) | $\frac{dy}{dx} = 5x^4 + 40x$ | M1 A1 A1 | 3 | One of these powers correct One of these terms correct All correct (no + c etc) |
| (b) | $x = -2$ $\frac{dy}{dx} = 5 \times (-2)^4 + (40 \times -2)$ $\frac{dy}{dx} = 5 \times 16 + (40 \times -2) = 0$ $\Rightarrow P$ is stationary point | M1 A1 | | Substitute $x = -2$ into their $\frac{dy}{dx}$ CSO Shown = 0 plus statement eg "st pt", "as required", "grad = 0" etc |
| (c)(i) | Or their $\frac{dy}{dx} = 0 \Rightarrow x^4 = k$ $x^2 = -8 \Rightarrow x = -2$ | (M1) (A1) | 2 | CSO $x = 0$ need not be considered |
| (ii) | $\frac{d^2y}{dx^2} = 20x^3 + 40$ $= 20 \times (-2)^3 + 40$ $= (-160 + 40) = -120$ | B1 ✓ M1 A1 | 3 | Correct ft their $\frac{dy}{dx}$ Subst $x = -2$ into their second derivative CSO |
| (d) | Maximum (value) their c(f) answer must be < 0 Other valid methods acceptable provided "maximum" is the conclusion (When $x = 1$) $y = 13$ When $x = 1$, $\frac{dy}{dx} = 5 + 40$ $y = (\text{their } 45)x + k$ OE Tangent has equation $y - 13 = 45(x - 1)$ | E1 ✓ B1 M1 m1 A1 | 1 4 | Accept minimum if their c(f) answer > 0 and correctly interpreted Parts (i) and (ii) may be combined by candidate but -120 must be seen to award A1 in part (c)(i) Sub $x = 1$ into their $\frac{dy}{dx}$ ft their $\frac{dy}{dx}$ CSO OE $y = 45x + c$, $c = -32$ |
| | Total | | 13 | |

| Q | Solution | Marks | Total | Comments |
|---------|---|--|-----------|---|
| 4(a)(i) | $p(3) = 27 - 3 + 6$ (Remainder) = 30 Or long division up to remainder Quotient = $x^2 + 3x + 8$ and remainder = 30 clearly stated or indicated | M1 A1 (M1) | | p(3) attempted |
| (ii) | $p(-2) = -8 + 2 + 6$ $p(-2) = 0 \Rightarrow x + 2$ is factor Minimum statement required "factor" | M1 A1 | 2 | p(-2) attempted : NOT long division Shown = 0 plus statement May make statement <i>first</i> such as " $x+2$ is a factor if $p(-2) = 0$ " |
| (iii) | $b = -2$ $c = 3$ or long division/comparing coefficients | B1 B1 | | No working required for B1 + B1 Try to mark first using B marks |
| (iv) | $b^2 - 4ac = (-2)^2 - 4 \times 3$ $b^2 - 4ac = -8$ (or < 0) \Rightarrow no (other) real roots Or $(x-1)^2 + 2 > 0$ therefore no real roots Or $(x-1)^2 = -2$ has no real roots | (M1) (A1) | 2 | Award M1 if B0 earned and a clear method is used Must write final answer in this form if long division has been used to get A1 |
| (b)(i) | $(y_B =) 6$ | M1 A1 | 1 | Discriminant correct from their quadratic M0 if $b = -1, c = 6$ used (using cubic eqn) CSO All values must be correct plus statement |
| (ii) | $\frac{x^4 - x^2 + 6x}{4 - 2}$ $\left[\begin{matrix} -10 \\ -2 \end{matrix} \right] = 0 - (4 - 2 - 12)$ $= 10$ | (M1) (A1) | 2 | Completion of square for their quadratic Shown to be positive plus statement regarding no real roots |
| (iii) | Area of $\Delta = \frac{1}{2} \times 2 \times 6$ = 6 Shaded region area = $10 - 6 = 4$ | B1 M1 A1 A1 m1 A1 M1 | 5 | Condone (0, 6) One term correct Another term correct All correct (ignore + c or limits) F(-2) attempted CSO Clearly from F(0) - F(-2) Condone - 2 and fit their y_B value Or $\int_{-2}^0 (3x+6)dx$ and attempt to integrate Must be positive allow -6 converted to +6 CSO 10 must come from correct working |
| | Total | | 17 | |

| Q | Solution | Marks | Total | Comments |
|---------|--|------------|-----------|---|
| 5(a)(i) | $C(5, -12)$ | B1 | 1 | $\pm\sqrt{69}$ or ± 13 as final answer scores B0 |
| (ii) | Radius = 13 (or $\sqrt{169}$) $(-5)^2 + 12^2$ or $25 + 144$ $= 169 \Rightarrow$ circle passes through O | B1 | 1 | Correct arithmetic plus statement Eg "O lies on circle", "as required" etc |
| (b)(i) | Sketch  | B1 | | Freehand circle through origin and cutting positive x-axis with centre in 4 th quadrant Condone value 10 missing or incorrect |
| (ii) | $25 + (p + 12)^2 = 169$ $(p + 12) = \pm 12$ $p = -24$ | M1 A1 | 3 | Or doubling their y_C -coordinate Condone use of y instead of p SC B2 for correct value of p stated or marked on diagram |
| (c)(i) | grad AC = $\frac{-12+7}{5+7}$ $= -\frac{5}{12}$ | M1 | | correct expression, but fit their C |
| (ii) | grad tangent = $\frac{12}{5}$ $y+7 = \frac{12}{5}(x+7)$ $\Rightarrow 12x - 5y + 49 = 0$ | A1 B1 ✓ | 2 | Condone $\frac{5}{-12}$ their grad AC fit "their $\frac{12}{5}$ " must be tangent and not AC |
| | Total | | 11 | OE with integer coefficients with all terms on one side of the equation |

| Q | Solution | Marks | Total | Comments |
|--------------|---|----------------|-----------|---|
| 6(a)(i) | $(x-4)^2 + 1$ or $p = 4$ or $q = 1$ | B1 B1 | 2 | ISW for $p = -4$ if $(x-4)^2$ seen |
| (ii) | (Minimum value is) 1 | B1✓ | 1 | Correct or FT "their q " (NOT coords) |
| (iii) | (Minimum occurs when $x =$) 4 | B1✓ | 1 | Correct or FT "their p " - may use calculus Condone ($p, **$) for this B1 mark |
| (b)(i) | $(x-5)^2 = x^2 - 10x + 25$ | B1 | 1 | Condone one slip in one bracket May be seen under $\sqrt{\quad}$ sign |
| (ii) | $(x-5)^2 + (7-x-4)^2$ $= (x-5)^2 + (3-x)^2$ $= x^2 - 10x + 25 + 9 - 6x + x^2$ $AB^2 = 2x^2 - 16x + 34$ $= 2(x^2 - 8x + 17)$ | M1 A1 A1 | 3 | From a fully correct expression AG CSO |
| (iii) | Minimum $AB^2 = 2 \times$ "their (a)(ii)" Minimum $AB = \sqrt{2}$ | M1 A1 | 2 | Or use of their $x = 4$ in expression Or use of their $B(4, 3)$ and $A(5, 4)$ in distance formula M0 if calculus used Answer only of $2 \times$ "their (a)(ii)", scores M1, A0 |
| Total | | | 10 | |

| Q | Solution | Marks | Total | Comments |
|--------------|--|-----------------|-----------|---|
| 7(a) | $k(x^2 + 3) = 2x + 2$ $\Rightarrow kx^2 - 2x + 3k - 2 = 0$ | B1 | 1 | AG OE all terms on one side and = 0 |
| (b)(i) | Discriminant $= (-2)^2 - 4k(3k - 2)$ $= 4 - 12k^2 + 8k$ Two distinct real roots $\Rightarrow b^2 - 4ac > 0$ $4 - 12k^2 + 8k > 0$ $\Rightarrow 12k^2 - 8k - 4 < 0$ $\Rightarrow 3k^2 - 2k - 1 < 0$ | M1 A1 B1✓ | | Condone one slip (including x is one slip) Condone 2^2 or 4 as first term condone recovery from missing brackets "their discriminant in terms of k " > 0 Not simply the statement $b^2 - 4ac > 0$ Change from > 0 to < 0 and divide by 4 AG CSO |
| (ii) | $(3k + 1)(k - 1)$ Critical values 1 and $-\frac{1}{3}$ Use of sign diagram or sketch  | M1 A1 | 4 | Correct factors or correct use of formula May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working If previous A1 earned, sign diagram or sketch must be correct for M1 Otherwise, M1 may be earned for an attempt at the sketch or sign diagram using their critical values. |
| | $\Rightarrow -\frac{1}{3} < k < 1$ or $1 > k > -\frac{1}{3}$ condone $-\frac{1}{3} < k$ AND $k < 1$ for full marks but not OR or " " instead of AND | A1 | 4 | Full marks for correct final answer with or without working \leq loses final A mark <i>Answer only of</i> $1 < k < -\frac{1}{3}$ or $k < -\frac{1}{3}; k < 1$ etc scores M1, A1, M0 since the correct critical values are evident <i>Answer only of</i> $\frac{1}{3} < k < 1$ etc where critical values are not both correct gets M0, M0 |
| Total | | | 9 | |
| TOTAL | | | 75 | |