General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

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1 The straight line L has equation $y = 3x - 1$ and the curve C has equation

$$y = (x+3)(x-1)$$

(a) Sketch on the same axes the line L and the curve C, showing the values of the intercepts on the x-axis and the y-axis. *(5 marks)*

(b) Show that the x-coordinates of the points of intersection of L and C satisfy the equation $x^2 - x - 2 = 0$. *(2 marks)*

(c) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

2 It is given that $x = \sqrt{3}$ and $y = \sqrt{12}$.

Find, in the simplest form, the value of:

(a) xy ; *(1 mark)*

(b) $\dfrac{y}{x}$; *(2 marks)*

(c) $(x+y)^2$. *(3 marks)*

3 Two numbers, x and y, are such that $3x + y = 9$, where $x \geqslant 0$ and $y \geqslant 0$.

It is given that $V = xy^2$.

(a) Show that $V = 81x - 54x^2 + 9x^3$. *(2 marks)*

(b) (i) Show that $\dfrac{dV}{dx} = k(x^2 - 4x + 3)$, and state the value of the integer k. *(4 marks)*

(ii) Hence find the two values of x for which $\dfrac{dV}{dx} = 0$. *(2 marks)*

(c) Find $\dfrac{d^2V}{dx^2}$. *(2 marks)*

(d) (i) Find the value of $\dfrac{d^2V}{dx^2}$ for each of the two values of x found in part (b)(ii). *(1 mark)*

(ii) Hence determine the value of x for which V has a maximum value. *(1 mark)*

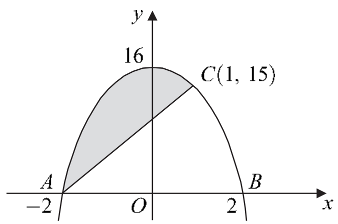
(iii) Find the maximum value of V. *(1 mark)*

4 (a) Express $x^2 - 3x + 4$ in the form $(x-p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(b) Hence write down the minimum value of the expression $x^2 - 3x + 4$. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 3x + 4$. *(3 marks)*

5 The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line AC. *(3 marks)*

(b) (i) Find $\int_{-2}^{1} (16 - x^4)\,dx$. *(5 marks)*

(ii) Hence calculate the area of the shaded region bounded by the curve and the line AC. *(3 marks)*

6 The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. *(2 marks)*

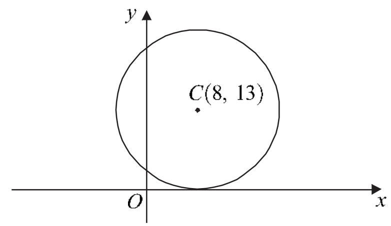
(b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of linear factors. *(3 marks)*

(c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of k. *(1 mark)*

(ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x-axis. *(3 marks)*

7 The circle S has centre $C(8, 13)$ and touches the x-axis, as shown in the diagram.



(a) Write down an equation for S, giving your answer in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(2 marks)*

(b) The point P with coordinates $(3, 1)$ lies on the circle.

(i) Find the gradient of the straight line passing through P and C. *(1 mark)*

(ii) Hence find an equation of the tangent to the circle S at the point P, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(4 marks)*

(iii) The point Q also lies on the circle S, and the length of PQ is 10. Calculate the shortest distance from C to the chord PQ. *(3 marks)*

8 The quadratic equation $(k+1)x^2 + 4kx + 9 = 0$ has real roots.

(a) Show that $4k^2 - 9k - 9 \geqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 - Jan 2007 – Answers

Question 1:

$Line\ L: y = 3x - 1$

$Curve\ C: y = (x+3)(x-1) = x^2 + 2x - 3$

$a)$ The line crosses the y-axis at $(0,-1)$ and the x-axis at $(\frac{1}{3}, 0)$

The curve crosses the y-axis at $(0,-3)$ and the x-axis at $(-3,0)$ and $(1,0)$

$b)$ Solve simultaneously $\begin{cases} y = 3x-1 \\ y = x^2+2x-3 \end{cases}$ *by identification*

$(y=)\ x^2 + 2x - 3 = 3x - 1$

$x^2 - x - 2 = 0$

$c)\quad x^2 - x - 2 = (x-2)(x+1) = 0$

$x = 2 \ or \ x = -1$

$and\ y = 3x - 1 \qquad y = 5 \ or \ y = -4$

The line and the curve cross at $(2,5)$ and $(-1,-4)$

Exam report

Question 2:

$x = \sqrt{3} \ and \ y = \sqrt{12}$

$a)\ xy = \sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$

$b)\ \frac{y}{x} = \frac{\sqrt{12}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$

$c)\ (x+y)^2 = \left(\sqrt{3} + \sqrt{12}\right)^2 = 3 + 12 + 2\sqrt{36} = 27$

Exam report

Question 3:

$3x + y = 9 \quad x \geq 0, y \geq 0$

$V = xy^2$

$a)\ y = 9 - 3x \quad so \ V = xy^2 = x(9-3x)^2$

$V = x\left(81 + 9x^2 - 54x\right)$

$V = 9x^3 - 54x^2 + 81x$

$b)\ i)\ \frac{dV}{dx} = 9 \times 3x^2 - 54 \times 2x + 81 = 27x^2 - 108x + 81$

$\frac{dV}{dx} = 27\left(x^2 - 4x + 3\right)$

$ii)\ \frac{dV}{dx} = 0 \ when \ x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \ or \ x = 1$

$c)\ \frac{d^2V}{dx^2} = 27(2x-4) = 54x - 108$

$d)\ i)\ \frac{d^2V}{dx^2}(x=3) = 27(2\times3-4) = 27\times2 = 54 > 0$

$\frac{d^2V}{dx^2}(x=1) = 27(2\times1-4) = 27\times-2 = -54 < 0$

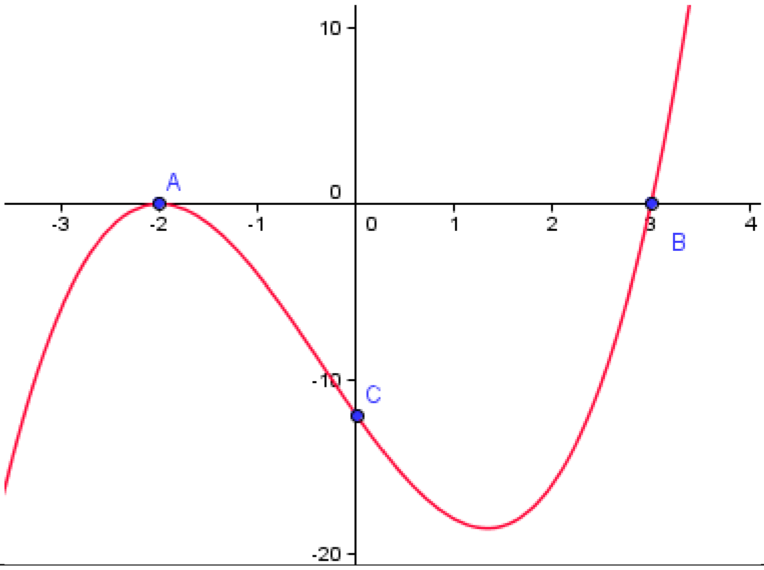
$ii)$ There is a maximum for $x = 1$

$iii)\ For \ x = 1,\ V = 81 - 54 + 9 = 36$

Exam report

Question 4:	Exam report
a) $x^2 - 3x + 4 = \left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 4 = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$ b) For all x, $\left(x - \frac{3}{2}\right)^2 \geq 0$ so $\left(x - \frac{3}{2}\right)^2 + \frac{7}{4} \geq \frac{7}{4}$ The minimum value is $\frac{7}{4}$ c) $x \xrightarrow[\text{1.5 units in } x\text{-dir}]{\text{translation}} x - \frac{3}{2} \xrightarrow{f} \left(x - \frac{3}{2}\right)^2 \xrightarrow[\frac{7}{4}\text{ units in } y\text{-dir}]{\text{translation}} \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$ $y = x^2$ *is mapped* into $y = x^2 - 3x + 4$ by the translation vector $\begin{bmatrix} \frac{3}{2} \\ \frac{7}{4} \end{bmatrix}$	

Question 5:	Exam report
$y = 16 - x^4$ $A(-2,0),\ B(2,0)$ *and* $C(1,15)$ a) *Gradient of* $AC = m_{AC} = \frac{15-0}{1+2} = 5$ *Equation of* $AC: y - 0 = 5(x+2)$ $y = 5x + 10$ b) i) $\int_{-2}^{1} (16 - x^4)dx = \left[16x - \frac{1}{5}x^5\right]_{-2}^{1} = \left(16 - \frac{1}{5}\right) - \left(-32 + \frac{32}{5}\right)$ $= 48 - \frac{33}{5} = 48 - 6\tfrac{3}{5} = 41\tfrac{2}{5}$ ii) Call the point $H(1,0)$ the area of the shaded region is $(\text{area beneath the curve}) - (\text{area of triangle } AHC)$ $41\tfrac{2}{5} - \frac{1}{2} \times 3 \times 15 = 41\tfrac{2}{5} - 22\tfrac{1}{2} = 18\tfrac{9}{10}$	

Question 6:	Exam report
$p(x)=x^3+x^2-8x-12$ $a)$ The remainder of the division by $(x-1)$ is $p(1)$ $p(1)=1+1-8-12=-18$ $b)i)$ $p(-2)=(-2)^3+(-2)^2-8\times(-2)-12$ $=-8+4+16-12=0$ -2 is a root of p, so $(x+2)$ is a factor of p. $ii)$ $x^3+x^2-8x-12=(x+2)(x^2-x-6)$ $=(x+2)(x-3)(x+2)$ $=(x+2)^2(x-3)$ $c)i)$ By substituting x by 0, we have $p(0)=-12$ The curve passes through the point $(0,-12)$ $ii)$ The curve crosses the $x-axis$ at $(3,0)$ and it is tangent to it at $(-2,0)$.  	

Question 7:

Circle S has centre $C(8,13)$ and touches the x-axis

The radius is 13 (y-coordinate of C)

$a)$ Equation of S: $(x-8)^2+(y-13)^2=13^2$

$b)$ $P(3,1)$ lies on the circle

$i)\ gradient = m_{PC} = \frac{13-1}{8-3} = \frac{12}{5}$

$ii)$ The tangent at P is perpendicular to the radius PC

the gradient of the tangent is $-\frac{1}{m_{PC}} = -\frac{5}{12}$

The equation of the tangent is : $y-1=-\frac{5}{12}(x-3)$

$12y-12=-5x+15$

$5x+12y=27$

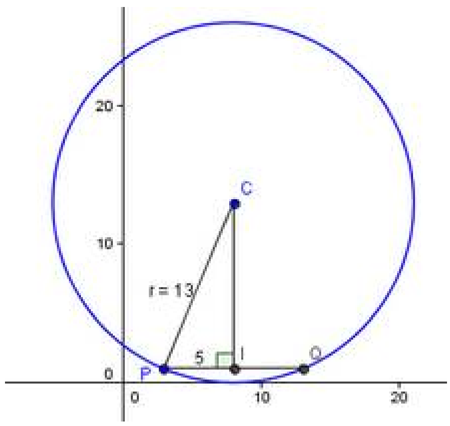
$iii)$ Call I the midpoint of PQ. The triangle PIC is a right-angled triangle and the shortest distance from C to the chord PQ is the distance CI.

Using pythagoras' theorem: $CI^2 = CP^2 - PI^2$

$=13^2-5^2=169-25=144$

$CI=\sqrt{144}=12$

Exam report



Question 8:

$(k+1)x^2+4kx+9=0$ *has real roots*

which means that the discriminant ≥ 0.

$a)\ (4k)^2-4\times(k+1)\times 9\geq 0$

$16k^2-36k-36\geq 0 \quad (\div 4)$

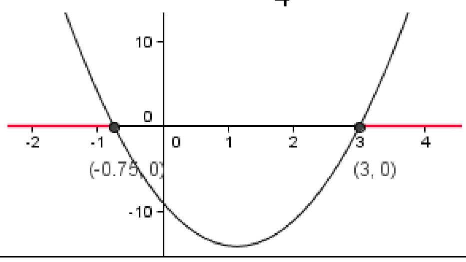
$4k^2-9k-9\geq 0$

$b)\ 4k^2-9k-9\geq 0$

$(4k+3)(k-3)\geq 0$

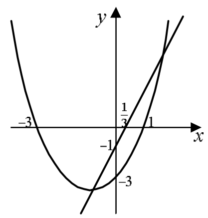
critical values $-\frac{3}{4}$ *and* 3

$(4k+3)(k-3)\geq 0 \ \ for\ \ k\leq -\frac{3}{4}\ or\ k\geq 3$

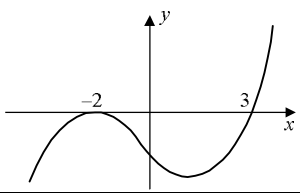


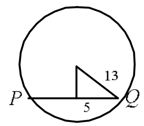
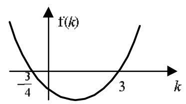
Exam report

GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	59	51	43	35	28

Q	Solution	Marks	Total	Comments
1(a)	L: straight line with positive gradient and negative intercept on y-axis	B1		Line must cross both axes but need not reach the curve
	cutting at $\left(\frac{1}{3},0\right)$ and $(0,-1)$ (intercepts **stated** or marked on sketch)	B1		Condone 0.33 or better for $\frac{1}{3}$
	C: **attempt at parabola** $\cup$ or $\cap$ through $(-3,0)$ and $(1,0)$ or values –3 and 1 stated as intercepts on x-axis	B1		
	$\cup$ shaped graph – vertex below x-axis and cutting x-axis twice	M1		
	through $(0,-3)$ and minimum point to left of y-axis	A1	5	(y-intercept or coordinates marked)
(b)	$(x+3)(x-1)=3x-1$	M1		
	$x^2+3x-x-3-3x+1=0$ $\Rightarrow x^2-x-2=0$	A1	2	AG; must have "= 0" and no errors
(c)	$(x-2)(x+1)=0$	M1		$(x\pm1)(x\pm2)$ or use of formula (one slip)
	$\Rightarrow x=2,\ -1$	A1		correct values imply M1A1
	Substitute one value of x to find y	m1		
	Points of intersection (2, 5) and (–1,–4)	A1	4	May say $x=2, y=5$ etc SC: (2, 5) $\Rightarrow$B2 (–1,–4)$\Rightarrow$B2 without working
	Total		11	
2(a)	$xy=6$	B1	1	B0 for $\sqrt{36}$ or ± 6
(b)	$\frac{y}{x}=\frac{2\sqrt{3}}{\sqrt{3}}$ or $\sqrt{\frac{12}{3}}$ or $\sqrt{\frac{4}{1}}$ or $\frac{\sqrt{12}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$	M1		Allow M1 for ± 2
	$=2$	A1	2	
(c)	$x^2+2xy+y^2$ or $(\sqrt{3}+2\sqrt{3})^2$ correct	M1		or $\left(\sqrt{3}+\sqrt{12}\right)\left(\sqrt{3}+\sqrt{12}\right)$ expanded as 4 terms – no more than one slip
	Correct with 2 of $x^2, y^2, 2xy$ simplified $3+2\sqrt{36}+12$ or $3^2\times3$ or $\left(3\sqrt{3}\right)^2$	A1		Correct but unsimplified – one more step
	$=27$	A1	3	
	Total		6	

Q	Solution	Marks	Total	Comments
3(a)	$V=x(9-3x)^2$	M1		Attempt at V in terms of x (condone slip when rearranging formula for $y = 9 - 3x$) or $(9-3x)^2=81-54x+9x^2$
	$V=x(81-54x+9x^2)$ $=81x-54x^2+9x^3$	A1	2	AG; no errors in algebra
(b)(i)	$\frac{dV}{dx}=81-108x+27x^2$	M1 A1 A1		One term correct Another correct All correct (no + c etc)
	$=27(x^2-4x+3)$	A1	4	CSO; all algebra and differentiation correct
(ii)	$(x-3)(x-1)$ or $(27x-81)(x-1)$ etc	M1		"Correct" factors or correct use of formula
	$\Rightarrow x=1, 3$	A1	2	SC: B1,B1 for $x = 1, x = 3$ found by inspection (provided no other values)
(c)	$\frac{d^2V}{dx^2}=-108+54x$ (condone one slip)	M1		ft their $\frac{dV}{dx}$ (may have cancelled 27 etc)
		A1	2	CSO; all differentiation correct
(d)(i)	$x=3\Rightarrow\frac{d^2V}{dx^2}=54$; $x=1\Rightarrow\frac{d^2V}{dx^2}=-54$	B1√	1	ft their $\frac{d^2V}{dx^2}$ and their **two** x-values
(ii)	$(x=)$ 1 (gives maximum value)	E1	1	Provided their $\frac{d^2V}{dx^2}<0$
(iii)	$V_{max}=36$	B1	1	CAO
	Total		13	
4(a)	$\left(x-\frac{3}{2}\right)^2$	B1		Must have $(\ )^2$ $p=1.5$
	$+\frac{7}{4}$	B1	2	$q=1.75$
(b)	Minimum value is $\frac{7}{4}$	B1√	1	ft their q or correct value
(c)	**Translation** (and **no other** transformation stated)	E1		(**not** shift, move, transformation etc)
	through $\begin{bmatrix}\frac{3}{2}\\ \frac{7}{4}\end{bmatrix}$ (or equivalent in words)	M1		M1 for one component correct or ft their p or q values
		A1	3	CSO; condone 1.5 right and 1.75 up etc
	Total		6	

Q	Solution	Marks	Total	Comments
5(a)	Grad $AC = \frac{15}{3} = 5$	B1		OE
	Equation of AC: $y = m(x+2)$ or $(y-15) = m(x-1)$	M1		Or use of $y = mx + c$ with (–2, 0) or (1, 15) correctly substituted for x and y
	$y = 5x + 10$	A1	3	OE eg $y - 15 = 5(x-1)$, $y = 5(x+2)$
(b)(i)	$\left[16x - \frac{x^5}{5}\right]$	M1 A1 A1		Raise one power by 1 One term correct All correct
	$\left(16 - \frac{1}{5}\right) - \left(-32 + \frac{32}{5}\right)$	m1		$F(1) - F(-2)$ attempted
	$= 41\frac{2}{5}$ (or 41.4, $\frac{207}{5}$ etc)	A1	5	CSO; withhold if $+c$ added
(ii)	Area $\Delta = \frac{1}{2} \times 3 \times 15$ or $22\frac{1}{2}$ or 22.5	B1		Or $\int_{-2}^{1}(5x+10)\,dx = 22.5$
	Shaded area = "their (b)(i) answer" – correct triangle	M1		Condone "difference" if $\Delta > \int$
	$\Rightarrow$ shaded area $= 18\frac{9}{10}$	A1	3	CSO; OE (18.9 etc)
	Total		**11**	
6(a)	Remainder $= p(1) = 1 + 1 - 8 - 12$ $= -18$	M1 A1	2	Use of p(1) NOT long division
(b)(i)	$p(-2) = -8 + 4 + 16 - 12$ $= 0 \Rightarrow (x+2)$ is factor	M1 A1	2	NOT long division $p(-2)$ shown $= 0$ **and** statement
(ii)	Quad factor by comparing coefficients or $(x^2 + kx \pm 6)$ by inspection	M1		Or full long division or attempt at Factor Theorem using $f(\pm 3)$
	$p(x) = (x+2)(x^2 - x - 6)$	A1		Correct quadratic factor or $(x-3)$ shown to be factor by Factor Theorem
	$p(x) = (x+2)^2(x-3)$ or $(x+2)(x+2)(x-3)$	A1	3	CSO; SC: B1 for $(x+2)(x^{***})(x-3)$ by inspection or without working
(c)(i)	$(k =)\ -12$	B1	1	Condone $y = -12$ or (0, –12)
(ii)		M1 A1 A1	 3	Cubic shape (one max and one min) Maximum at $(-2, 0)$ and through (3, 0) – at least one of these values marked "correct" graph as shown (touching smoothly at –2, 3 marked and minimum to right of y-axis)
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$(x-8)^2 + (y-13)^2$	B1		**Exactly this** with + and squares
	$= 13^2$	B1	2	Condone 169
(b)(i)	grad $PC = \frac{12}{5}$	B1	1	Must simplify $\frac{-12}{-5}$
(ii)	grad of tangent $= \frac{-1}{\text{grad } PC} = -\frac{5}{12}$	B1√		Condone $-\frac{1}{2.4}$ etc
	tangent has equation $y - 1 = -\frac{5}{12}(x-3)$	M1 A1		ft gradient but M0 if using grad PC Correct – but not in required final form
	$5x + 12y = 27$ OE	A1	4	MUST have integer coefficients
(iii)	half chord = 5	B1		Seen or stated
	 $d^2 = (\text{their } r)^2 - 5^2$ (provided $r > 5$)	M1		Pythagoras used correctly $d^2 = 13^2 - 5^2$
	Distance = 12	A1	3	CSO
	Total		**10**	
8(a)	$b^2 - 4ac = 16k^2 - 36(k+1)$	M1		Condone one slip
	Real roots: discriminant $\geqslant 0$ $\Rightarrow 16k^2 - 36k - 36 \geqslant 0$	B1		
	$\Rightarrow 4k^2 - 9k - 9 \geqslant 0$	A1	3	AG (watch signs)
(b)	$(4k+3)(k-3)$	M1		Or correct use of formula (unsimplified)
	critical points $(k =)\ -\frac{3}{4}$, 3	A1		Not in a form involving surds Values may be seen in inequalities etc
	 sketch	M1		Or sign diagram
	$k \geqslant 3$, $k \leqslant -\frac{3}{4}$	A1	4	NMS full marks
				Condone use of word "and" but final answer in a form such as $3 \leqslant k \leqslant -\frac{3}{4}$ scores A0
	Total		**7**	
	TOTAL		**75**	