AQA

General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MPC1

Unit Pure Core 1

Monday 14 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The point A has coordinates $(-3, 2)$ and the point B has coordinates $(7, k)$.

The line AB has equation $3x + 5y = 1$.

(a) (i) Show that $k = -4$. *(1 mark)*

(ii) Hence find the coordinates of the midpoint of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) A line which passes through the point A is perpendicular to the line AB. Find an equation of this line, giving your answer in the form $px + qy + r = 0$, where p, q and r are integers. *(3 marks)*

(d) The line AB, with equation $3x + 5y = 1$, intersects the line $5x + 8y = 4$ at the point C. Find the coordinates of C. *(3 marks)*

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2 A bird flies from a tree. At time t seconds, the bird's height, y metres, above the horizontal ground is given by

$$y = \frac{1}{8}t^4 - t^2 + 5, \quad 0 \leqslant t \leqslant 4$$

(a) Find $\frac{dy}{dt}$. *(2 marks)*

(b) (i) Find the rate of change of height of the bird in metres per second when $t = 1$. *(2 marks)*

(ii) Determine, with a reason, whether the bird's height above the horizontal ground is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the value of $\frac{d^2y}{dt^2}$ when $t = 2$. *(2 marks)*

(ii) Given that y has a stationary value when $t = 2$, state whether this is a maximum value or a minimum value. *(1 mark)*

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3 (a) (i) Express $\sqrt{18}$ in the form $k\sqrt{2}$, where k is an integer. *(1 mark)*

(ii) Simplify $\frac{\sqrt{8}}{\sqrt{18} + \sqrt{32}}$. *(3 marks)*

(b) Express $\frac{7\sqrt{2} - \sqrt{3}}{2\sqrt{2} - \sqrt{3}}$ in the form $m + \sqrt{n}$, where m and n are integers. *(4 marks)*

4 (a) (i) Express $x^2 - 6x + 11$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions. *(2 marks)*

(b) A curve has equation $y = x^2 - 6x + 11$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) Sketch the curve, indicating the value of y where the curve crosses the y-axis. *(3 marks)*

(iii) Describe the geometrical transformation that maps the curve with equation $y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$. *(3 marks)*

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5 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 - 4x^2 - 3x + 18$$

(a) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x + 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x - 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as a product of linear factors. *(3 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 3x + 18$, stating the values of x where the curve meets the x-axis. *(3 marks)*

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6 The gradient, $\frac{\mathrm{d}y}{\mathrm{d}x}$, of a curve at the point (x, y) is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 10x^4 - 6x^2 + 5$$

The curve passes through the point $P(1, 4)$.

(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(3 marks)*

(b) Find the equation of the curve. *(5 marks)*

7 A circle with centre $C(-3, 2)$ has equation

$$x^2 + y^2 + 6x - 4y = 12$$

(a) Find the y-coordinates of the points where the circle crosses the y-axis. *(3 marks)*

(b) Find the radius of the circle. *(3 marks)*

(c) The point $P(2, 5)$ lies outside the circle.

(i) Find the length of CP, giving your answer in the form $\sqrt{n}$, where n is an integer. *(2 marks)*

(ii) The point Q lies on the circle so that PQ is a tangent to the circle. Find the length of PQ. *(2 marks)*

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8 A curve has equation $y = 2x^2 - x - 1$ and a line has equation $y = k(2x - 3)$, where k is a constant.

(a) Show that the x-coordinate of any point of intersection of the curve and the line satisfies the equation

$$2x^2 - (2k + 1)x + 3k - 1 = 0$$ *(1 mark)*

(b) The curve and the line intersect at two distinct points.

(i) Show that $4k^2 - 20k + 9 > 0$. *(3 marks)*

(ii) Find the possible values of k. *(4 marks)*

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1(a) (i)	$21+5k=1$			condone $3\times7+5k=1$
	$\Rightarrow k=-4$	B1	1	**AG** condone $y=-4$
(ii)	$(x=)\ 2$	B1		
	$(y=)\ -1$	B1	2	midpoint coords are (2, –1)
(b)	$y=\frac{1}{5}-\frac{3}{5}x$	M1		obtaining $y=a\pm\frac{3}{5}x$ or $\frac{\Delta y}{\Delta x}=\frac{-4-2}{7--3}$ or $\frac{-1-2}{2--3}$ or $\frac{-4--1}{7-2}$ condone **one** sign error in expression
	(Gradient AB =) $-\frac{3}{5}$	A1	2	allow – 0.6 , $\frac{6}{-10}$ etc for A1 & condone error in rearranging if gradient is correct .
(c)	Perp grad = $\frac{5}{3}$	M1		–1/ "their" grad AB
	$y-2=\frac{5}{3}(x+3)$ *or* $y=\frac{5}{3}x+c,\quad c=7$ etc	A1		correct equation in any form (must simplify $x--3$ to $x+3$ *or* c to a single term equivalent to 7)
	$5x-3y+21=0$	A1	3	or any multiple of this with integer coefficients –terms in any order but all terms on one side of equation
(d)	$3x+5y=1$ and $5x+8y=4$ $\Rightarrow Px=Q$ or $Ry=S$	M1		must use **correct** pair of equations **and** attempt to eliminate y (or x) (generous)
	$x=12$	A1		
	$y=-7$	A1	3	(12, –7)
	Total		**11**	

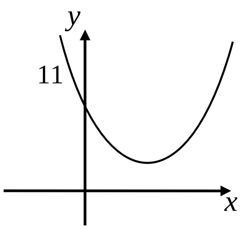
MPC1 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\left(\frac{dy}{dt}=\right)\frac{4t^3}{8}-2t$	M1 A1	2	one of these terms correct all correct (no + *c* etc)
(b)(i)	$t=1 \Rightarrow \frac{dy}{dt}=\frac{4}{8}-2$	M1		Correctly sub $t = 1$ into their $\frac{dy}{dt}$
	$=-1\frac{1}{2}$	A1cso	2	must have $\frac{dy}{dt}$ correct (watch for t^3 etc)
(ii)	$\frac{dy}{dt}<0$			must have used $\frac{dy}{dt}$ in part (b)(i)
	$\Rightarrow$ (height is) **decreasing** (when $t = 1$)	E1√	1	must state that " $\frac{dy}{dt}<0$ " or "$-1.5 < 0$" or the equivalent in words FT their value of $\frac{dy}{dt}$ with appropriate explanation and conclusion
(c)(i)	$\left(\frac{d^2y}{dt^2}=\right)\quad \frac{4}{8}\times 3t^2-2$	M1		Correctly differentiating their $\frac{dy}{dt}$ even if wrongly simplified
	$\left(t=2,\quad \frac{d^2y}{dt^2}=\right)\quad 4$	A1cso	2	Both derivatives correct and simplified to 4
(ii)	$\Rightarrow$ minimum	E1√	1	FT their numerical value of $\frac{d^2y}{dt^2}$ from part (c) (i)
	Total		**8**	

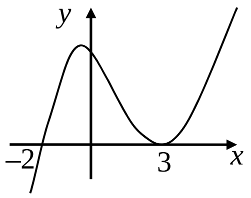
MPC1 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\sqrt{18} = 3\sqrt{2}$	B1	1	Condone $k = 3$
(ii)	$\dfrac{2\sqrt{2}}{3\sqrt{2}+4\sqrt{2}}$	M1		attempt to write each term in form $n\sqrt{2}$ with at least 2 terms correct
		A1		correct unsimplified
	$=\dfrac{2}{7}$	A1	3	
				or $\times\dfrac{\sqrt{2}}{\sqrt{2}}$ M1 integer terms $=\dfrac{4}{6+8}$ A1 $=\dfrac{2}{7}$ A1
(b)	$\dfrac{7\sqrt{2}-\sqrt{3}}{2\sqrt{2}-\sqrt{3}}\times\dfrac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$	M1		
	(numerator =) $14\times 2-2\sqrt{6}+7\sqrt{6}-3$	m1		correct unsimplified but must simplify $\left(\sqrt{2}\right)^2$, $\left(\sqrt{3}\right)^2$ and $\sqrt{2}\times\sqrt{3}$ correctly
	(denominator = 8 – 3 =) 5	B1		must be seen or identified as denominator
				giving $\dfrac{25+5\sqrt{6}}{5}$
	(Answer =) $5+\sqrt{6}$	A1cso	4	$m = 5$, $n = 6$
	Total		**8**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$(x-3)^2$	M1		or $p = 3$ seen
	$(x-3)^2 + 2$	A1	2	
(ii)	$(x-3)^2 = -2$	M1		FT their positive value of q
	No (real) square root of –2 therefore equation has no real solutions	A1cso	2	**not** use of discriminant for graphical approach see below to see if SC1 can be awarded
(b)(i)	$x =$ ‘their’ p *or* $y =$ ‘their’ q	M1		or $x = 3$ found using calculus
	Vertex is at (3, 2)	A1cao	2	
(ii)		B1		y intercept = 11 ***stated*** or ***marked on y-*** axis (as y intercept of any graph)
		M1		∪ shape (generous)
		A1	3	above x-axis , vertex in first quadrant crossing y-axis into second quadrant
(iii)	Translation	E1		and no other transformation
	through $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$	M1		FT negative of BOTH ‘their’ vertex coords
		A1	3	**both** components **correct** for A1; may describe in words or use a column vector
	Total		**12**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\mathrm{p}(-1) = (-1)^3 - 4\times(-1)^2 - 3(-1) + 18$	M1		p(–1) attempted **not** long division
	$(= -1 - 4 + 3 + 18) = 16$	A1	2	
(b)(i)	$\mathrm{p}(3) = 3^3 - 4\times 3^2 - 3\times 3 + 18$	M1		p(3) attempted **not** long division
	$\mathrm{p}(3) = 27 - 36 - 9 + 18 = 0 \Rightarrow x - 3$ is a factor	A1	2	shown = 0 plus statement
(ii)	Quadratic factor $(x^2 - x + c)$ or $(x^2 + bx - 6)$	M1		$-x$ or –6 term by inspection *or* full long division by $x-3$ *or* comparing coefficients *or* p(–2) attempted
	Quadratic factor $\left(x^2 - x - 6\right)$	A1		correct quadratic factor (or x+2 shown to be factor by Factor Theorem)
	$[\mathrm{p}(x) =]\ (x-3)(x-3)(x+2)$	A1	3	or $[\mathrm{p}(x) =]\ (x-3)^2(x+2)$ must see product of factors
(c)		M1		cubic curve with one maximum and one minimum
		A1		meeting x-axis at –2 and touching x-axis at 3
	Final A1 is dependent on previous A1 and can be withheld if curve has very poor curvature beyond x = 3, V shape at x = 3 etc	A1	3	graph as shown , going beyond $x = -2$ but condone max on or to right of y-axis
	Total		**10**	

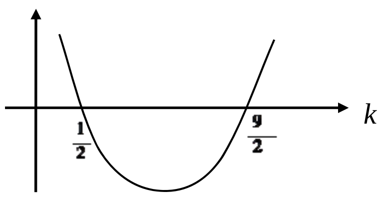
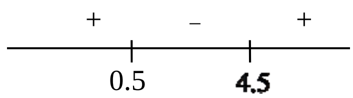
MPC1 (cont)

Q	Solution	Marks	Total	Comments
6(a)	(Gradient = 10 – 6 + 5) = 9	B1		correct gradient from sub x =1 into $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$y-4=\text{"}their\ 9\text{"}(x-1)$ or $y=\text{"}their\ 9\text{"}x+c$ **and** attempt to find c using $x=1$ and $y=4$	M1		must attempt to use given expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ **and** must be attempting tangent and not normal **and** coordinates must be correct
	$y=9x-5$	A1	3	condone $y=9x+c,\ldots\quad c=-5$
(b)	$(y=)\frac{10}{5}x^5-\frac{6}{3}x^3+5x+C$	M1		one term correct
		A1		another term correct
		A1		all integration correct including + C
	$4 = 2 - 2 + 5 + C$ $\Rightarrow C=-1$	m1		substituting both x =1 and y = 4 **and** attempting to find C
	$y=2x^5-2x^3+5x-1$	A1cso	5	must have y = ... and coefficients simplified
	Total		**8**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$x=0 \Rightarrow y^2-4y-12 \ (=0)$	M1		sub $x=0$ & correct quadratic in y *or* $(y-2)^2=16$ *or* $(y-2)^2-16=0$
	$(y-6)(y+2) \quad (=0)$	A1		correct factors *or* formula as far as $\frac{4\pm\sqrt{64}}{2}$ *or* $y-2=\pm\sqrt{16}$
	$\Rightarrow y=-2,\ 6$	A1	3	condone (0, –2) & (0, 6)
(b)	$(x+3)^2-9+(y-2)^2-4\,(=12)$	M1		***correct*** sum of square terms **and** attempt to complete squares (or multiply out) PI by next line
	$\left(r^2=\right) \quad 9+4+12$	A1		$\left(r^2=\right)25$ seen on RHS
	$\left(\Rightarrow r=\right) \ 5$	A1	3	$r=\sqrt{25}$ or $r=\pm5$ scores A0
(c)(i)	$\left(CP^2=\right) \quad (2--3)^2+(5-2)^2$	M1		condone one sign slip within one bracket
	$\Rightarrow\left(CP=\right) \quad \sqrt{34}$	A1	2	$n=34$
(ii)	$PQ^2=CP^2-r^2=34-25$	M1		Pythagoras used correctly with values FT "their" r and CP
	$\left(\Rightarrow PQ=\right) \quad 3$	A1	2	
	Total		**10**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$2x^2 - x - 1 = 2kx - 3k$ $2x^2 - x - 1 - 2kx + 3k = 0$ OE $\Rightarrow 2x^2 - (2k+1)x + 3k - 1 = 0$	B1	1	equated and multiplied out and all 5 terms on one side and = 0 **AG** (correct with no trailing = signs etc)
(b)(i)	$(2k+1)^2 - 4 \times 2(3k-1)$	M1		clear attempt at $b^2 - 4ac$
	$(2k+1)^2 - 4 \times 2(3k-1) > 0$	B1		discriminant > 0 which must appear before the printed answer
	$4k^2 + 4k + 1 - 24k + 8 > 0$			
	$\Rightarrow 4k^2 - 20k + 9 > 0$	A1cso	3	**AG** (all working correct with no missing brackets etc)
(ii)	$4k^2 - 20k + 9 = (2k-9)(2k-1)$	M1		correct factors or correct use of formula as far as $\dfrac{20 \pm \sqrt{256}}{8}$
	critical values are $\dfrac{1}{2}$ and $\dfrac{9}{2}$	A1		condone $\dfrac{4}{8}$, $\dfrac{36}{8}$ etc here but must combine sums of fractions
		M1		sketch or sign diagram including values 
	$k < \dfrac{1}{2}$, $k > \dfrac{9}{2}$ ***Take their final line as their answer***	A1	4	fractions must be simplified condone use of OR but not AND
	Total		**8**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - January 2013 exams

A-level

Code	Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points A*	A	B	C	D	E
LAW03	LAW UNIT 3	80	66	60	54	48	43	38
MD01	MATHEMATICS UNIT MD01	75	-	63	57	52	47	42
MD02	MATHEMATICS UNIT MD02	75	68	62	55	49	43	37
MFP1	MATHEMATICS UNIT MFP1	75	-	69	61	54	47	40
MFP2	MATHEMATICS UNIT MFP2	75	67	60	53	47	41	35
MFP3	MATHEMATICS UNIT MFP3	75	68	62	55	48	41	34
MFP4	MATHEMATICS UNIT MFP4	75	68	61	53	45	37	30
MM1B	MATHEMATICS UNIT MM1B	75	-	58	52	46	40	35
MM2B	MATHEMATICS UNIT MM2B	75	66	59	52	46	40	34
MPC1	MATHEMATICS UNIT MPC1	75	-	64	58	52	46	40
MPC2	MATHEMATICS UNIT MPC2	75	-	62	55	48	41	35
MPC3	MATHEMATICS UNIT MPC3	75	69	63	56	49	42	36
MPC4	MATHEMATICS UNIT MPC4	75	58	53	48	43	38	34
MS1A	MATHEMATICS UNIT MS1A	100	-	78	69	60	52	44
MS/SS1A/W	*MATHEMATICS UNIT S1A - WRITTEN*	*75*		*58*				*34*
MS/SS1A/C	*MATHEMATICS UNIT S1A - COURSEWORK*	*25*		*20*				*10*
MS1B	MATHEMATICS UNIT MS1B	75	-	60	54	48	42	36
MS2B	MATHEMATICS UNIT MS2B	75	70	66	58	50	42	35
MEST1	MEDIA STUDIES UNIT 1	80	-	54	47	40	33	26
MEST2	MEDIA STUDIES UNIT 2	80	-	63	54	45	36	28
MEST3	MEDIA STUDIES UNIT 3	80	68	58	48	38	28	18
MEST4	MEDIA STUDIES UNIT 4	80	74	68	56	45	34	23