General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MPC1

Unit Pure Core 1

Friday 13 January 2012 9.00 am to 10.30 am

For this paper you must have: • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.	

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The point A has coordinates $(6, -4)$ and the point B has coordinates $(-2, 7)$.

(a) Given that the point O has coordinates $(0, 0)$, show that the length of OA is less than the length of OB. *(3 marks)*

(b) (i) Find the gradient of AB. *(2 marks)*

(ii) Find an equation of the line AB in the form $px + qy = r$, where p, q and r are integers. *(3 marks)*

(c) The point C has coordinates $(k, 0)$. The line AC is perpendicular to the line AB. Find the value of the constant k. *(3 marks)*

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2 (a) Factorise $x^2 - 4x - 12$. *(1 mark)*

(b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes. *(4 marks)*

(c) (i) Express $x^2 - 4x - 12$ in the form $(x - p)^2 - q$, where p and q are positive integers. *(2 marks)*

(ii) Hence find the minimum value of $x^2 - 4x - 12$. *(1 mark)*

(d) The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

Find an equation of the new curve. You need not simplify your answer. *(2 marks)*

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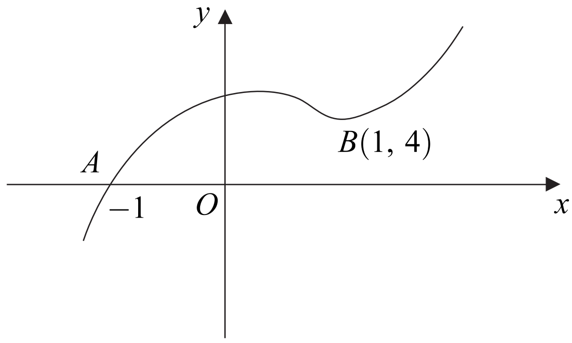
3 (a) (i) Simplify $\left(3\sqrt{2}\right)^2$. *(1 mark)*

(ii) Show that $\left(3\sqrt{2} - 1\right)^2 + \left(3 + \sqrt{2}\right)^2$ is an integer and find its value. *(4 marks)*

(b) Express $\dfrac{4\sqrt{5} - 7\sqrt{2}}{2\sqrt{5} + \sqrt{2}}$ in the form $m - \sqrt{n}$, where m and n are integers. *(4 marks)*

4 The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(a) Given that $y = x^5 - 3x^2 + x + 5$, find:

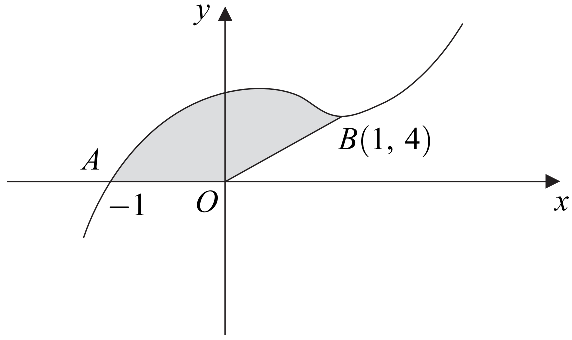
(i) $\dfrac{dy}{dx}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(b) Find an equation of the tangent to the curve at the point $A(-1, 0)$. *(2 marks)*

(c) Verify that the point B, where $x = 1$, is a minimum point of the curve. *(3 marks)*

(d) The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(i) Find $\displaystyle\int_{-1}^{1} (x^5 - 3x^2 + x + 5)\, dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve between A and B and the line segments AO and OB. *(2 marks)*

Turn over ▶

5 The polynomial $p(x)$ is given by $p(x) = x^3 + cx^2 + dx - 12$, where c and d are constants.

(a) When $p(x)$ is divided by $x + 2$, the remainder is -150.

Show that $2c - d + 65 = 0$. *(3 marks)*

(b) Given that $x - 3$ is a factor of $p(x)$, find another equation involving c and d. *(2 marks)*

(c) By solving these two equations, find the value of c and the value of d. *(3 marks)*

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6 A rectangular garden is to have width x metres and length $(x + 4)$ metres.

(a) The perimeter of the garden needs to be greater than 30 metres.

Show that $2x > 11$. *(1 mark)*

(b) The area of the garden needs to be less than 96 square metres.

Show that $x^2 + 4x - 96 < 0$. *(1 mark)*

(c) Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*

(d) Hence determine the possible values of the width of the garden. *(1 mark)*

7 A circle with centre C has equation $x^2 + y^2 + 14x - 10y + 49 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

(3 marks)

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) Sketch the circle. *(2 marks)*

(d) A line has equation $y = kx + 6$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$. *(2 marks)*

(ii) The equation $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ has equal roots. Show that

$$12k^2 - 7k - 12 = 0$$

(3 marks)

(iii) Hence find the values of k for which the line is a tangent to the circle. *(2 marks)*

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

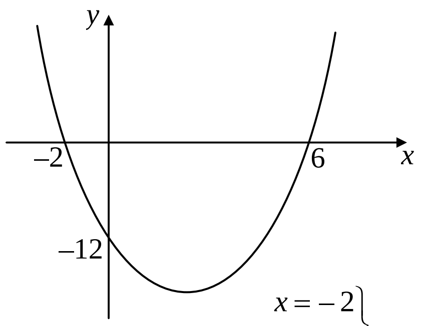
MPC1

Q	Solution	Marks	Total	Comments
1(a)	$(OA^2 =)\, 6^2 + (-4)^2$; $(OB^2 =)\, (-2)^2 + 7^2$	M1		either correct PI by 52 or 53 seen
	$(OA^2 =)\, 52$ **and** $(OB^2 =) 53$ or $(OA =)\sqrt{52}$ **and** $(OB =)\sqrt{53}$	A1		both correct values 52 or $\sqrt{52}$ **and** 53 or $\sqrt{53}$ seen
	$OA = \sqrt{52}$ and $OB = \sqrt{53}$ $\Rightarrow OA < OB$	A1	3	or $OA^2 = 52$ *and* $OB^2 = 53$ correct working + concluding statement involving *OA* and/or *OB*
(b)(i)	grad $AB = \dfrac{7+4}{-2-6}$	M1		condone one sign error
	$= -\dfrac{11}{8}$	A1	2	
(ii)	$y - -4 =$ 'their grad *AB*'$(x - 6)$ or $y - 7 =$ 'their grad *AB*'$(x - -2)$	M1		or $y =$ 'their grad *AB*' $x + c$ and attempt to find c using $x = 6$, $y = -4$ or $x = -2$, $y = 7$
	$y + 4 = -\dfrac{11}{8}(x - 6)$ OE	A1		any correct form *eg* $y = -\dfrac{11}{8}x + \dfrac{34}{8}$ but must simplify $- -$ to +
	$\Rightarrow 11x + 8y = 34$	A1	3	condone $8y + 11x = 34$ or any multiple of these equations
(c)	(grad *AC* =) $\dfrac{8}{11}$	B1$\checkmark$		FT -1 / 'their grad *AB*'
	$\dfrac{4}{k-6} = \text{'their } \dfrac{8}{11}\text{'}$ OE $\Rightarrow 2k - 12 = 11$	M1		equating gradients; LHS must be correct and RHS is "attempt" at perp grad to *AB*
	$\Rightarrow k = \dfrac{23}{2}$	A1cso	3	$k = 11.5$ OE
	Total		**11**	

(c) ***Alternative:*** Eqn *AC* : $(y + 4) = \text{'their}\dfrac{8}{11}\text{'}(x - 6)$ B1$\checkmark$ $(11y = 8x - 92)$ **AND** must sub $y = 0$ for M1

or $(y - 0) = \text{'their}\dfrac{8}{11}\text{'}(x - k)$ B1$\checkmark$ **AND** must sub $x = 6$, $y = -4$ for M1

MPC1 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$(x-6)(x+2)$	B1	1	ISW for $x=6$, $x=-2$ etc
(b)				
	$\left.\begin{array}{l} x=-2 \\ x=6 \end{array}\right\}$	B1√		correct x values *or* FT 'their' factors (x-intercepts stated ***or*** marked on sketch) may be seen in (a)
	$y=-12$	B1		(stated ***or*** –12 marked on sketch)
	$\cup$ – shaped curve	M1		approximately
	"correct" shape in all 4 quadrants with minimum to right of y-axis	A1	4	
(c)(i)	$(x-2)^2$	M1		$p=2$
	$(x-2)^2-16$	A1	2	$p=2$ and $q=16$
(ii)	(Minimum value is) –16	B1√	1	FT ' their $-q$ '
(d)	Replacing each x by $x+3$ **OR** adding 2 to their quadratic	M1		in original equation or 'their' completed square or factorised form or replacing y by $y-2$
	$y=\left[(x+3)^2-4(x+3)-12\right]+2$ or $y=(x+1)^2-14$ or $y=x^2+2x-13$ or $y-2=(x-3)(x+5)$	A1	2	OE any correct equation in x and y **unsimplified**
	Total		**10**	

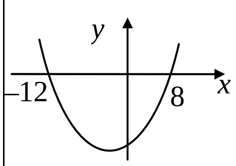
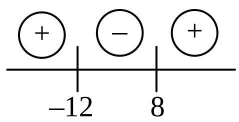
MPC1 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\left(3\sqrt{2}\right)^2 = 18$	B1	1	
(ii)	$\left(3\sqrt{2}-1\right)^2 = \text{'}\mathit{their}\ 18\text{'} - 3\sqrt{2} - 3\sqrt{2} + 1$	M1		FT their $\left(3\sqrt{2}\right)^2$
	$= 18 - 3\sqrt{2} - 3\sqrt{2} + 1$	A1		$\left(=19-6\sqrt{2}\right)$
	$\left(3+\sqrt{2}\right)^2 = 9 + 3\sqrt{2} + 3\sqrt{2} + 2$	B1		$\left(=11+6\sqrt{2}\right)$
	$\Rightarrow$ Sum = 30	A1cso	4	
(b)	$\dfrac{4\sqrt{5}-7\sqrt{2}}{2\sqrt{5}+\sqrt{2}} \times \dfrac{2\sqrt{5}-\sqrt{2}}{2\sqrt{5}-\sqrt{2}}$	M1		
	Numerator = $8\left(\sqrt{5}\right)^2 - 4\sqrt{5}\sqrt{2} - 14\sqrt{5}\sqrt{2} + 7\left(\sqrt{2}\right)^2$	m1		correct unsimplified $\left(=54-18\sqrt{10}\right)$
	Denominator = $\left(2\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^2$ = 18	B1		must be seen as denominator
	$\Rightarrow$ Answer = $3 - \sqrt{10}$	A1cso	4	
	Total		**9**	

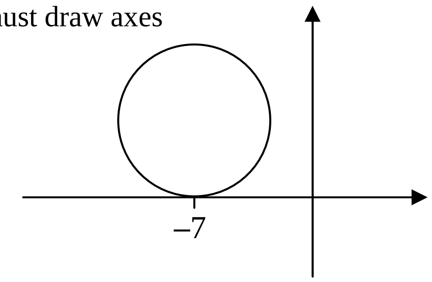
MPC1 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\left(\frac{dy}{dx}=\right)\ 5x^4-6x+1$	M1 A1 A1	3	one term correct another term correct all correct (no + c etc)
(ii)	$\left(\frac{d^2y}{dx^2}=\right)\ 20x^3-6$	B1√	1	FT 'their' $\frac{dy}{dx}$
(b)	$x=-1 \Rightarrow \frac{dy}{dx}=5(-1)^4-6(-1)+1\ \ (=12)$ $\Rightarrow\ y=12(x+1)$	M1 A1cso	2	must sub $x=-1$ into 'their' $\frac{dy}{dx}$ any correct form with $(x--1)$ simplified condone $y=12x+c,\ \ c=12$
(c)	$x=1 \Rightarrow \frac{dy}{dx}=5-6+1$ $\frac{dy}{dx}=0 \Rightarrow$ stationary point when $x=1$, $\frac{d^2y}{dx^2}=14$ $\Rightarrow$ (B is a) minimum (point)	M1 A1cso E1	3	sub $x=1$ into their $\frac{dy}{dx}$ shown $=0$ plus correct statement or $\frac{d^2y}{dx^2}=20-6>0$ $\Rightarrow$ (B is a) minimum (point) must have correct $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for E1
(d)(i)	$\frac{x^6}{6}-\frac{3x^3}{3}+\frac{x^2}{2}+5x$ $\left[\frac{1}{6}-1+\frac{1}{2}+5\right]-\left[\frac{1}{6}+1+\frac{1}{2}-5\right]$ $=\ 8$	M1 A1 A1 m1 A1cso	5	one term correct another term correct all correct (may have $+c$) 'their' F(1) – F(–1) with powers of 1 and –1 evaluated correctly
(ii)	'their answer to part (i)' – 2 $\Rightarrow$ Area = 6	M1 A1cso	2	
	Total		**16**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$p(-2) = (-2)^3 + (-2)^2 c + (-2)d - 12$	M1		p(–2) attempted *or* long division by x+2 as far as remainder
	‘their’ $-8 + 4c - 2d\ - 12 = -150$	m1		putting expression for remainder = –150
	$\Rightarrow\ 2c - d + 65 = 0$	A1cso	3	**AG** terms all on one side in any order (check that there are no errors in working)
(b)	$p(3) = 3^3 + 3^2 c + 3d - 12$	M1		p(3) attempted *or* long division by x–3 as far as remainder
	$9c + 3d + 15 = 0$	A1	2	any correct equation with terms collected eg $3c + d = -5$
(c)	$\left.\begin{array}{l} 2c - d + 65 = 0 \\ 3c + d + \ 5 = 0 \end{array}\right\} \Rightarrow 5c = -70$	M1		Elimination of c or d
	$\Rightarrow\ c = -14\ ,\ d = 37$ OE	A1 A1	 3	value of c ***or*** d correct unsimplified both c and d correct unsimplified
	Total		**8**	
6(a)	Sides are x and $x + 4$ $\Rightarrow\ x + x + x + 4 + x + 4 > 30$ *or* $2x + 2x + 8 > 30$ *or* $2(2x + 4) > 30$ *or* $4x + 8 > 30$			must see this line OE
	$(\Rightarrow 4x > 22)$ $\Rightarrow 2x > 11$	B1	1	**AG** (be convinced) condone 11 < 2x
(b)	$x(x + 4) < 96$ $\Rightarrow x^2 + 4x - 96 < 0$	 B1	 1	must see this line OE **AG**
(c)	$(x + 12)(x - 8)$	M1		correct factors or correct quadratic equation formula
	Critical values 8, –12	A1		
	 or 	M1		sketch or sign diagram
	$\Rightarrow -12 < x < 8$	A1cso	4	accept $x < 8$ **AND** $x > -12$ but **not** $x < 8$ **OR** $x > -12$ **nor** $x < 8$ **,** $x > -12$
(d)	$5\frac{1}{2} < x < 8$	B1	1	
	Total		**7**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\left(x+7\right)^2+\left(y-5\right)^2$	M1		one term correct ; condone $(x--7)^2$
		A1		both terms correct with squares and plus sign between terms
	$\left(x+7\right)^2+\left(y-5\right)^2 = 5^2$	A1cao	3	condone 25 for 5^2
(b)(i)	$C\left(-7, 5\right)$	B1√		correct or FT 'their' circle equation
(ii)	$r = 5$	B1√	2	correct or FT 'their' $r^2 > 0$ condone $\sqrt{25}$ etc but not $\pm\sqrt{25}$
(c)	must draw axes 	M1		freehand circle with C correct or FT 'their C' for quadrant of centre
		A1	2	circle touching x-axis at -7 with -7 marked (need not show 5 on y-axis) but circle must not touch y-axis
(d)(i)	$x^2+(kx+6)^2+14x-10(kx+6)+49=0$			clear attempt to sub $y = kx+6$ into original or 'their' circle equation …
	$x^2+k^2x^2+12kx+36+14x$ $-10kx-60+49=0$	M1		…***and*** attempt to multiply out
	$(1+k^2)x^2+2kx+14x+25=0$ $\Rightarrow (k^2+1)x^2+2(k+7)x+25=0$	A1cso	2	**AG** condone $x^2(1+k^2)+2x(7+k)+\ldots$ etc
(ii)	Equal roots '$b^2-4ac=0$'	B1		allow statement alone if discriminant in terms of k attempted
	$\left[2\left(k+7\right)\right]^2-4\times 25\left(k^2+1\right)$	M1		discriminant (condone one slip)
	$4\left\{k^2+14k+49-25k^2-25\right\}=0$ $-24k^2+14k+24=0$			
	$\Rightarrow 12k^2-7k-12=0$	A1	3	**AG** all working correct but = 0 must appear before last line
(iii)	$\left(4k+3\right)\left(3k-4\right)$	M1		correct factors or correct use of formula as far as $k=\dfrac{7\pm\sqrt{49+576}}{24}$
	$\Rightarrow k=-\dfrac{3}{4},\ k=\dfrac{4}{3}$ OE are values of k for which line is a tangent	A1	2	
	Total		**14**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - January 2012 exams

A-level

Code	Title	Max. Scaled Mark	A*	A	B	C	D	E
			Scaled Mark Grade Boundaries and A* Conversion Points					
LAW02	LAW UNIT 2	94	-	73	66	59	52	46
LAW03	LAW UNIT 3	80	69	63	57	51	45	40
MD01	MATHEMATICS UNIT MD01	75	-	62	56	50	44	39
MFP1	MATHEMATICS UNIT MFP1	75	-	67	60	53	46	39
MM1A	MATHEMATICS UNIT MM1A	100	no candidates were entered for this unit					
MM1B	MATHEMATICS UNIT MM1B	75	-	59	52	46	40	34
MPC1	MATHEMATICS UNIT MPC1	75	-	61	55	49	43	37
MS1A	MATHEMATICS UNIT MS1A	100	-	74	65	56	47	38
MS/SS1A/W	*MATHEMATICS UNIT S1A - WRITTEN*	*75*		*54*				*28*
MS/SS1A/C	*MATHEMATICS UNIT S1A - COURSEWORK*	*25*		*20*				*10*
MS1B	MATHEMATICS UNIT MS1B	75	-	56	49	42	36	30
MD02	MATHEMATICS UNIT MD02	75	69	64	57	50	44	38
MFP2	MATHEMATICS UNIT MFP2	75	59	52	45	38	31	25
MM2B	MATHEMATICS UNIT MM2B	75	69	63	55	47	39	32
MPC2	MATHEMATICS UNIT MPC2	75	-	66	59	52	46	40
MS2B	MATHEMATICS UNIT MS2B	75	69	63	55	47	40	33
MFP3	MATHEMATICS UNIT MFP3	75	67	60	52	44	37	30
MPC3	MATHEMATICS UNIT MPC3	75	64	57	50	43	37	31
MFP4	MATHEMATICS UNIT MFP4	75	60	54	48	42	37	32
MPC4	MATHEMATICS UNIT MPC4	75	63	57	51	45	39	33
MEST1	MEDIA STUDIES UNIT 1	80	-	55	47	40	33	26
MEST2	MEDIA STUDIES UNIT 2	80	-	63	54	45	36	28
MEST3	MEDIA STUDIES UNIT 3	80	67	57	47	37	27	18
MEST4	MEDIA STUDIES UNIT 4	80	74	68	56	45	34	23
PHIL1	PHILOSOPHY UNIT 1	90	-	55	49	43	37	32