General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MPC1

Unit Pure Core 1

Monday 10 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The curve with equation $y = 13 + 18x + 3x^2 - 4x^3$ passes through the point P where $x = -1$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Show that the point P is a stationary point of the curve and find the other value of x where the curve has a stationary point. *(3 marks)*

(c) (i) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

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2 (a) Simplify $(3\sqrt{3})^2$. *(1 mark)*

(b) Express $\dfrac{4\sqrt{3} + 3\sqrt{7}}{3\sqrt{3} + \sqrt{7}}$ in the form $\dfrac{m + \sqrt{21}}{n}$, where m and n are integers. *(4 marks)*

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3 The line AB has equation $3x + 2y = 7$. The point C has coordinates $(2, -7)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) The line which passes through C and which is parallel to AB crosses the y-axis at the point D. Find the y-coordinate of D. *(3 marks)*

(b) The line with equation $y = 1 - 4x$ intersects the line AB at the point A. Find the coordinates of A. *(3 marks)*

(c) The point E has coordinates $(5, k)$. Given that CE has length 5, find the two possible values of the constant k. *(3 marks)*

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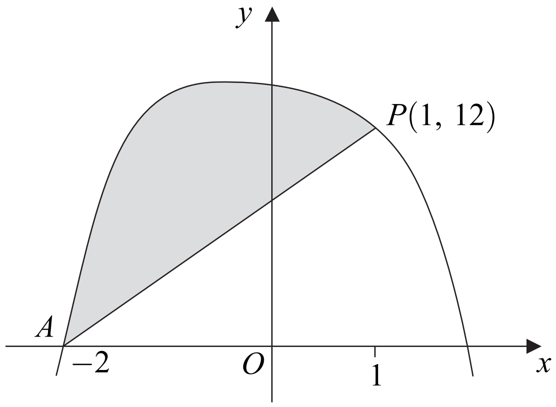
4 The curve sketched below passes through the point $A(-2, 0)$.



The curve has equation $y = 14 - x - x^4$ and the point $P(1, 12)$ lies on the curve.

(a) (i) Find the gradient of the curve at the point P. *(3 marks)*

(ii) Hence find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(2 marks)*

(b) (i) Find $\int_{-2}^{1} (14 - x - x^4)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve $y = 14 - x - x^4$ and the line AP. *(2 marks)*

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5 (a) (i) Sketch the curve with equation $y = x(x-2)^2$. *(3 marks)*

(ii) Show that the equation $x(x-2)^2 = 3$ can be expressed as

$$x^3 - 4x^2 + 4x - 3 = 0$$ *(1 mark)*

(b) The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 4x - 3$.

(i) Find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x)$ in the form $(x-3)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(c) Hence show that the equation $x(x-2)^2 = 3$ has only one real root and state the value of this root. *(3 marks)*

Turn over ▶

6 A circle has centre $C(-3,\ 1)$ and radius $\sqrt{13}$.

(a) (i) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

(ii) Hence find the equation of the circle in the form

$$x^2 + y^2 + mx + ny + p = 0$$

where m, n and p are integers. *(3 marks)*

(b) The circle cuts the y-axis at the points A and B. Find the distance AB. *(3 marks)*

(c) (i) Verify that the point $D(-5,\ -2)$ lies on the circle. *(1 mark)*

(ii) Find the gradient of CD. *(2 marks)*

(iii) Hence find an equation of the tangent to the circle at the point D. *(2 marks)*

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7 (a) (i) Express $4 - 10x - x^2$ in the form $p - (x+q)^2$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = 4 - 10x - x^2$. *(1 mark)*

(b) The curve C has equation $y = 4 - 10x - x^2$ and the line L has equation $y = k(4x - 13)$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$x^2 + 2(2k+5)x - (13k+4) = 0$$ *(1 mark)*

(ii) Given that the curve C and the line L intersect in two distinct points, show that

$$4k^2 + 33k + 29 > 0$$ *(3 marks)*

(iii) Solve the inequality $4k^2 + 33k + 29 > 0$. *(4 marks)*

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1(a)	$\frac{dy}{dx} = 18 + 6x - 12x^2$	M1 A1 A1	 3	one of these terms correct another term correct all correct (no + c etc) (penalise + c once only in question)
(b)	$18 + 6x - 12x^2 = 0$	M1		putting their $\frac{dy}{dx} = 0$, PI by attempt to solve or factorise
	$6\ (3 - 2x)(x + 1)\quad (= 0)$	m1		attempt at factors of **their quadratic** or use of quadratic equation formula
	$x = -1,\ x = \frac{3}{2}$ OE	A1	3	must see both values unless $x = -1$ is verified separately If M1 not scored, award SC B1 for verifying that $x = -1$ leads to $\frac{dy}{dx} = 0$ and a further SC B2 for finding $x = \frac{3}{2}$ as other value
(c)(i)	$\frac{d^2y}{dx^2} = 6 - 24x$	B1√		FT their $\frac{dy}{dx}$ but $\frac{d^2y}{dx^2}$ must be correct if 3 marks earned in part (a)
	When $x = -1$, $\frac{d^2y}{dx^2} = 6 - (24 \times -1)$	M1		Sub $x = -1$ into 'their' $\frac{d^2y}{dx^2}$
	$\frac{d^2y}{dx^2} = 30$	A1cso	3	
(ii)	Minimum point	E1√	1	must have a value in (c)(i) FT "maximum" if their value of $\frac{d^2y}{dx^2} < 0$
	Total		**10**	

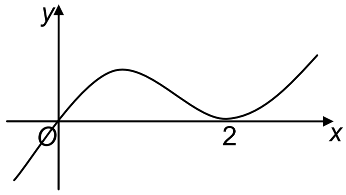
MPC1 (cont)

Q	Solution	Marks	Total	Comments
2(a)	27	B1	1	
(b)	$\dfrac{4\sqrt{3}+3\sqrt{7}}{3\sqrt{3}+\sqrt{7}} \times \dfrac{3\sqrt{3}-\sqrt{7}}{3\sqrt{3}-\sqrt{7}}$	M1		
	(Numerator =) $36 + 9\sqrt{21} - 4\sqrt{21} - 21$	m1		expanding numerator condone one slip or omission
	(Denominator =) 20 $\dfrac{15+5\sqrt{21}}{20}$	B1		must be seen as denominator
	$= \dfrac{3+\sqrt{21}}{4}$	A1cso	4	$m = 3, n = 4$ condone $\dfrac{3}{4}+\dfrac{\sqrt{21}}{4}$
	Total		**5**	
3(a)(i)	$y = \dfrac{1}{2}(7-3x)$	M1		attempt at $y = \ldots$ or use of 2 correct points using $\dfrac{\Delta y}{\Delta x}$
	$\Rightarrow$ gradient $= -\dfrac{3}{2}$	A1	2	condone slip in rearranging if gradient is correct
(ii)	$y =$ 'their grad' $x + c$ and substitution of $x = 2$, $y = -7$	M1		or using $3x + 2y = k$ with $x = 2$, $y = -7$ and attempt to find k or $y - -7 =$ 'their grad'$(x-2)$
	$y = -\dfrac{3}{2}x + c, \quad c = -4$	A1		correct equation in any form $y+7 = -\dfrac{3}{2}(x-2)$, $3x + 2y + 8 = 0$, etc
	$(x = 0 \Rightarrow)\ y = -4$	A1cso	3	or y-intercept = –4 or $D(0, -4)$
(b)	$3x + 2(1-4x) = 7$, $y = 1 - \dfrac{4}{3}(7-2y)$	M1		elimination of y (or x) (condone one slip)
	$x = -1$	A1		one coordinate correct
	$y = 5$	A1	3	other coordinate correct coordinates of $A(-1, 5)$
(c)	$(5-2)^2 + (k+7)^2 = 5^2$ (or $k + 7 = 4$ or $k + 7 = -4$)	M1		condone one sign slip within one bracket
	$k = -3$	A1		one correct value of k
	or $k = -11$	A1	3	both correct (and no other values)
	Total		**11**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{dy}{dx} = -1 - 4x^3$	M1 A1		one of these terms correct all correct (no + c)
	(When $x = 1$, grad =) –5	A1cso	3	(Check that $\frac{dy}{dx}$ is actually correct!)
(ii)	$y - 12 = \text{'their grad'}(x-1)$	M1		any form of equation through (1, 12) and attempt at c if using $y = mx + c$
	$y = -5x + 17$ (or $y = 17 - 5x$)	A1√	2	FT their gradient Condone $y = -5x + c$, $c = 17$ etc
(b)(i)	$14x - \frac{x^2}{2} - \frac{x^5}{5}$	M1 A1 A1		one of these terms correct another term correct all correct (may have + c)
	$[\ ]_{-2}^{1} =$			
	$\left(14 - \frac{1}{2} - \frac{1}{5}\right) - \left(-28 - 2 + \frac{32}{5}\right)$	m1		F(1) and F(–2) attempted
	= 36.9 OE	A1	5	Condone recovery to this value
(ii)	Area $\Delta = \frac{1}{2} \times 3 \times 12$ = 18	M1		Correct area of triangle unsimplified
	⇒ shaded area = 18.9	A1cso	2	
	Total		**12**	

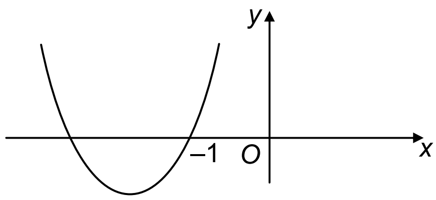
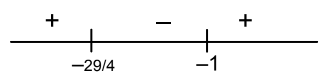
MPC1 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)		M1 A1 A1	 3	cubic curve with one max and one min (either way up) curve touching positive x-axis (either way up) correct graph passing through O and touching x-axis at 2
(ii)	$x\left(x^2-4x+4\right)=3$ $\Rightarrow x^3-4x^2+4x-3=0$	B1	1	AG (must have = 0)
(b)(i)	$\text{p}(-1)=(-1)^3-4(-1)^2+4(-1)-3$ $(=-1-4-4-3)$ $=-12$	M1 A1	 2	p(–1) attempted (condone one slip) or full long division to remainder must indicate remainder $=-12$ if long division used
(ii)	$\text{p}(3)=3^3-4\times3^2+4\times3-3$ $\text{p}(3)=27-36+12-3$ $\text{p}(3)=0\Rightarrow x-3$ is factor	M1 A1	 2	p(3) attempted (condone one slip) NOT long division shown = 0 **plus statement**
(iii)	Either $b=-1$ (coefficient of x correct) or $c=1$ (constant term correct) $\text{p}(x)=(x-3)\left(x^2-x+1\right)$	M1 A1	 2	allow M1 for full attempt at long division or comparing coefficients if neither b nor c is correct
(c)	Discriminant of 'their quadratic' $=(-1)^2-4$ Discriminant = –3 (or <0) $\Rightarrow$ no real roots (Only real root is $x=$) 3	M1 A1cso B1	 3	numerical expression must be seen must have correct quadratic and statement and all working correct
	Total		**13**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$(x+3)^2+(y-1)^2$	B1		condone $(x--3)^2$
	$= 13$	B1	2	condone $\left(\sqrt{13}\right)^2$
(ii)	$x^2+6x+9+y^2-2y+1$	M1		attempt to multiply out both of 'their' brackets; must have x and y terms
	$x^2+y^2+6x-2y$	A1		both $m = 6$ and $n = -2$
	$-3=0$	A1	3	All correct, $p = -3$ and ... = 0
(b)	$x=0 \Rightarrow y^2-2y-3=0$			putting $x = 0$ PI
	$\Rightarrow (y-3)(y+1)=0$	M1		and attempt to solve or factorise
	$y=3,\ y=-1$	A1		
	$\Rightarrow$ Distance $AB=3+1=4$	A1cso	3	**OR** Pythagoras $d^2=13-3^2$ M1 $d = 2$ A1 distance = $2\times2 = 4$ A1
(c)(i)	$(-5+3)^2+(-2-1)^2=4+9$			Substitution $x=-5$, $y=-2$ into any correct circle equation
	$=13$			
	$\Rightarrow$ D lies on circle	B1	1	convincing verification **plus statement**
(ii)	grad CD = $\dfrac{1+2}{-3+5}$	M1		condone one sign slip
	$=\dfrac{3}{2}$ (or 1.5)	A1	2	not $\dfrac{-3}{-2}$
(iii)	Perpendicular gradient $=-\dfrac{2}{3}$	M1		ft their grad CD or $m_1m_2=-1$ stated
	Tangent has equation $y+2=-\dfrac{2}{3}(x+5)$	A1	2	any form of correct equation eg $2x + 3y +16 =0$ $y=-\dfrac{2}{3}x+c,\ c=-\dfrac{16}{3}$
	Total		**13**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	(–) $(x+5)^2$	M1		$q = 5$; condone $(-x-5)^2$
	$29 - (x+5)^2$	A1	2	$p = 29$ and $q = 5$
(ii)	$x = -5$ is line of symmetry	B1√	1	FT $x = -$ 'their q' or correct
(b)(i)	$4 - 10x - x^2 = k(4x-13)$			
	$\Rightarrow x^2 + 4kx + 10x - 13k - 4 = 0$			Must see both these lines OE
	$\Rightarrow x^2 + 2(2k+5)x - (13k+4) = 0$	B1	1	AG all correct working and = 0
(ii)	2 distinct roots $\Rightarrow b^2 - 4ac > 0$	B1		stated or used (must be > 0)
	Discriminant = $4(2k+5)^2 + 4(13k+4)$	M1		condone one slip (may be within formula)
	$4(4k^2 + 20k + 25 + 13k + 4) > 0$			or $16k^2 + 132k + 116 > 0$
	$\Rightarrow 4k^2 + 33k + 29 > 0$	A1	3	AG > 0 must appear before final line
(iii)	$(4k+29)(k+1)$	M1		correct factors or correct unsimplified quadratic equation formula $\dfrac{-33 \pm \sqrt{33^2 - 4\times4\times29}}{8}$
	$k = -\dfrac{29}{4}$, $k = -1$	A1		condone $k = -\dfrac{58}{8}$, -7.25 etc but not left with square roots etc as above
$-\dfrac{29}{4}$		M1		sketch or sign diagram including values 
	$k < -\dfrac{29}{4}$, $k > -1$ ***Take their final line as their answer***	A1	4	condone use of **OR** but not **AND**
	Total		**11**	
	TOTAL		**75**	

Scaled mark unit grade boundaries - January 2011 exams

A-level

Code	Title	Max. Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points A*	A	B	C	D	E
MD01	GCE MATHEMATICS UNIT D01	75	-	61	55	49	43	37
MFP1	GCE MATHEMATICS UNIT FP1	75	-	63	56	49	43	37
MM1A	GCE MATHEMATICS UNIT M1A	100	no candidates were entered for this unit					
MM1B	GCE MATHEMATICS UNIT M1B	75	-	61	53	46	39	32
MPC1	GCE MATHEMATICS UNIT PC1	75	-	56	49	42	36	30
MS1A	GCE MATHEMATICS UNIT S1A	100	-	84	74	64	54	44
MS/SS1A/W	*GCE MATHEMATICS UNIT S1A - WRITTEN*	*75*		*64*				*34*
MS/SS1A/C	*GCE MATHEMATICS UNIT S1A - COURSEWORK*	*25*		*20*				*10*
MS1B	GCE MATHEMATICS UNIT S1B	75	-	61	53	46	39	32
MD02	GCE MATHEMATICS UNIT D02	75	69	63	56	50	44	38
MFP2	GCE MATHEMATICS UNIT FP2	75	67	60	51	42	34	26
MM2B	GCE MATHEMATICS UNIT M2B	75	63	55	47	40	33	26
MPC2	GCE MATHEMATICS UNIT PC2	75	-	61	54	47	40	33
MS2B	GCE MATHEMATICS UNIT S2B	75	66	59	52	45	38	31
XMCA2	GCE MATHEMATICS UNIT XMCA2	125	105	93	81	70	59	48
MFP3	GCE MATHEMATICS UNIT FP3	75	66	59	52	45	38	31
MPC3	GCE MATHEMATICS UNIT PC3	75	66	59	52	45	38	31
MFP4	GCE MATHEMATICS UNIT FP4	75	63	55	47	40	33	26
MPC4	GCE MATHEMATICS UNIT PC4	75	68	61	54	47	41	35
MEST1	GCE MEDIA STUDIES UNIT 1	80	-	56	49	42	35	28
MEST2	GCE MEDIA STUDIES UNIT 2	80	-	63	54	45	36	28
MEST3	GCE MEDIA STUDIES UNIT 3	80	69	58	47	37	27	17
MEST4	GCE MEDIA STUDIES UNIT 4	80	72	65	53	42	31	20
PHIL1	GCE PHILOSOPHY UNIT 1	90	-	54	48	42	37	32
PHIL2	GCE PHILOSOPHY UNIT 2	90	-	62	56	50	44	38