General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MPC1

Unit Pure Core 1

Monday 11 January 2010 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $p(x)$ is given by $p(x) = x^3 - 13x - 12$.

(a) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(3 marks)*

2 The triangle ABC has vertices $A(1, 3)$, $B(3, 7)$ and $C(-1, 9)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence show that angle ABC is a right angle. *(2 marks)*

(b) (i) Find the coordinates of M, the mid-point of AC. *(2 marks)*

(ii) Show that the lengths of AB and BC are equal. *(3 marks)*

(iii) Hence find an equation of the line of symmetry of the triangle ABC. *(3 marks)*

3 The depth of water, y metres, in a tank after time t hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{dy}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value. *(4 marks)*

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$. *(1 mark)*

4 (a) Show that $\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}$ is an integer and find its value. *(3 marks)*

(b) Express $\dfrac{2\sqrt{7}-1}{2\sqrt{7}+5}$ in the form $m+n\sqrt{7}$, where m and n are integers. *(4 marks)*

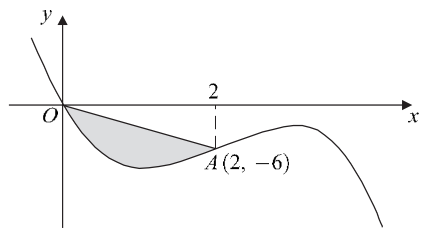
5 (a) Express $(x-5)(x-3)+2$ in the form $(x-p)^2+q$, where p and q are integers. *(3 marks)*

(b) (i) Sketch the graph of $y=(x-5)(x-3)+2$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(ii) Write down an equation of the tangent to the graph of $y=(x-5)(x-3)+2$ at its vertex. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y=x^2$ onto the graph of $y=(x-5)(x-3)+2$. *(3 marks)*

6 The curve with equation $y=12x^2-19x-2x^3$ is sketched below.



The curve crosses the x-axis at the origin O, and the point $A\,(2, -6)$ lies on the curve.

(a) (i) Find the gradient of the curve with equation $y=12x^2-19x-2x^3$ at the point A. *(4 marks)*

(ii) Hence find the equation of the normal to the curve at the point A, giving your answer in the form $x+py+q=0$, where p and q are integers. *(3 marks)*

(b) (i) Find the value of $\displaystyle\int_0^2 (12x^2-19x-2x^3)\,dx$. *(5 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

7 A circle with centre C has equation $x^2+y^2-4x+12y+15=0$.

(a) Find:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(2 marks)*

(b) Explain why the circle lies entirely below the x-axis. *(2 marks)*

(c) The point P with coordinates $(5, k)$ lies outside the circle.

(i) Show that $PC^2=k^2+12k+45$. *(2 marks)*

(ii) Hence show that $k^2+12k+20>0$. *(1 mark)*

(iii) Find the possible values of k. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 - Jan 2010 – Answers

Question 1:	Exam report
$p(x)=x^3-13x-12$ $a)\ p(-3)=(-3)^3-13\times(-3)-12$ $p(-3)=-27+39-12=0$ -3 is a root of p, so $(x+3)$ is a factor of p $b)\ p(x)=x^3-13x-12=(x+3)(x^2-3x-4)$ $=(x+3)(x-4)(x+1)$	In part (a), most candidates realised the need to find the value of f(x) when x = –3. However, it was also necessary, after showing that f(–3) = 0, to write a statement that the zero value implied that x + 3 was a factor. It was good to see quite a large number of candidates being aware of this but others lost a valuable mark. In part (b), some candidates used long division effectively to find the quadratic factor and, although this was the most successful method, some were confused by the lack of an x^2 term; others used the method of comparing coefficients or found the terms of the quadratic by inspection; a number used the Factor Theorem to find another linear factor, but seldom found both of the remaining factors. Very able candidates were able to write down the correct product of three linear factors but many more were unsuccessful when they tried to do this without any discernible method.

Question 2:	Exam report
$A(1,3)\ ,\ B(3,7)\ ,\ C(-1,9)$ $a)\,i)$ Gradient of $AB=m_{AB}=\frac{7-3}{3-1}=2$ $ii)$ Gradient of BC $=m_{BC}=\frac{9-7}{-1-3}=-\frac{1}{2}$ $m_{BC}\times m_{AB}=2\times-\frac{1}{2}=-1$ BC and AB are perpendicular so the traingle ABC is a right-angled triangle $b)\,i)$ Mid-point of AC $=M\left(\frac{1-1}{2},\frac{3+9}{2}\right)=M(0,6)$ $ii)\ AB=\sqrt{(3-1)^2+(7-3)^2}=\sqrt{4+16}=\sqrt{20}$ $BC=\sqrt{(-1-3)^2+(9-7)^2}=\sqrt{16+4}=\sqrt{20}$ $length\ \ AB=length\ \ BC$ $iii)$ The triangle ABC is an isosceles right-angled triangle The line of symmetry is the line BM. gradient of $BM=m_{BM}=\frac{7-6}{3-0}=\frac{1}{3}$ $Equation\ of\ \ BM:\ y-7=\frac{1}{3}(x-3)$ $3y-21=x-3$ $x-3y+18=0$	In part (a)(i), although some made arithmetic errors when finding the gradient of AB, the majority of answers were correct. It was necessary to reduce fractions such as 4/2 in order to score full marks. In part (a)(ii), those who chose to use Pythagoras' Theorem, calculating lengths of sides to prove that the triangle was right angled, scored no marks here. The word "hence" indicated that the gradient of AB needed to be used in the proof that angle ABC was a right angle. A large number of those using gradients failed to score full marks on this part of the question. It was not sufficient to show that the gradient of BC was –1/2 and then to simply say "therefore ABC is a right angle"; an explanation that the product of the gradients was equal to – 1 was required. In part (b)(i), most candidates were able to find the correct coordinates of the mid point, although a few transposed the coordinates and others subtracted, rather than added, the coordinates before halving the results. In part (b)(ii), it was rare to see a solution with all mathematical statements correct. Too often candidates wrote things like AB = $2^2+4^2=20=\sqrt{20}$ and, although this was not penalised on this occasion, examiners in the future might not be quite so generous. It was surprising how many candidates did not know the distance formula. Some wrote down vectors but, unless their lengths were calculated, no marks were scored. In part (b)(iii), many candidates found an equation of the wrong line. The line of symmetry was actually BM, although some chose an equivalent method using the gradient of a line perpendicular to AC. The most successful candidates often used an equation of the form $y-y_1=m(x-x_1)$; far too often those using y = mx + c were unable to find the correct value of c, usually because of poor arithmetic.

Question 3:	Exam report
$y=\frac{1}{8}t^4-2t^2+4t \qquad 0\le t\le 4$ $a)\,i)\ \frac{dy}{dt}=\frac{1}{2}t^3-4t+4$ $ii)\ \frac{d^2y}{dt^2}=\frac{3}{2}t^2-4$ $b)\ \frac{dy}{dt}(t=2)=\frac{1}{2}\times(2)^3-4\times(2)+4$ $=4-8+4=0$ At $t=2,\ \frac{dy}{dx}=0,$ there is a stationary point. $\frac{d^2y}{dt^2}(t=2)=\frac{3}{2}\times(2)^2-4=6-4=2>0$ The stationary point is a MINIMUM. $c)\,i)$ The rate of change is $\frac{dy}{dt}(t=1)=\frac{1}{2}-4+4=\frac{1}{2}=0.5\,m/s$ $ii)\ \frac{dy}{dt}(t=1)>0,$ *so* the depth is INCREASING at $t=1$.	In part (a), almost all candidates were able to find the first and second derivative correctly, although there was an occasional arithmetic slip and some could not cope with the fraction term. In part (b), those who substituted t = 2 into $\frac{dy}{dx}$ did not always explain that $\frac{dy}{dx}$ = 0 is the condition for a stationary point. Some assumed that a stationary point occurred when t = 2, went straight to the test for maximum or minimum and only scored half the marks. It was advisable to use the second derivative test here; those who considered values of $\frac{dy}{dx}$ on either side of t = 2 usually reached an incorrect conclusion because of the proximity of another stationary point. In part (c)(i), the concept of 'rate of change' was not understood by many who failed to realise the need to substitute t = 1 into $\frac{dy}{dx}$. Some candidates wrongly substituted t = 1 into the initial expression for y or into their expression for $\frac{d^2y}{dx^2}$ and these candidates were unable to score any marks at all on this part. Even those who used $\frac{dy}{dx}$ sometimes made careless arithmetic errors when adding three numbers. In part (c)(ii), it was not enough to simply write the word "increasing": some explanation about $\frac{dy}{dx}$ being positive was also required. Some candidates erroneously found the value of the second derivative when t = 1 or calculated the value of y on either side of t = 1.

Question 4:	Exam report
$a)\ \frac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}=\frac{5\sqrt{2}+3\sqrt{2}}{2\sqrt{2}}=\frac{8\sqrt{2}}{2\sqrt{2}}=\frac{8}{2}=4$ $b)\ \frac{2\sqrt{7}-1}{2\sqrt{7}+5}=\frac{2\sqrt{7}-1}{2\sqrt{7}+5}\times\frac{2\sqrt{7}-5}{2\sqrt{7}-5}=\frac{28-10\sqrt{7}-2\sqrt{7}+5}{28-25}$ $=\frac{33-12\sqrt{7}}{3}=11-4\sqrt{7}$	In part (a), there were far more mistakes than had been anticipated; for example, $\sqrt{50}=2\sqrt{5}$ and $\sqrt{18}=2\sqrt{3}$. It was also common to see poor cancelling such as $\frac{8\sqrt{2}}{2\sqrt{2}}=4\sqrt{2}$. Examiners had to take care that totally incorrect work leading to the correct answer was not rewarded. For instance, $\frac{\sqrt{50}+\sqrt{18}}{\sqrt{8}}=\frac{2\sqrt{5}+2\sqrt{9}}{2\sqrt{4}}=\frac{10+6}{4}=4$ was seen on a number of occasions. In part (b), it was pleasing to see that most candidates were familiar with the technique for rationalising the denominator in this type of problem and, although there were some who made slips when multiplying out the two brackets in the numerator, particularly when trying to calculate $2\sqrt{7}\times2\sqrt{7}$, many obtained the correct answer in the given form and it was good to see most getting the final step correct by dividing **both** terms by 3.

Question 5:

$a)\ (x-5)(x-3)+2 = x^2-8x+15+2 = x^2-8x+17$

$x^2-8x+17 = (x-4)^2-16+17 = (x-4)^2+1$

$b)\ i)$ The minimum point/vertex is $(4,1)$

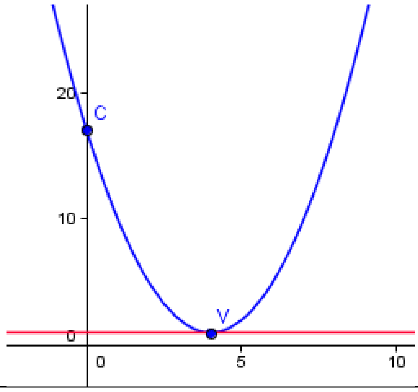
The curve crosses the y-axis at $(0,17)$

$ii)$ At the vertex, the tangent to the curve is "horizontal"

$y=1$

$c)\ x \xrightarrow{\text{translation 4 units in x-dir}} x-4 \xrightarrow{f} (x-4)^2 \xrightarrow{\text{trnslation 1 unit in y-dir}} (x-4)^2+1$

The transformation is a translation vector $\begin{bmatrix}4\\1\end{bmatrix}$.



Exam report

In part (a), the slightly unusual form of the initial quadratic caused problems to those who forgot to add 2 after multiplying out the brackets. The first term in the completion of the square was done successfully by most candidates, although the answer $(x-4)^2-3$ was seen almost as often as the correct form. It was unfortunate that an error at this stage was seldom picked up by candidates when they went on to sketch the curve.

In part (b)(i), the sketch was usually of the correct shape but many drew a parabola with the vertex in the wrong quadrant, or the y-intercept was incorrect. The question did ask for the coordinates of the minimum point and this was not always stated by candidates.

In part (b)(ii), a few candidates immediately wrote down the correct equation for the tangent, whereas many felt the need to differentiate in order to find the gradient of the curve at the vertex. Many candidates seemed unaware that the vertex was actually the minimum point and that the tangent at the vertex would be parallel to the line $y = 0$.

In part (c), the more able candidates earned full marks. The term translation was required and, although there seemed to be an improvement in candidates' using the correct word, it was still common to see words such as "shift" or "move" being used instead. Others thought that simply writing down a vector was enough. A very commonly stated, but incorrect, vector was $\begin{bmatrix}-4\\1\end{bmatrix}$

Question 6:

$y = 12x^2-19x-2x^3 \qquad O(0,0)\ and\ A(2,-6)$

$a)\ i)\ \dfrac{dy}{dx} = 24x-19-6x^2$

$\dfrac{dy}{dx}(x=2) = m_A = 24\times 2-19-6\times 2^2 = 5$

The gradient of the curve at A is 5.

$ii)$ The gradient of the normal at A is $-\dfrac{1}{m_A} = -\dfrac{1}{5}$

The eqaution of the normal: $y+6 = -\dfrac{1}{5}(x-2)$

$5y+30 = -x+2$

$x+5y+28=0$

$b)\ i)\ \int_0^2 (12x^2-19x-2x^3)dx = \left[4x^3-\dfrac{19}{2}x^2-\dfrac{1}{2}x^4\right]_0^2$

$=(32-38-8)-(0) = -14$

$ii)$ The area comprised between the curve and the x-axis is 14.

The area of the triangle is $\dfrac{1}{2}\times 2\times 6 = 6$.

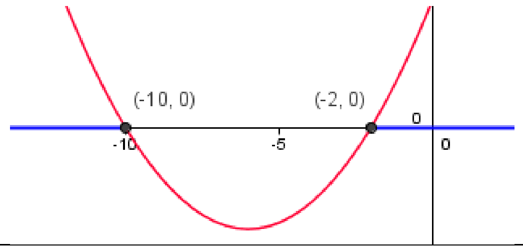
The area of the shaded region is $14-6\ = 8$

Exam report

In part (a)(i), many candidates did not realise that differentiation was required in order to find the gradient of the curve, but instead erroneously used the coordinates of O and A. Some tried to rearrange the terms but usually made sign errors in doing so. In part (a)(ii), those who had the correct gradient in part (a)(i) were usually successful in finding the correct equation of the normal, though not everyone followed through to the required form, and sign errors were common. However, most obtained at least a method mark here, unless they found the equation of the tangent. The main casualties were once again those who always use the y = mx + c form for the equation of a straight line.

In part (b)(i), most candidates were well drilled in integration and scored full marks, although some wrote down terms with incorrect denominators. The limits 2 and 0 were usually substituted correctly, but it was incredible how many could not evaluate 32 – 38 – 8 without a calculator. The correct value of the integral was –14, but far too many thought that they had to change their answer to +14 and so lost a mark. A small number of candidates differentiated or substituted into the expression for y rather than the integrated function.

In part (b)(ii), there was still some apparent confusion about area when a region lies below the x-axis. The area of the triangle was 6 units and hence the area of the shaded region was 14 – 6 = 8 but, not surprisingly, there were all kinds of combinations of positive and negative quantities seen here. It was worrying to see so many candidates failing to calculate the triangle area, with several finding the length of OA instead. Some able candidates found the equation of OA and the area under it by integration, but this was not the expected method.

Question 7:	Exam report
$x^2+y^2-4x+12y+15=0$ $(x-2)^2-4+(y+6)^2-36+15=0$ $(x-2)^2+(y+6)^2=25$ $a)i)\ Centre\ C(2,-6)$ $ii)\ radius\ r=\sqrt{25}=5$ $b)$ The distance from C to the x-axis is 6, which is more that the radius 5. c) $P(5,k)$ *lies outside the circle* $i)\ PC^2=(2-5)^2+(-6-k)^2=9+36+k^2+12k$ $PC^2=k^2+12k+45$ $ii)\ P$ lies outside the circle so $PC>5\ or\ PC^2>25$ $we\ have\ then\ \ k^2+12k+45>25$ $k^2+12k+20>0$ $iii)\ k^2+12k+20>0$ $(k+2)(k+10)>0$ $critical\ values: -10\ and\ -2$ $(k+2)(k+10)>0\ \ for\ \ k<-10\ or\ k>-2$  	In part (a), most candidates found at least one of the correct coordinates for the centre C, with the most common error being at least one sign error or writing the centre as (4, –12). However, the correct value for the radius was not so common, with $\sqrt{15}$ being frequently seen. In part (b), most explanations involving a comparison of the y-coordinate of the centre and the radius of the circle earned at least one mark. In order to score the second mark, some of the better answers explained why the y-coordinate of the “highest” point of the circle was –1, but most comments were insufficient. Those who proved that the circle did not intersect the x-axis needed to state that the y-coordinate of the centre was negative in order to score full marks. Part (c)(i) was very poorly done. Many candidates tried to use their circle equation and essentially faked the given result instead of correctly using the distance formula. Once again, if candidates are asked to “show that ...” then the full equation needs to be seen in the final line of the proof and an essential part of the working was to see a statement such as $PC^2 = 3^2 + (k + 6)^2$, leading to the printed answer. In part (c)(ii), the word “hence” indicated that the expression for PC^2 in part (c)(i) needed to be used. Candidates needed to realise that the point P lies outside the circle when PC > r and that using this result immediately leads to the given inequality. The inequality sign here caused confusion with many looking for a discriminant. In part (c)(iii), many candidates scored only the two marks available for finding the critical values –2 and –10. No doubt some would have benefited from practising the solution of inequalities of this type by drawing a suitable sketch, or by familiarising themselves with the technique of using a sign diagram as indicated in previous mark schemes.

GRADE BOUNDARIES

Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	62	54	47	40	33

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
−*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

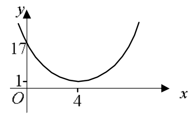
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

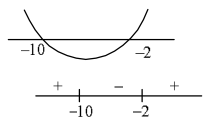
Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$p(-3)=(-3)^3-13(-3)-12$	M1		must attempt $p(-3)$ NOT long division
	$=-27+39-12$ $=0 \quad \Rightarrow x+3$ is factor	A1	2	shown $=0$ plus statement
(b)	$(x+3)(x^2+bx+c)$	M1		Full long division, comparing coefficients or by inspection either $b=-3$ or $c=-4$
	(x^2-3x-4) obtained	A1		or M1A1 for either $(x-4)$ or $(x+1)$ clearly found using factor theorem
	$(x+3)(x-4)(x+1)$	A1	3	CSO; must be seen as a product of 3 factors NMS full marks for correct product SC B1 for $(x+3)(x-4)(\ )$ or $(x+3)(x+1)(\ )$ or $(x+3)(x+4)(x-1)$ NMS
	Total		**5**	
2(a)(i)	grad $AB=\dfrac{7-3}{3-1}$	M1		$\dfrac{\Delta y}{\Delta x}$ correct expression, possibly implied
	$=2$ (must simplify 4/2)	A1	2	
(ii)	grad $BC=\dfrac{7-9}{3+1}=-\dfrac{2}{4}$	M1		Condone one slip **NOT** Pythagoras or cosine rule etc
	grad AB × grad BC = −1 $\Rightarrow \angle ABC=90°$ or AB & BC perpendicular	A1	2	convincingly proved plus statement SC B1 for −1/(their grad AB) or statement that $m_1m_2=-1$ for perpendicular lines if M0 scored
(b)(i)	$M\ (0, 6)$	B2	2	B1 + B1 each coordinate correct
(ii)	$(AB^2=)\quad (3-1)^2+(7-3)^2$ $(BC^2=)\quad (3+1)^2+(7-9)^2$	M1		either expression correct, simplified or unsimplified
	$AB^2=2^2+4^2$ or $BC^2=4^2+2^2$ or $\sqrt{20}$ found as a length	A1		Must see either $AB^2=$.., or $BC^2=$...,
	$AB^2=BC^2 \Rightarrow AB=BC$ or $AB=\sqrt{20}$ and $BC=\sqrt{20}$	A1	3	
(iii)	grad $BM=\dfrac{7-6}{3-0}$ or −1/(grad AC) attempted	M1		ft their M coordinates
	$=\dfrac{1}{3}$	A1		correct gradient of line of symmetry
	BM has equation $y=\dfrac{1}{3}x+6$	A1	3	CSO, any correct form
	Total		**12**	

Q	Solution	Marks	Total	Comments
3(a)(i)	$\frac{dy}{dt}=\frac{4t^3}{8}-4t+4$	M1 A1 A1	3	one term correct another term correct all correct (no + c etc) unsimplified
(ii)	$\frac{d^2y}{dt^2}=\frac{12t^2}{8}-4$	M1		ft one term "correct"
		A1	2	correct unsimplified (penalise inclusion of $+c$ once only in question)
(b)	$t=2$; $\frac{dy}{dt}=4-8+4$	M1		Substitute $t=2$ into their $\frac{dy}{dt}$
	$\frac{dy}{dt}=0 \Rightarrow$ stationary value	A1		CSO; shown = 0 plus statement
	$t=2$; $\frac{d^2y}{dt^2}=6-4=2$	M1		Sub $t=2$ into their $\frac{d^2y}{dt^2}$
	$\Rightarrow y$ has MINIMUM value	A1	4	CSO
(c)(i)	$t=1$; $\frac{dy}{dt}=\frac{1}{2}-4+4$	M1		Substitute $t=1$ into their $\frac{dy}{dt}$
	$=\frac{1}{2}$	A1	2	OE; CSO NMS full marks if $\frac{dy}{dt}$ correct
(ii)	$\frac{dy}{dt}>0 \Rightarrow$ (depth is) INCREASING	E1√	1	allow decreasing if states that their $\frac{dy}{dt}<0$ **Reason must be given not just the word increasing or decreasing**
	Total		12	
4(a)	$\sqrt{50}=5\sqrt{2}$; $\sqrt{18}=3\sqrt{2}$; $\sqrt{8}=2\sqrt{2}$ At least two of these correct	M1		or $\times\frac{\sqrt{8}}{\sqrt{8}}$ or $\left(\times\frac{\sqrt{2}}{\sqrt{2}}\right)$ or $\sqrt{\frac{25}{4}}+\sqrt{\frac{9}{4}}$
	$\frac{5\sqrt{2}+3\sqrt{2}}{2\sqrt{2}}$	A1		any **correct** expression all in terms of $\sqrt{2}$ or with denominator of 8, 4 or 2 simplifying numerator eg $\frac{\sqrt{400}+\sqrt{144}}{8}$
	Answer $=4$	A1	3	CSO
(b)	$\frac{(2\sqrt{7}-1)(2\sqrt{7}-5)}{(2\sqrt{7}+5)(2\sqrt{7}-5)}$	M1		OE
	numerator $=4\times7-2\sqrt{7}-10\sqrt{7}+5$	m1		expanding numerator (condone one error or omission)
	denominator $=3$	B1		(seen as denominator)
	Answer $=11-4\sqrt{7}$	A1	4	
	Total		7	

Q	Solution	Marks	Total	Comments
5(a)	$x^2-8x+15+2$	B1		Terms in x must be collected, PI
	their $(x-4)^2$ $(+k)$	M1		ft $(x-p)^2$ for their quadratic
	$=(x-4)^2+1$	A1	3	ISW for stating $p=-4$ if correct expression seen
(b)(i)		M1		∪ shape in any quadrant (generous)
		A1		correct with min at (4, 1) stated or 4 and 1 marked on axes condone within first quadrant only
		B1	3	crosses y-axis at (0, 17) stated or 17 marked on y-axis
(ii)	$y=k$	M1		$y=$ constant
	$y=1$	A1	2	Condone $y=0x+1$
(c)	Translation (*not shift, move etc*)	E1		and no other transformation
	with vector $\begin{bmatrix}4\\1\end{bmatrix}$	M1		One component correct or ft either their p or q
		A1	3	CSO; condone 4 across, 1 up; or two separate vectors etc
	Total		11	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\frac{dy}{dx} = 24x - 19 - 6x^2$	M1		2 terms correct
		A1		all correct (no + c etc)
	when $x = 2$, $\frac{dy}{dx} = 48 - 19 - 24$	m1		
	$\Rightarrow$ gradient = 5	A1	4	CSO
(ii)	grad of normal $= -\frac{1}{5}$	B1√		ft their answer from (a)(i)
	$y + 6 = \left(\textit{their} - \frac{1}{5}\right)(x - 2)$ or $y = \left(\textit{their} - \frac{1}{5}\right)x + c$ and c evaluated using $x = 2$ and $y = -6$	M1		ft grad of their normal using **correct** coordinates BUT must not be tangent condone omission of brackets
	$x + 5y + 28 = 0$	A1	3	CSO; condone all on one side in different order
(b)(i)	$\frac{12}{3}x^3 - \frac{19}{2}x^2 - \frac{2}{4}x^4$	M1		one term correct
		A1		another term correct
		A1		all correct (ignore $+c$ or limits)
	$= 32 - 38 - 8$	m1		F(2) attempted
	$= -14$	A1	5	CSO; withhold A1 if changed to +14 here
(ii)	Area $\Delta = \frac{1}{2} \times 2 \times 6 = 6$	B1		condone -6
	Shaded region area $= 14 - 6$	M1		difference of $\pm\left|\int\right| \pm\left|\Delta\right|$
	$= 8$	A1	3	CSO
	Total		**15**	

Q	Solution	Marks	Total	Comments
7(a)(i)	$x = \pm 2$ or $y = \pm 6$ or $(x-2)^2 + (y+6)^2$	M1		
	$C(2, -6)$	A1	2	correct
(ii)	$(r^2 =)\, 4 + 36 - 15$	M1		(RHS =) *their* $(-2)^2$ + *their* $(6)^2 - 15$
	$\Rightarrow r = 5$	A1	2	Not ± 5 *or* $\sqrt{25}$
(b)	explaining why $\left|y_C\right| > r$; $6 > 5$	E1		Comparison of y_C and r, eg $-6 + 5 = -1$ or indicated on diagram
	full convincing argument, but must have correct y_C and r	E1	2	Eg "highest point is at $y = -1$" scores E2 E1: showing no real solutions when $y = 0$ +E1 stating centre or any point below x-axis
(c)(i)	$(PC^2 =)\quad (5-2)^2 + (k+6)^2$			ft their C coords
	$= 9 + k^2 + 12k + 36$	M1		and attempt to multiply out
	$PC^2 = k^2 + 12k + 45$	A1	2	**AG** CSO (must see $PC^2 =$ at least once)
(ii)	$PC > r \Rightarrow PC^2 > 25$ $\Rightarrow k^2 + 12k + 20 > 0$	B1	1	**AG** Condone $k^2 + 12k + 45 > 25$ $\Rightarrow k^2 + 12k + 20 > 0$
(iii)	$(k+2)(k+10)$	M1		**Correct** factors or **correct** use of formula
	$k = -2$, $k = -10$ are critical values	A1		May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working.
	Use of sketch or sign diagram:  	M1		If previous A1 earned, sign diagram or sketch must be correct for M1, otherwise M1 may be earned for an attempt at the sketch or sign diagram using their critical values.
	$\Rightarrow k > -2,\ k < -10$	A1	4	$k \geqslant -2,\ k \leqslant -10$ loses final A mark
	Condone $k > -2$ OR $k < -10$ for full marks but not AND instead of OR Take final line as their answer			*Answer only* of $k > -2,\ k > -10$ *etc* scores M1, A1, M0 since the critical values are evident. *Answer only* of $k > 2,\ k < -10$ *etc* scores M0, M0 since the critical values are not both correct.
	Total		**13**	
	TOTAL		**75**	