

Friday 9 January 2009 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examinating Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary workings, otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The points A and B have coordinates $(1, 6)$ and $(5, -2)$ respectively. The mid-point of AB is M .
 - (a) Find the coordinates of M . (2 marks)
 - (b) Find the gradient of AB , giving your answer in its simplest form. (2 marks)
 - (c) A straight line passes through M and is perpendicular to AB .
 - (i) Show that this line has equation $x - 2y + 1 = 0$. (3 marks)
 - (ii) Given that this line passes through the point $(k, k + 5)$, find the value of the constant k . (2 marks)
- 2 (a) Factorise $2x^2 - 5x + 3$. (1 mark)
 - (b) Hence, or otherwise, solve the inequality $2x^2 - 5x + 3 < 0$. (3 marks)
- 3 (a) Express $\frac{7 + \sqrt{5}}{3 + \sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers. (4 marks)
 - (b) Express $\sqrt{45} + \frac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. (3 marks)
- 4 (a) (i) Express $x^2 + 2x + 5$ in the form $(x + p)^2 + q$, where p and q are integers. (2 marks)
 - (ii) Hence show that $x^2 + 2x + 5$ is always positive. (1 mark)
 - (b) A curve has equation $y = x^2 + 2x + 5$.
 - (i) Write down the coordinates of the minimum point of the curve. (2 marks)
 - (ii) Sketch the curve, showing the value of the intercept on the y -axis. (2 marks)
 - (c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 2x + 5$. (3 marks)

- 5 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

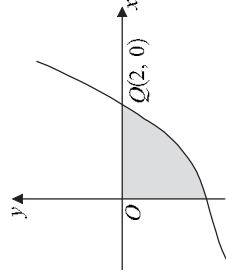
$$x = \frac{1}{2}t^4 - 20t^2 + 66t, \quad 0 \leq t \leq 4$$

- (a) Find:
- $\frac{dx}{dt}$; (3 marks)
 - $\frac{d^2x}{dt^2}$. (2 marks)
- (b) Verify that x has a stationary value when $t = 3$, and determine whether this stationary value is a maximum value or a minimum value. (4 marks)
- (c) Find the rate of change of x with respect to t when $t = 1$. (2 marks)
- (d) Determine whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$. (2 marks)

- 6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

- Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. (2 marks)
- Express $p(x)$ in the form $(x - 2)(x^2 + ax + b)$, where a and b are constants. (2 marks)

- (b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x -axis at the point $Q(2, 0)$.



- Find the gradient of the curve C at the point Q . (4 marks)
- Hence find an equation of the tangent to the curve C at the point Q . (2 marks)
- Find $\int (x^3 + x - 10) dx$. (3 marks)
- Hence find the area of the shaded region bounded by the curve C and the coordinate axes. (2 marks)


- 7 A circle with centre C has equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

- (a) Express this equation in the form $(x - a)^2 + (y - b)^2 = r^2$ (3 marks)
- (b) Write down:
- the coordinates of C ;
 - the radius of the circle. (2 marks)
- (c) The point D has coordinates $(7, -2)$.
- Verify that the point D lies on the circle. (1 mark)
 - Find an equation of the normal to the circle at the point D , giving your answer in the form $mx + ny = p$, where m , n and p are integers. (3 marks)
- (d) (i) A line has equation $y = kx$. Show that the x -coordinates of any points of intersection of the line and the circle satisfy the equation $(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$ (2 marks)
- (ii) Find the values of k for which the equation has equal roots. (5 marks)
- (iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (d)(ii). (1 mark)

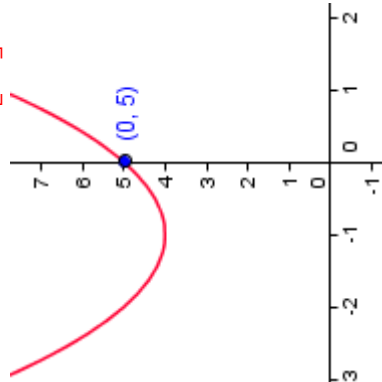
END OF QUESTIONS

AQA – Core 1 - Jan 2009 – Answers

Question 1:	Exam report
<p>a) $A(1, 6)$ $B(5, -2)$</p> <p>The mid-point $M\left(\frac{5+1}{2}, \frac{-2+6}{2}\right) = M(3, 2)$</p> <p>b) Gradient of $AB = m_{AB} = \frac{-2-6}{5-1} = -2$</p> <p>c) i) The gradient of the perpendicular to AB is $-\frac{1}{m_{AB}} = \frac{1}{2}$</p> <p>The equation of the perpendicular bisector is:</p> $y - 2 = \frac{1}{2}(x - 3)$ $2y - 4 = x - 3$ $x - 2y + 1 = 0$ <p>ii) Substitute x by k and y by $k + 5$ in the equation:</p> $k - 2(k + 5) + 1 = 0$ $k - 2k - 10 + 1 = 0$ $k = -9$	<p>In part (a) most candidates were able to find the correct coordinates of the mid point, although a few transposed the coordinates and others subtracted rather than adding the coordinates before halving the results. Full marks were only awarded in part (b) for a gradient of -2 and quite a few candidates did not give their answer in this simplest form.</p> <p>In part (c)(i) most candidates realised that the product of the gradients should be -1. However, not all were able to calculate the negative reciprocal. Others used an incorrect point such as A or B and therefore found an equation of the wrong line. The most successful used an equation of the form $y - y_1 = m(x - x_1)$ as flagged above. The printed answer helped most candidates to be successful in finding the correct equation of the line.</p> <p>In part (c)(ii) most candidates made an attempt at this part of the question, but the failure to use brackets for the second term caused the majority to find an incorrect value for k. Others foolishly tried to substitute $x = k$ and $y = k + 5$ into their own incorrect line equation rather than using the printed answer from part (c)(i).</p>

Question 2:	Exam report
<p>a) $2x^2 - 5x + 3 = (2x - 3)(x - 1)$</p> <p>b) Critical values $\frac{3}{2}$ and 1</p> <p>$2x^2 - 5x + 3 < 0$ for $1 < x < \frac{3}{2}$</p> 	<p>In part (a) it was quite alarming to see the number of candidates who were unable to factorise this quadratic. Most candidates scored only a single mark in part (b) for attempting to find the critical values. Many would benefit from practising the solution of inequalities of this type by drawing a suitable sketch or by familiarising themselves with the technique of using a sign diagram as indicated in previous mark schemes</p>

Question 3:	Exam report
<p>a) $\frac{7 + \sqrt{5}}{3 + \sqrt{5}} = \frac{7 + \sqrt{5}}{3 + \sqrt{5}} \times \frac{3 - \sqrt{5}}{3 - \sqrt{5}} = \frac{21 - 7\sqrt{5} + 3\sqrt{5} - 5}{9 - 5}$</p> $= \frac{16 - 4\sqrt{5}}{4} = 4 - \sqrt{5}$ <p>b) $\sqrt{45} + \frac{20}{\sqrt{5}} = \sqrt{9 \times 5} + \frac{20\sqrt{5}}{5} = 3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$</p>	<p>In part (a) it was pleasing to see that most candidates were familiar with the technique for rationalising the denominator in this type of problem and, although there were some who made slips when multiplying out the two brackets in the numerator, most obtained the correct answer in the given form.</p> <p>In part (b) the term $\sqrt{45}$ was usually expressed as $3\sqrt{5}$, but the term $\frac{20}{\sqrt{5}}$ caused far more difficulties than expected. Consequently, the final correct answer was only obtained by the better candidates.</p>

Question 4:	Exam report
<p>a) i) $y = x^2 + 2x + 5 = (x+1)^2 - 1 + 5 = (x+1)^2 + 4$</p> <p>ii) For all x, $(x+1)^2 \geq 0$ so $(x+1)^2 + 4 \geq 4$ y is always positive.</p> <p>b) i) The minimum point is $(-1, 4)$</p> <p>ii) The curve crosses the y-axis at $(0, 5)$</p> <p>c) $x \xrightarrow[\text{-1 unit in } x\text{-dir}]{\text{translation}} x+1 \xrightarrow[\text{4 unit in } y\text{-dir}]{\text{translation}} (x+1)^2 \xrightarrow{\text{translation}} (x+1)^2 + 4$</p> <p>summary: Translation vector $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$</p> 	<p>In part (a)(i) the completion of the square was done successfully by most candidates, although occasionally the value 6 was seen instead of 4 for q.</p> <p>In part (a)(ii) it was necessary to comment on both parts of the expression; $(x+1)^2 \geq 0$ and hence adding 4 implies that $(x+1)^2 + 4 > 0$ for all values of x. Because of the word “hence”, an argument based on algebra rather than the features of a curve was required; for instance, an answer explaining that $(x+1)^2 + 4$ has a minimum value of 4 was acceptable, but a statement about the curve having a minimum point at $(-1, 4)$ was not.</p> <p>In part (b)(i) most candidates were able to write down the correct minimum point. A few chose to use differentiation but sometimes made arithmetic slips in finding the coordinates of the stationary point.</p> <p>In part (b)(ii) those with the correct minimum point were usually able to produce a correct sketch, although the value of the y-intercept was sometimes missing. Some credit was given to candidates with an incorrect minimum point, usually $(1, 4)$, provided their graph was consistent with this minimum point.</p> <p>The more able candidates earned full marks in part (c). The term translation was required but generally the wrong word was used or it was accompanied by another transformation such as a stretch. A very common (but incorrect) vector stated was $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.</p>

Question 5:	Exam report
<p>$x = \frac{1}{2}t^4 - 20t^2 + 66t \quad 0 \leq t \leq 4$</p> <p>a) i) $\frac{dx}{dt} = \frac{1}{2} \times 4t^3 - 20 \times 2t + 66$ $\frac{dx}{dt} = 2t^3 - 40t + 66$</p> <p>ii) $\frac{d^2x}{dt^2} = 6t^2 - 40$</p> <p>b) Verify that $\frac{dx}{dt}(t=3) = 0$.</p> <p>$\frac{dx}{dt}(t=3) = 2 \times 3^3 - 40 \times 3 + 66$ $= 54 - 120 + 66 = 0$</p> <p>There is a stationary point when $t = 3$.</p> <p>$\frac{d^2x}{dt^2}(t=3) = 6 \times 3^2 - 40 = 54 - 40 = 14 > 0$</p> <p>This point is a MINIMUM.</p> <p>c) The rate of change is $\frac{dx}{dt}(t=1) = 2 - 40 + 66 = 28$</p> <p>d) $\frac{dx}{dt}(t=2) = 2 \times 2^3 - 40 \times 2 + 66 = 16 - 80 + 66 = 2 > 0$</p> <p>The distance is INCREASING when $t = 2$.</p>	<p>In part (a) almost all candidates were able to find the first and second derivative correctly, although there was an occasional arithmetic slip and some could not cope with the fraction term.</p> <p>Those who substituted $t = 3$ into $\frac{dx}{dt}$ in part (b) did not always explain that $\frac{dx}{dt}$ is the condition for a stationary point. Some assumed that a stationary point occurred when $t = 3$ and went straight to the test for maximum or minimum and only scored half the marks. It was advisable to use the second derivative test here; those who considered values of $\frac{dx}{dt}$ on either side of $t = 3$ usually reached an incorrect conclusion because of the proximity of another stationary point.</p> <p>In part (c) the concept of “rate of change” was not understood by many. Approximately equal numbers of candidates substituted $t = 1$ into the expression for $\frac{dx}{dt}$ or $\frac{d^2x}{dt^2}$ and so only about half of the candidates were able to score any marks on this part. Those who used $\frac{dx}{dt}$ often made careless arithmetic errors when adding three numbers.</p> <p>In part (d), as in part (c), candidates did not realise which expression to use and many wrongly selected the second derivative. It is a general weakness that candidates do not realise that the sign of the first derivative indicates whether a function is increasing or decreasing at a particular point.</p>

Question 6:

a) $p(x) = x^3 + x - 10$

i) $p(2) = 2^3 + 2 - 10 = 8 + 2 - 10 = p(2) = 0$

2 is a root of p , so $(x - 2)$ is a factor of p .

ii) $x^3 + x - 10 = (x - 2)(x^2 + 2x + 5)$

b) i) The gradient of the curve at Q is $\frac{dy}{dx}(x = 2)$

$$\frac{dy}{dx} = 3x^2 + 1 \text{ and for } x = 2, \frac{dy}{dx} = m_Q = 13$$

ii) The equation of the tangent at Q is :

$$y - 0 = 13(x - 2)$$

$$y = 13x - 26$$

iii) $\int (x^3 + x - 10)dx = \frac{1}{4}x^4 + \frac{1}{2}x^2 - 10x + c$

iv) The curve is below the x-axis,

so the area of the shaded part is

$$-\int_0^2 (x^3 + x - 10)dx = \left[-\frac{1}{4}x^4 - \frac{1}{2}x^2 + 10x \right]_0^2 = (-4 - 2 + 20) - (0) = 14$$

$$\text{Area} = 14$$

Exam report

In part (a)(i) the majority of candidates realised the need to find the value of $f(x)$ when $x = 2$. However, it was also necessary, after showing that $f(2) = 0$, to write a statement that the zero value implied that $x - 2$ was a factor. It was good to see more candidates being aware of this. In part (a)(ii), those who used inspection were the most successful here. Methods involving long division or equating coefficients usually contained algebraic errors.

In part (b)(i) a surprising number of candidates failed to see the need to differentiate in order to find the gradient at Q. Those who attempted to

find $\frac{dy}{dx}$ sometimes wrote it as $3x^2 + x$, but usually were aware of the need to substitute $x = 2$.

In part (b)(ii) those who had the correct gradient in part (b)(i) were usually successful in finding the correct equation of the tangent, and most obtained at least a method mark here.

In part (b)(iii) most were well drilled in integration and earned full

marks, although some wrote $\frac{x^4}{4} + \frac{x^2}{2} - 10$ and others gave

$$\frac{x^4}{4} + \frac{x^2}{2} - \frac{10^2}{2} \text{ as their answer.}$$

For part (b)(iv) the correct limits were usually used, although many sign/arithmetic slips occurred after substitution of the numbers 0 and 2 and it was incredible how many could not evaluate $6 - 20$ without a calculator. Very few candidates realised the need to show clearly that, although the integral from 0 to 2 gave a value of -14 , the area of the shaded region was 14. A separate statement was needed and those who simply wrote $4 + 2 - 20 = -14 = 14$ did not score full marks. Those more able candidates who made a statement about the region being entirely below the x-axis and who subsequently evaluated the integral from 2 to 0 correctly scored full marks.

Question 7:

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

$$a) (x-3)^2 - 9 + (y+5)^2 - 25 + 9 = 0$$

$$(x-3)^2 + (y+5)^2 = 5^2$$

b) i) The centre $C(3, -5)$

ii) radius $r = 5$

c) $D(7, -2)$

i) Substitute x and y by 7 and -2 :

$$(x-3)^2 + (y+5)^2 = (7-3)^2 + (-2+5)^2 = 16 + 9 = 25$$

D belongs to the circle.

ii) The normal to to circle at D is the line CD

$$m_{CD} = \frac{-2+5}{7-3} = \frac{3}{4}$$

The equation of the normal: $y + 2 = \frac{3}{4}(x - 7)$

$$4y + 8 = 3x - 21$$

$$3x - 4y = 29$$

$$\begin{cases} y = kx \\ (x-3)^2 + (y+5)^2 = 25 \end{cases}$$

d) i) Solve simultaneously

by substitution, we have $(x-3)^2 + (kx+5)^2 = 25$

$$x^2 + 9 - 6x + k^2x^2 + 25 + 10kx = 25$$

$$(1+k^2)x^2 + (10k-6)x + 9 = 0$$

$$(1+k^2)x^2 + 2(5k-3)x + 9 = 0$$

ii) This equation has equal roots if the discriminant $= 0$.

$$(10k-6)^2 - 4 \times (1+k^2) \times 9 = 0$$

$$100k^2 + 36 - 120k - 36k^2 - 36 = 0$$

$$64k^2 - 120k = 0$$

$$k(64k - 120) = 0$$

$$k = 0 \text{ or } k = \frac{120}{64} = \frac{15}{8}$$

iii) When $k = 0$ or $\frac{15}{8}$, the line is **tangent** to the circle.

Exam report

In part (a) most candidates found the correct values of a and b , but correct values for r^2 were not so common. Some sloppiness was again evident with candidates failing to write squared outside the brackets or omitting the plus sign between the terms on the left hand side. It was common to see things such as $25 = 25 = \sqrt{25} = 5^2$ and this could be penalised in the future.

In part (b) the coordinates of the centre, C and the radius r , although not always correct, usually gained full credit when following through from part (a).

In part (c)(i) most candidates attempted to verify that the point D was on the circle, although some, who had obviously worked a previous examination question, were keen to show that the distance from C to D was less than the radius and that D lay inside the circle. This verification was marked fairly strictly and the argument had to be correct including a final concluding statement. Those who simply wrote $4^2 + 3^2 = 25$, for example, did not earn the mark.

In part (c)(ii) many candidates found the gradient of CD and then assumed they had to find the negative reciprocal of this since the question asked for the normal at D . Reference to a sketch might have prevented this incorrect assumption.

In part (d)(i) most candidates made errors by not using brackets; the expression kx^2 was seen almost as often as the correct form k^2x^2 after substituting $y = kx$ into their circle equation.

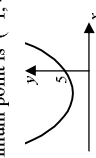
In part (d)(ii), although there were some correct solutions seen, the discriminant often contained algebraic slips and the condition for equal roots was rarely stated. Often it was several lines into the working before an “ $= 0$ ” appeared and many times this was omitted entirely. The value $k = 0$ was often ignored in otherwise correct solutions, but it was more common to see a three term quadratic because of previous algebraic errors.

In part (d)(iii) several candidates realised that the line would be a tangent for each of the two values of k , but many completely missed the point and talked about transformations, often giving vectors in their answer.

GRADE BOUNDARIES

Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	62	54	46	39	32

Q	Solution	Marks	Total	Comments
1(a)	$M(3, 2)$	B1 B1	2	B1 for each coordinate
(b)	Gradient $AB = \frac{-2-6}{5-1} = \left(\frac{-8}{4}\right) = -2$	M1 A1	2	May use coords of M instead of A or B - condone one slip CSO Answer must be simplified to -2
(c) (i)	Gradient of perpendicular = $\frac{1}{2}$ $\Rightarrow y-2 = \frac{1}{2}(x-3)$ $\Rightarrow 2y-4 = x-3 \Rightarrow x-2y+1=0$ AG	B1 ✓ M1 A1	3	ft "their" -1/gradient AB attempt at perp to AB, ft their M coords CSO Must write down the printed answer
(ii)	$k-2(k+5)+1=0$ or $\frac{(k+5)-2}{k-3} = \frac{1}{2}$ $\Rightarrow k = -9$	M1 A1	2	Sub into given line equation or correct expression involving gradients Condone omission of brackets or use of x Condone $x = -9$ (Full marks for correct answer without working)
Total			9	
2(a)	$(x-1)(2x-3)$	B1	1	$(1-x)(3-2x)$ or $2(x-1)(x-1.5)$ etc
(b)	Critical values are 1, $1\frac{1}{2}$ Sign diagram or sketch $\Rightarrow 1 < x < 1\frac{1}{2}$	B1 ✓ M1 A1	3	Correct or ft their factors from (a) $\begin{array}{c} + & & - & & + \\ & 1 & & & 1\frac{1}{2} \end{array}$ Full marks for correct inequality without working
Total			4	
3(a)	$\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ Numerator = $21+3\sqrt{5}-7\sqrt{5}-\sqrt{5}^2$ Denominator = $9-5=4$ Answer = $4-\sqrt{5}$	M1 m1 B1 A1	4	Multiply by $\frac{3-\sqrt{5}}{3-\sqrt{5}}$ or $\frac{\sqrt{5}-3}{\sqrt{5}-3}$ Condone one slip $16-4\sqrt{5}$ (Or $5-9 = -4$ from other conjugate) CSO
(b)	$\sqrt{45} = 3\sqrt{5}$ $\frac{20}{\sqrt{5}} = \frac{20\sqrt{5}}{5}$ Sum = $7\sqrt{5}$	B1 M1 A1	3	May score if combined as one expression Must have 5 in denominator
Total			7	

Q	Solution	Marks	Total	Comments
4(a)(i)	$(x+1)^2 + 4$	B1 B1	2	$p = 1$ $q = 4$
(ii)	$(x+1)^2 \geq 0 \Rightarrow (x+1)^2 + 4 > 0$ $(\Rightarrow x^2 + 2x + 5 > 0$ for all values of x)	E1	1	Condone if they say $(x+1)^2$ positive and adding 4 so always positive
(b)(i)	$x = -1$ or $y = 4$	M1	2	ft their $x = -p$ or $y = q$
(ii)	Minimum point is $(-1, 4)$ 	A1 B1	2	Sketch roughly as shown
(c)	Translation (not shift, move etc) through $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ (or 1 left, 4 up etc)	B1 E1 M1	2	y-intercept 5 or (0, 5) marked or stated and NO other transformation stated
		A1	3	either component correct or ft their $-p, q$ correct translation M1, A1 independent of E mark
Total			10	
5(a)(i)	$\frac{dx}{dt} = 2t^3 - 40t + 66$	M1	1	one term correct
(ii)	$\frac{d^2x}{dt^2} = 6t^2 - 40$	A1 A1	3	another term correct all correct unsimplified (no + c etc) ft one term correct
(b)	$\frac{dx}{dt} = 54 - 120 + 66 = 0 \Rightarrow$ stationary value	M1 A1 ✓	2	ft all "correct", 2 terms equivalent substitute $t = 3$ into their $\frac{dx}{dt}$ CSO shown = 0 (54 or 2×27 seen) and statement
(c)	Substitute $t = 3$ into $\frac{d^2x}{dt^2}$ ($= 14$) $\frac{d^2x}{dt^2} > 0 \Rightarrow$ minimum value Substitute $t = 1$ into their $\frac{dx}{dt}$ $\frac{dx}{dt} = 28$	M1 A1	4	CSO; all values (if stated) must be correct must be their $\frac{dx}{dt}$ NOT $\frac{d^2x}{dt^2}$ etc ft their $\frac{dx}{dt}$ when $t = 1$
(d)	Substitute $t = 2$ into their $\frac{dx}{dt}$ $= 16 - 80 + 66 = 2 (> 0)$ \Rightarrow increasing when $t = 2$	M1 A1 ✓	2	must be their $\frac{dx}{dt}$ NOT $\frac{d^2x}{dt^2}$ or x Interpreting their value of $\frac{dx}{dt}$ Allow decreasing if their $\frac{dx}{dt} < 0$
Total			13	

Q	Solution	Marks	Total	Comments
6(a)(i)	$p(2) = 8 + 2 - 10$ $\Rightarrow p(2) = 0 \Rightarrow (x-2)$ is factor	M1 A1	2	Must find $p(2)$ NOT long division Shown = 0 plus a statement
(ii)	Attempt at long division (genericus) $p(x) = (x-2)(x^2 + 2x + 5)$	M1 A1	2	Obtaining a quotient $x^2 + cx + d$ or equating coefficients (full method) $a=2, b=5$ by inspection B1, B1
(b)(i)	$\frac{dy}{dx} = 3x^2 + 1$ When $x=2$ $\frac{dy}{dx} = 3 \times 4 + 1$ Therefore gradient at Q is 13	M1 A1 m1	4	One term correct All correct – no +c etc Sub $x=2$ into their $\frac{dy}{dx}$
(ii)	$y = 13(x-2)$	A1	4	CSO
(iii)	$\int \dots dx = \frac{x^4}{4} + \frac{x^2}{2} - 10x(+c)$	M1 A1 A1	3	Tangent (NOT normal) attempted fit their gradient answer from (b)(i) CSO; correct in any form one term correct second term correct all correct (condone no +c)
(iv)	$[4 + 2 - 20] - [0] = -14$ Area of shaded region = 14	M1 A1	2	F(2) attempted and possibly F(0) Must have earned M1 in (b)(iii) CSO; separate statement following correct evaluation of limits
Total			15	

Q	Solution	Marks	Total	Comments
7(a)(i)	$(x-3)^2 + (y+5)^2$ $= 25 - 9 + 9 = 25 \quad (= 5^2)$	B1 B1 B1	3	One term correct LHS correct with + and squares Condone RHS = 25
(b)(i) (ii)	$C(3, -5)$ Radius = 5	B1✓ B1✓	2	Correct or fit their RHS provided > 0
(c)(i)	$(7-3)^2 + (-2+5)^2 = 16+9 = 25$ $\Rightarrow D$ lies on circle <i>Must see statement</i>	B1	1	Or sub'n of (7, -2) in original equation $7^2 + (-2)^2 - 42 - 20 + 9 = 0$ Or sub $x=7$ into eqn & showing $y = -2$ etc
(ii)	Attempt at gradient of CD as normal $\text{grad } CD = \frac{-2 - (-5)}{7 - 3} = \frac{3}{4}$ $y+2 = \frac{3}{4}(x-7)$ or $y+5 = \frac{3}{4}(x-3)$ $\Rightarrow 3x - 4y = 29$	M1 A1 A1	3	withhold if subsequently uses $m_1 m_2 = -1$ $\frac{\Delta y}{\Delta x}$ (condone one slip) FT their centre C Correct equation in any form $y = \frac{3}{4}x - \frac{29}{4}$ CSO <i>Integer</i> coefficients Condone $4y - 3x + 29 = 0$ etc
(d)(i)	$y = kx$ sub'd into original circle equation $x^2 + (kx)^2 - 6x + 10kx + 9 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$ AG	M1 A1	2	or using their completed square form and multiplying out CSO must see at least previous line for A1 any error such as $kx^2 = \dots = k^2 x^2$ gets A0
(ii)	$4(5k-3)^2 - 36(k^2+1)$ $-64k^2 - 120k$ Equal roots: $4(5k-3)^2 - 36(k^2+1) = 0$ $8k^2 - 15k = 0$ $\Rightarrow k = 0, k = \frac{15}{8}$	M1 A1 B1 m1 A1	5	Discriminant in k (can be seen in quad formula) Condone one slip or $8k^2 - 15k = 0$ OE $b^2 - 4ac = 0$ clearly stated or evident by an equation in k with at most 2 slips. Attempt to solve <i>their</i> quadratic or linear equation if k has been cancelled OE but must have $k=0$
(iii)	(Line is a) tangent (to the circle)	E1	1	If " $=0$ " is not seen but correct values of k are found, candidate will lose B1 mark but may earn all other marks
Total			17	Line touches circle at one point
TOTAL			75	