General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Wednesday 9 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$.

(a) Find the coordinates of the mid-point of BC. *(2 marks)*

(b) (i) Find the gradient of AB, in its simplest form. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $x + qy = r$, where q and r are integers. *(2 marks)*

(iii) Find an equation of the line passing through C which is parallel to AB. *(2 marks)*

(c) Prove that angle ABC is a right angle. *(3 marks)*

2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the x-coordinate of M. *(3 marks)*

(c) (i) Find $\frac{d^2y}{dx^2}$. *(1 mark)*

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. *(2 marks)*

(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$. *(2 marks)*

3 (a) Express $5\sqrt{8} + \frac{6}{\sqrt{2}}$ in the form $n\sqrt{2}$, where n is an integer. *(3 marks)*

(b) Express $\frac{\sqrt{2} + 2}{3\sqrt{2} - 4}$ in the form $c\sqrt{2} + d$, where c and d are integers. *(4 marks)*

4 A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$.

(a) By completing the square, express this equation in the form

$$x^2 + (y - b)^2 = k$$ *(2 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle, leaving your answer in surd form. *(1 mark)*

(c) A line has equation $y = 2x$.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation $x^2 - 4x + 4 = 0$. *(2 marks)*

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, P. *(3 marks)*

(d) Prove that the point $Q(-1, 4)$ lies inside the circle. *(2 marks)*

5 (a) Factorise $9 - 8x - x^2$. *(2 marks)*

(b) Show that $25 - (x + 4)^2$ can be written as $9 - 8x - x^2$. *(1 mark)*

(c) A curve has equation $y = 9 - 8x - x^2$.

(i) Write down the equation of its line of symmetry. *(1 mark)*

(ii) Find the coordinates of its vertex. *(2 marks)*

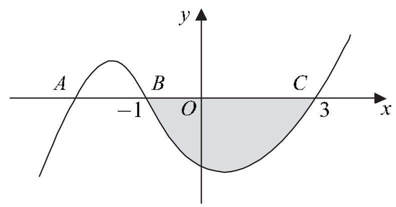
(iii) Sketch the curve, indicating the values of the intercepts on the x-axis and the y-axis. *(3 marks)*

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A. *(1 mark)*

(ii) Find $\int_{-1}^{3} (x^3 - 7x - 6)\,dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. *(1 mark)*

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B. *(3 marks)*

(v) Hence find an equation of the normal to the curve at the point B. *(3 marks)*

7 The curve C has equation $y = x^2 + 7$. The line L has equation $y = k(3x + 1)$, where k is a constant.

(a) Show that the x-coordinates of any points of intersection of the line L with the curve C satisfy the equation

$$x^2 - 3kx + 7 - k = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points. Show that

$$9k^2 + 4k - 28 > 0$$ *(3 marks)*

(c) Solve the inequality $9k^2 + 4k - 28 > 0$. *(4 marks)*

END OF QUESTIONS

AQA – Core 1 – Jan 2008 – Answers

Question 1:	Exam report
$A(-2,3)\ \ B(4,1)\ \ C(2,-5)$ $a)$ Mid-point of BC $\left(\frac{4+2}{2},\frac{1-5}{2}\right)=(3,-2)$ $b)\,i)\ m_{AB}=\frac{1-3}{4+2}=-\frac{2}{6}=-\frac{1}{3}$ $ii)\ Eq:\ y-3=-\frac{1}{3}(x+2)$ $3y-9=-x-2$ $x+3y=7$ $iii)$ this line has the same gradient $-\frac{1}{3}$ $Eq:\ y+5=-\frac{1}{3}(x-2)$ $3y+15=-x+2$ $x+3y=-13$ $c)$ Let's work out the gradient of the line BC: $m_{BC}=\frac{-5-1}{2-4}=\frac{-6}{-2}=3$ $m_{AB}\times m_{BC}=-\frac{1}{3}\times 3=-1$ *Conclusion*: the line AB and BC are perpendicular the triangle ABC is a right-angled triangle.	In part (a), apart from a few sign errors, it was pleasing to see that most candidates were able to find the correct mid-point. However, those who insisted on subtracting the coordinates before dividing by 2 would do well to learn the formula in the first bullet point above. Quite a few candidates found the mid-point of *AB* instead of *BC*, and this was generously treated as a misread. In part (b)(i), many ignored the request to simplify the gradient, but most were successful in writing the gradient of *AB* as -1/3. In part (b)(ii), almost all candidates managed to write down a correct equation for the line *AB*, but careless arithmetic prevented many from obtaining the required form of $x + 3y = 7$. Some were content to give a final answer that was not in the required form, thus losing a mark. In part (b)(iii), some candidates immediately used $m1 \times m2 = -1$ to find the gradient of the parallel line and scored no marks. Many who used the formula $y = mx + c$ for the equation of the straight line through *C* parallel to *AB* made arithmetic slips and did not obtain a correct final equation. In part (c), the most common approach, and the one expected, was to use gradients in order to prove that angle *ABC* was a right angle. Some simply assumed the result, stating that since the gradient of *AB* was -1/3 then *BC* had gradient 3. It was necessary to show, by considering the differences of the coordinates that *BC* had gradient 3. Far too many simply found the two gradients and wrote "therefore the lines *BC* and *AB* are perpendicular". Since this was a proof, it was expected that the product of the two gradients would be shown to equal –1 before a statement was made about angle *ABC* being a right angle. Some were successful in proving the result using Pythagoras' Theorem, but many attempts were incomplete with several candidates writing $\sqrt{40}+\sqrt{40}=\sqrt{80}$ or other inaccurate statements. Others used the cosine rule, and one or two used the scalar product of two vectors in order to prove the result. A surprising number confused "isosceles" with "right-angled" and, having found two equal sides, stated that the result was proved.

Question 2:	Exam report
$y=x^4-32x+5$ $a)\ \frac{dy}{dx}=4x^3-32$ $b)\ M$ is a stationary point. Let's solve $\frac{dy}{dx}=0$ $4x^3-32=0$ $x^3=8$ $x=2$ $c)\,i)\ \frac{d^2y}{dx^2}=12x^2$ $ii)\ \frac{d^2y}{dx^2}(x=2)=12\times 2^2=48>0$ M is a minimum $d)\ \frac{dy}{dx}(x=0)=-32<0$ The curve is decreasing.	In part (a), most candidates were able to find the correct expression for $\frac{dy}{dx}$, although there were some who left + 5 in their answer or added +C. In part (b), It had been expected that candidates would solve the equation $\frac{dy}{dx}=0$ and obtain the equation $x^3 = 8$ and hence deduce that x = 2 . It seemed, however, that many were unable to formulate an appropriate equation, but merely spotted the correct answer: x = 2. This was not penalised on this occasion, provided that the candidate stated clearly that the x-coordinate of M was equal to 2. In part (c)(i), the expression for $\frac{d^2y}{dx^2}$ was usually correct. In part (c)(ii), although the method was left open, most candidates found the value of the second derivative when x = 2 and correctly concluded that M was a minimum point. In part (c)(iii), some candidates were not aware of the need to find the value of $\frac{dy}{dx}$ when x = 0 in order to ascertain whether the curve was increasing or decreasing at that point.

Question 3:	Exam report
$a)\ 5\sqrt{8}+\frac{6}{\sqrt{2}}=5\sqrt{4\times2}+\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=5\times2\sqrt{2}+\frac{6\sqrt{2}}{2}$ $=10\sqrt{2}+3\sqrt{2}=13\sqrt{2}$ $b)\ \frac{\sqrt{2}+2}{3\sqrt{2}-4}=\frac{\sqrt{2}+2}{3\sqrt{2}-4}\times\frac{3\sqrt{2}+4}{3\sqrt{2}+4}=\frac{6+4\sqrt{2}+6\sqrt{2}+8}{9\times2-16}$ $=\frac{14+10\sqrt{2}}{2}=7+5\sqrt{2}$	Candidates did not always approach part (a) of the question with confidence. Several wrote $5\sqrt{8}=5\sqrt{4\times2}=7\sqrt{2}\ or\ 5+2\sqrt{2}$, others tried to rationalise $\frac{6}{\sqrt{2}}$ by simply multiplying the denominator by $\sqrt{2}$. Consequently, it was quite common to see only one of the two terms expressed correctly in the form $k\sqrt{2}$. It was quite strange, though, to see many obtaining an answer of $13\sqrt{2}$ from completely wrong working; clearly this was not given any credit. Some combined the two terms with a common denominator but often with an incorrect numerator. In part (b), it was not uncommon to see the denominator and numerator multiplied by different surds and the usual errors occurred as candidates tried to multiply out brackets. A few multiplied top and bottom by the conjugate of the numerator. Nevertheless, this part of the question seemed to be answered much better than similar questions in previous years, despite the fairly difficult denominator.

Question 4:	Exam report
$x^2+y^2-10y+20=0$ $a)\ x^2+(y-5)^2-25+20=0$ $x^2+(y-5)^2=5$ $b)\ i)\ C(0,5)$ $ii)\ r=\sqrt{5}$ $c)\ y=2x$ $i)$ solve simultaneously $\begin{cases} y=2x \\ x^2+y^2-10y+20=0 \end{cases}$ *Substitute y by 2x* : $x^2+(2x)^2-10\times(2x)+20=0$ $5x^2-20x+20=0$ $x^2-4x+4=0$ $ii)\ x^2-4x+4=(x-2)^2=0$ $x=2$ is a repeated root The line $y=2x$ is tangent to the circle. $d)\ Q(-1,4)$ $x^2+y^2-10y+20=(-1)^2+4^2-10\times4+20$ $=1+16-40+20=-3<5$ Q is inside the circle	In part (a), it was only necessary to complete the square for the y-terms. As a result, there were probably fewer errors this year expressing the left-hand side of the equation of the circle as (y – 5)^2. However, the right hand side was often written as $\sqrt{5}$, –5 or – 45 instead of 5. In part (b), quite a number who had the correct circle equation in part (a) wrote the coordinates of the centre as (5, 0) or (0, –5). Generous follow through marks were awarded for the radius provided the right-hand side of the equation had a positive value. The wording in the question reassured most, though, that the radius was $\sqrt{5}$. In part (c)(i), those with poor algebraic skills, often writing 2x^2 instead of (2x)^2, struggled to establish the given quadratic equation. Also, quite a few made errors in their working but miraculously wrote down the given equation on their final line. A surprising number derived an equation in y. Quite a few simply solved the given quadratic equation in this part and thus failed to show an understanding of what was required. In part (c)(ii), it was necessary to state that the equation had a repeated root of x = 2, or to use the zero value of the discriminant to show that the equation had equal roots, and hence to conclude that the line was a tangent to the circle. In part (d), far too many simply substituted the coordinates of the point Q into the equation of the circle obtaining a nonsensical statement such as "–3 = 0 so the point lies inside the circle". It was necessary to see that the distance CQ was being calculated and then concluded that this distance was less than the radius of the circle, and hence the point Q must lie inside the circle.

Question 5:	Exam report
$a)\ 9-8x-x^2=(9+x)(1-x)$ $b)\ 25-(x+4)^2=25-(x^2+8x+16)$ $=-x^2-8x+9$ $c)\ y=25-(x+4)^2$ $i)$ Line of symmetry : $x=-4$ $ii)$ *Vertex* $(-4,25)$ $iii)$ The graph crosses the x-axis at $(-9,0)$ and $(1,0)$ the y-axis at $(0,9)$	Candidates did not seem confident working with a quadratic expression where the coefficient of x^2 was negative. Throughout this question, candidates chose instead to work with the expression $x^2 + 8x - 9$, or the equation $x^2 + 8x - 9 = 0$, and lost quite a lot of marks. In part (a), a large number of candidates could not factorise the given quadratic correctly, a few clearly not even recognising what was required. In part (b), those who kept brackets in their working were usually successful in proving the identity. Some able candidates started with $9 - 8x - x^2$ and showed their skill in completing the square. In part (c), quite a large number of candidates seemed unfamiliar with the terms "line of symmetry" and "vertex" and certainly failed to see the link with part (b) of the question. Some stated that the coordinates of the maximum point were (–4, 25) and then wrote the coordinates of the vertex as something entirely different. The sketches were somewhat varied: some found the wrong x-intercepts and drew a curve through these points; those who had completely changed the question into $y = x^2 + 8x - 9$ had a U-shaped graph. Those who drew a graph with the vertex in the correct position and with the correct shape usually had the y-intercept marked correctly as 9. However some drew their curve with a maximum point on the y-axis.

Question 6:	Exam report
$a)\ p(x)=x^3-7x-6$ $i)$ *Work out* $p(-1)$: $(-1)^3-7\times(-1)-6=-1+7-6=0$ -1 is a root of p, so $(x+1)$ is a factor of p $ii)\ p(x)=(x+1)(x^2-x-6)=(x+1)(x-3)(x+2)$ $b)\ i)\ A(-2,0)\quad (root\ of\ \ p)$ $ii)\ \int_{-1}^{3}(x^3-7x-6)dx=\left[\frac{1}{4}x^4-\frac{7}{2}x^2-6x\right]_{-1}^{3}$ $=\left(\frac{81}{4}-\frac{63}{2}-18\right)-\left(\frac{1}{4}-\frac{7}{2}+6\right)$ $=\left(20\tfrac{1}{4}-31\tfrac{1}{2}-18\right)-\left(\frac{1}{4}-3\tfrac{1}{2}+6\right)$ $=20-28-24=-32$ $iii)$ *Area shaded* $=32$ $iv)$ The gradient of the curve at B is $\frac{dy}{dx}(x=-1)$ $\frac{dy}{dx}=3x^2-7\ \ and\ \ for\ \ x=-1,\ \frac{dy}{dx}=3-7=-4$ $v)$ The normal has gradient $-\frac{1}{m_B}=-\frac{1}{-4}=\frac{1}{4}$ The equation of the normal at B: $y-0=\frac{1}{4}(x+1)$ $y=\frac{1}{4}(x+1)$	In part (a)(i), a few candidates ignored the request to use the factor theorem and scored no marks for using long division. It was necessary to make a statement that " x + 1 is a factor", after showing that f(–1) = 0, in order to score full marks. Part (a)(ii) was not answered as well as similar questions in previous years. Perhaps the sketch lured some into trying to write down three factors without any further working, rather than using the intermediate step of showing that $p(x) = (x+1)(x^2 - x - 6)$ before writing p(x) as a product of three factors. Many who tried long division were flummoxed by there being no x^2 term. In part (b)(i), those who had the correct linear factors in part (a)(ii) usually wrote down correctly that A had coordinates (–2, 0), although some carelessly wrote the point as (0,–2). Many candidates simply found an indefinite integral in part (b)(ii) and then a definite integral in part (b)(iii). The two parts were generously treated holistically when candidates did this. The fractions once again caused problems to most candidates who are so used to having a calculator to do this work for them. It was very rare to see the correct answer of – 32 for the definite integral. In part (b)(iii), many lost out on an easy mark because they rolled their two sections into one: those who wrote "integral = –32 =32" gained full credit for part (b)(ii) but did not score the mark in part (b)(iii). It was necessary to give a positive value for the area of the region and to make this explicit. In anticipation of a lot of wrong answers in part (b)(ii), a follow through mark was awarded in part (b)(iii): for example, if a candidate's answer in part (b)(ii) was –20 and they concluded that the area was 20 in part (b)(iii), they scored the mark. In part (b)(iv), most candidates differentiated correctly, but quite a few thought that $3(-1)^2 - 7$ was equal to –10 and thus obtained the wrong gradient of the curve. In part (b)(v), a large number of candidates found the correct equation of the normal but some still confused tangents and normals and consequently thought that the gradient of the normal was equal to –4. It was quite common for weaker candidates to either negate their gradient or take the reciprocal but to fail to do both.

Question 7:	Exam report
$Curve\ C: y = x^2 + 7$ $Line\ L: y = k(3x+1)$ $a)$ By identification, $(y =)$ $x^2 + 7 = k(3x+1)$ $x^2 - 3kx + 7 - k = 0$ $b)$ There are two points of intersection which means that discriminant >0 $(-3k)^2 - 4 \times 1 \times (7-k) > 0$ $9k^2 + 4k - 28 > 0$ $c)\ 9k^2 + 4k - 28 = (9k-14)(k+2) > 0$ $critical\ values: \frac{14}{9}\ and\ -2$ $solutions: \ k < -2\ or\ k > \frac{14}{9}$	In part (a), some weaker candidates did not realise how to derive the given equation, and others made algebraic slips when proving the printed result, or failed to write "= 0". In part (b), the condition for two distinct points of intersection required candidates to use the condition that $b^2 - 4ac > 0$ at any early stage of their argument. Those who simply wrote "> 0" on their final line of working, without any previous reference to the discriminant being positive, failed to convince the examiners that they deserved full marks. In part (c), quite a number were unable to factorise the quadratic correctly and many resorted to using the quadratic equation formula to find the critical values. Where this was done correctly but left in surd form, it was given due credit except for the final mark. Very able candidates can write down the answer to the inequality once they have factorised the quadratic but far too many guessed at answers and an approach using a sign diagram or sketch is recommended. Candidates also need to realise that the final form of the answer cannot be written as $\frac{14}{9} < k < -2$.

GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	59	51	43	36	29

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

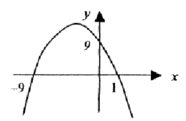
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks.**

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks.**

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	Mid-point of $BC = (3, -2)$	B1 B1	2	Either coordinate correct Both cords correct. Accept $x = 3$, $y = -2$
(b)(i)	$\frac{\Delta y}{\Delta x} = \frac{3-1}{-2-4}$	M1		$\pm\frac{2}{6}$ OE implies M1
	$= -\frac{1}{3}$	A1	2	
(ii)	$y - 3 =$ "their grad"$(x + 2)$ or $y - 1 =$ "their grad"$(x - 4)$	M1		Or $y = mx + c$ and correct attempt to find c
	Hence $x + 3y = 7$	A1	2	
(iii)	$y + 5 =$ "their grad AB"$(x - 2)$	M1		Or "their $x + qy = c$" and attempt to find c
	$y+5=-\frac{1}{3}(x-2)$ or $x + 3y + 13 = 0$	A1	2	OE
(c)	Grad $BC = 3$ (from $\frac{\Delta y}{\Delta x}=\frac{1+5}{4-2}$ OE)	B1		Or 2 lengths correct: $AB=\sqrt{40}; BC=\sqrt{40}; AC=\sqrt{80}$
	$m_1 m_2 = -1$ stated **or** grad $BC = 3$ and grad $AB - -\frac{1}{3}$ **or** grad $BC \times$ grad AB $(= 3\times -\frac{1}{3})$	M1		Or attempt at Pythagoras or Cosine Rule
	Product of gradients = –1 Hence AB and BC are perpendicular	A1 CSO	3	$AC^2 = AB^2 + BC^2 \Rightarrow \angle ABC = 90°$ Completing proof and statement
	Total		**11**	
2(a)	$\frac{dy}{dx} = 4x^3 - 32$	M1 A1 A1	3	Reduce one power by 1 One term correct All correct (no + c etc)
(b)	Stationary point $\Rightarrow \frac{dy}{dx} = 0$	M1		
	$\Rightarrow x^3 = 8$	A1√		$x^n = k$ following from their $\frac{dy}{dx}$
	$\Rightarrow x = 2$	A1	3	CSO
(c)(i)	$\frac{d^2y}{dx^2} = 12x^2$	B1√	1	FT their $\frac{dy}{dx}$
(ii)	When $x = 2$, $\frac{d^2y}{dx^2}$ considered	M1		Or complete test with $2 \pm \varepsilon$ using $\frac{dy}{dx}$
	$\Rightarrow$ minimum point	E1√	2	
(d)	Putting $x = 0$ into their $\frac{dy}{dx}$ $(= -32)$	M1		
	$\frac{dy}{dx} < 0 \Rightarrow$ decreasing	A1√	2	Allow "increasing" if their $\frac{dy}{dx} > 0$
	Total		**11**	

Q	Solution	Marks	Total	Comments
3(a)	$5\sqrt{8} = 10\sqrt{2}$	B1		Or $\frac{5\sqrt{16}+6}{\sqrt{2}}$ gets B1
	$\frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$ $(= 3\sqrt{2})$	M1		then M1 for rationalising; and A1 answer
	Answer = $13\sqrt{2}$	A1	3	$n = 13$
(b)	$\frac{\sqrt{2}+2}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4}$	M1		Multiplying top & bottom by $\pm(3\sqrt{2}+4)$
	Numerator = $6 + 6\sqrt{2} + 4\sqrt{2} + 8$	m1		Multiplying out (condone one slip)
	Denominator = 18 – 16 (= 2)	B1		
	Final answer = $5\sqrt{2}+7$	A1	4	
	Total		7	
4(a)	$x^2 + (y-5)^2$	B1		$b = 5$
	RHS = 5	B1	2	$k = 5$
(b)(i)	Centre (0, 5)	B1√	1	FT their b from part (a)
(ii)	Radius = $\sqrt{5}$	B1√	1	FT their k from part (a); RHS must be > 0
(c)(i)	$x^2 + 4x^2 - 20x + 20 = 0$	M1		May substitute into original or "their (a)"
	$\Rightarrow x^2 - 4x + 4 = 0$	A1	2	CSO; AG
(ii)	$(x-2)^2 = 0$ or $x = 2$	M1		
	Repeated root implies tangent	E1		Or $b^2 - 4ac$ shown = 0 plus statement
	Point of contact is $P(2, 4)$	A1	3	
(d)	$(CQ^2 =)\ 1^2 + 1^2$	M1		FT their C
	$\sqrt{2} < \sqrt{5} \Rightarrow Q$ lies inside circle	A1 CSO	2	CQ or CQ^2 OE must appear for A1
	Total		11	
5(a)	$(9+x)(1-x)$	M1		$\pm(9 \pm x)(1 \pm x)$
		A1	2	Correct factors
(b)	$25 - (x^2 + 8x + 16) = 9 - 8x - x^2$	B1	1	AG
(c)(i)	$x = -4$ is line of symmetry	B1	1	
(ii)	Vertex is (–4, 25)	B1,B1	2	
(iii)		M1		General ∩ shape
		B1		–9 and 1 marked on x-axis or stated
		A1	3	9 marked on y-axis and maximum to the left of y-axis Must continue below x-axis at both ends
	Total		9	

Q	Solution	Marks	Total	Comments
6(a)(i)	p(–1) = – 1 + 7 – 6	M1		Finding p(–1)
	= 0 therefore $x + 1$ is a factor	A1	2	Shown to = 0 **plus statement**
(ii)	$p(x) = (x+1)(x^2 - x - 6)$	M1		Long division/inspection (2 terms correct)
		A1		Quadratic factor correct
	$p(x) = (x+1)(x+2)(x-3)$	A1	3	May earn M1,A1 for correct second factor then A1 for $(x+1)(x+2)(x-3)$
(b)(i)	$A(-2, 0)$	B1	1	Condone $x = -2$
(ii)	$\frac{x^4}{4} - \frac{7x^2}{2} - 6x \quad (+c)$	M1		One term correct
		A1		Another term correct
	(may have + c or not)	A1		All correct unsimplified
	$\left[\frac{81}{4} - \frac{63}{2} - 18\right] - \left[\frac{1}{4} - \frac{7}{2} + 6\right]$	m1		F(3) – F(–1) attempted in correct order
	= – 32	A1	5	CSO; OE
(iii)	Area of shaded region = 32	B1√	1	FT their (b)(ii) but positive value needed
(iv)	$\frac{dy}{dx} = 3x^2 - 7$	M1		One term correct
		A1		All correct (no + c etc)
	When $x = -1$, gradient = – 4	A1	3	CSO
(v)	Gradient of normal = $\frac{1}{4}$	B1√		
	$y =$ "their gradient"$(x \pm 1)$	M1		Must be finding **normal**, not tangent
	$y = \frac{1}{4}(x+1)$	A1	3	CSO; any correct form eg $4y - x = 1$
	Total		18	
7(a)	$x^2 + 7 = k(3x+1) \Rightarrow x^2 - 3kx + 7 - k = 0$	B1	1	AG
(b)	$b^2 - 4ac = (-3k)^2 - 4(7-k)$	M1		Clear attempt at $b^2 - 4ac$ Condone slip in one term of expression
	(2 distinct roots when) $b^2 - 4ac > 0$	B1		Must involve k
	$9k^2 + 4k - 28 > 0$	A1	3	CSO; AG
(c)	$(9k - 14)(k + 2)$	M1		Factors or formula correct unsimplified
	Critical points –2 and $\frac{14}{9}$	A1		
	Sketch ∪ or sign diagram correct	M1		+ve | –ve | +ve ; –2 | $\frac{14}{9}$
	$k < -2,\ k > \frac{14}{9}$	A1	4	
	Total		8	
	TOTAL		75	