January 2006

Answer **all** questions.

1 (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)*

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*

2 The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x + 4y = 7$.

(a) (i) Show that $k = -2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

3 (a) (i) Express $x^2 - 4x + 9$ in the form $(x-p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. *(2 marks)*

(b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2 - 2x - 3 = 0$$

(1 mark)

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

4 The quadratic equation $x^2 + (m+4)x + (4m+1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

5 A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$

(3 marks)

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The point O has coordinates $(0, 0)$.

(i) Find the length of CO. *(2 marks)*

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. *(2 marks)*

6 The polynomial $p(x)$ is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Hence express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

7 The volume, V m^3, of water in a tank at time t seconds is given by

$$V = \tfrac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{dV}{dt}$; *(3 marks)*

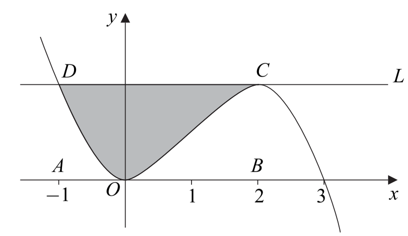
(ii) $\dfrac{d^2V}{dt^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in m^3 s^{-1}, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\dfrac{dy}{dx}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

END OF QUESTIONS

AQA – Core 1 - Jan 2006 – Answers

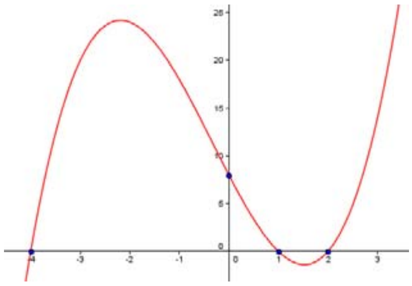
Question 1:	Exam report
$a)\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)=5-2\sqrt{5}+2\sqrt{5}-4=1$ $b)\sqrt{8}+\sqrt{18}=\sqrt{4\times 2}+\sqrt{9\times 2}=2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$	Many candidates earned full marks on this introductory question. (a) Most candidates multiplied out the two brackets to obtain four terms. The most common error occurred in the last term, which was sometimes seen as -2 instead of -4. Very few candidates recognised that it was the difference of two squares. (b) This part was less well done. Some candidates had problems simplifying $\sqrt{8}$ and $\sqrt{18}$ and wrote $2\sqrt{4}$ *and* $2\sqrt{9}$, for example. Some, having correctly converted both surds, added them incorrectly and so $6\sqrt{6}$ was quite common. A few candidates thought $\sqrt{8}+\sqrt{18}$ were equal to $\sqrt{26}$.

Question 2:	Exam report
$A(1,1)\quad B(5,k)$ $AB: 3x+4y=7$ $a)\,i)$ B belongs to the line so its coordinates satify the equation: $3\times 5+4\times k=7 \qquad 15+4k=7 \qquad k=-2$ $ii)\, I\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2}\right)=I\left(3,-\frac{1}{2}\right)$ $b)\, m_{AB}=\frac{y_B-y_A}{x_B-x_A}=\frac{-2-1}{5-1}=-\frac{3}{4}$ $c)\,i)\, m_{AC}=-\frac{1}{m_{AB}}=\frac{4}{3}$ $ii)\, AC: y-y_A=m_{AC}\left(x-x_A\right)$ $y-1=\frac{4}{3}(x-1)$ $3y-3=4x-4$ $4x-3y=1$ $iii)$ C belongs to the x-axis so $C(x_C,0)$ $4x_C-3\times 0=1 \qquad x_C=\frac{1}{4}$	(a)(i) Candidates used various methods to prove that k=-2. Some used the most direct method of substituting x = 5 into the given line equation and solving for y; some chose to verify that x = 5 and y =-2 satisfied the equation of the straight line. Others took a longer route; they found the gradient using (1,1) and (5,-2) and then found the equation passing through one of the points and proved it to be the given one. (a)(ii) Most candidates knew how to find the midpoint of a line. A few made a simplification error and wrote $\left(3,\frac{1}{2}\right)$ instead of $\left(3,-\frac{1}{2}\right)$.The common error amongst the weaker candidates was to subtract the coordinates instead of adding them. (b) Many candidates gave fully correct answers here. However, some, having obtained $\frac{-2-1}{5-1}$ wrote $\frac{3}{4}$ as a final answer. A few candidates used $\frac{x_1-x_2}{y_1-y_2}$. (c)(i) Most knew the gradient rule for perpendicular lines. However, not all could implement it since it involved the reciprocal of a fraction. (c)(ii) At least half of the candidates found the equation of the line passing through the midpoint of AB instead of through C. (iii) Most realised the need to substitute y = 0 into their AC equation and solve for x, so they at least earned the method mark. Even those with the correct equation did not always earn two marks. Some had difficulty in simplifying $\frac{1}{3}\div\frac{4}{3}$

Question 3:	Exam report
$i)\ x^2-4x+9=(x-2)^2-4+9=(x-2)^2+5$ $ii)$ for all $x, (x-2)^2\geq 0\ \ so\ \ (x-2)^2+5\geq 5$ The minimum y value is 5, obtained for $x=2$ $Min(2,5)$ $b)$ *Solve the equations* simultaneously: $\begin{cases} y=-2x+12 \\ y=x^2-4x+9 \end{cases}$ $This\ gives\ (y=)\,x^2-4x+9=-2x+12$ $x^2-2x-3=0$ $ii)\ x^2-2x-3=(x-3)(x+1)=0$ $x=3\ or\ x=-1$ $and\quad y=12-2x=6\ or\ y=14$ $(3,6)\ and\ (-1,14)$	(a)(i) Most candidates were familiar with the idea of .completing the square. and answered this part satisfactorily. There were occasional sign errors and +9-4 was not always evaluated correctly. (a)(ii) There were several correct answers although some wrote (5,-2) instead of (5,2). Some did not recognise the link between parts (i) and (ii) and chose to differentiate instead. This was a satisfactory, though more time-consuming, alternative method. Some earned no marks here as they wrote comments such as ".5 is the minimum", with no link to the y-coordinate being 5. (b)(i) This simple proof was usually well done. Occasionally the mark was lost due to the omission of " = 0"... (b)(ii) Many scored full marks here. Most factorised the equation and obtained the correct x-values. Some made no further progress, while a few substituted into the given quadratic equation and obtained y = 0, instead of using the equation of the line or curve to find the values of y. It was encouraging to see many factorising the quadratic correctly. Those who used the quadratic equation formula or completion of the square often made more errors than those who factorised

Question 4:	Exam report
$a)\ x^2+(m+4)x+(4m+1)=0\ has\ equal\ roots$ $so\ the$ discriminant $=0$: $(m+4)^2-4\times1\times(4m+1)=0$ $m^2+8m+16-16m-4=0$ $m^2-8m+12=0$ $b)\ m^2-8m+12=0$ $(m-6)(m-2)=0$ $m=6\ \ or\ \ m=2$	(a) There were several completely correct proofs here. Some lost the last mark by concentrating on the discriminant but failing to equate it to zero. There was a little fudging by some; for example, some who wrote -4(4*m*+1) = -16*m*+4 still managed to obtain the correct printed equation. Some of the weaker candidates found b^2-4ac using numerical values from the equation they were supposed to establish. (b) Almost all candidates found both values of *m* successfully. A few spotted just one answer and some factorised correctly and then wrote *m* = -2, *m* =-6, but they were in the minority.

Question 5:	Exam report
$a)\ x^2+y^2-8x+6y=11$ $(x-4)^2-16+(y+3)^2-9=11$ $(x-4)^3+(y+3)^2=36$ $b)i)\ Centre\ \ C(4,3)$ $ii)\ Radius\ \ r=\sqrt{36}=6$ $c)\ O(0,0)\quad C(4,3)$ $i)\ Length\ CO=\sqrt{(x_O-x_C)^2+(y_O-y_C)^2}$ $CO=\sqrt{4^2+3^2}=\sqrt{25}=CO=5$ $ii)\ Because\ CO<6, O$ lies INSIDE the circle.	(a) Completion of the squares in the circle equation was carried out well once more. The most common error was a sign slip usually in the second term. Another error lay in combining the constant terms, so answers such as -14 and 11 were seen for r^2. (b) Most earned the mark for the coordinates of the centre of the circle as this was a follow through mark. The mark for the radius was not always earned as some failed to take the square root or had an inappropriate answer such as a negative value for r^2. (c)(i) Most found *CO* to be 5. However, a few neglected to square -3 and 4 before adding and some subtracted 9 from 16. (c)(ii) This part was answered well with most realising the need to explain, using both lengths, why *O* lay inside or outside the circle. Some accompanied their explanations with diagrams, although this was not necessary.

Question 6:	Exam report
$a)\,i)\; p(x) = x^3 + x^2 - 10x + 8$ $p(2) = 2^3 + 2^2 - 10\times 2 + 8 = 8 + 4 - 20 + 8 = 0$ 2 *is a root of* p *so* $(x-2)$ *is a factor of* p. $ii)\; x^3 + x^2 - 10x + 8 = (x-2)(x^2+3x-4) = (x-2)(x+4)(x-1)$ $\begin{array}{r\|l} & x^2+3x-4 \\ \hline x-2 & x^3+x^2-10x+8 \\ & \underline{x^3-2x^2} \\ & 3x^2-10x \\ & \underline{3x^2-6x} \\ & -4x+8 \\ & \underline{-4x+8} \\ & 0 \end{array}$ $b)$ The graph cuts the axes at (2,0), (-4,0), (1,0) and (8,0). 	(a)(i) It was good to see that almost all candidates started correctly by evaluating p(2), though a few thought they needed to find p(-2) and others wrongly assumed that long division was the "factor theorem". It was necessary to write a conclusion or statement after showing that p(2) = 0 , in order to earn the second mark. (a)(ii) The most successful approach here was by using a quadratic factor (ax^2+bx+c), though long division also worked well for many. A surprising number who found the correct quadratic then factorised it wrongly. Those who tried the factor theorem again rarely spotted both factors. A few lost the final mark by failing to write p(*x*) as a product of factors. (b) Although there were many correct sketches, many lost a mark by failing to mark the point (0,8) on the *y*-axis. Candidates were expected to draw a cubic through their intercepts, to use an approximately linear scale and to continue the graph beyond the intercepts on the *x*-axis. It was common to see the negative values in the factors wrongly taken to be the roots and hence the intercepts on the *x*-axis.

Question 7:	Exam report
$a)\,i)\; \frac{dV}{dt} = \frac{1}{3}\times 6t^5 - 2\times 4t^3 + 3\times 2t = 2t^5 - 8t^3 + 6t$ $ii)\; \frac{d^2V}{dt^2} = 10t^4 - 24t^2 + 6$ $b)$ The rate of change is $\frac{dV}{dt}$ $so\; \frac{dV}{dt}(t=2) = 2\times 2^5 - 8\times 2^3 + 6\times 2 = 64 - 64 + 12 = 12m^3/s$ $c)\,i)\; \frac{dV}{dt}(t=1) = 2\times 1^5 - 8\times 1^3 + 6\times 1 = 2 - 8 + 6 = 0$ V has a stationary point when $t = 1$ $ii)\; \frac{d^2V}{dt^2}(t=1) = 10\times 1^4 - 24\times 1^2 + 6 = 10 - 24 + 6 = -8 < 0$ The stationary point is a MAXIMUM.	(a)(i) The differentiation was generally well done, though some candidates found the fractional coefficient problematic. Those who wrote $\frac{6}{3}t^5$ were not penalised in this part but writing $6.\frac{1}{3}$ generally led to errors later in the question. Some tried to avoid the fraction by considering 3*V* throughout the question, making errors, and suffered a heavy penalty. (a)(ii) Again, most applied the method correctly and here simplification of coefficients was necessary. A few failed to differentiate 6*t* or omitted it altogether. (b) This part was poorly done with many not recognising that the rate of change was $\frac{dV}{dt}$ and substituted *t* = 2 into the expression for *V* or $\frac{d^2V}{dt^2}$. Quite a lot of candidates made arithmetic errors. A few found two values of the expression and averaged them. (c)(i) Again, many failed to evaluate $\frac{dV}{dt}$ in order to verify that a stationary point occurred, but those who did generally obtained a value of zero. It was essential to include a relevant statement to earn both marks. (c)(ii) This required evaluation of the second derivative at *t* = 1 or an appropriate test. Candidates who tried to test the gradient on either side of 1 almost invariably failed, as the values used were too far away from the stationary point. A surprising number evaluated 10 -24+6 to be -20 thus losing the accuracy mark. Some appeared to be guessing and drew wrong conclusions about maxima or minima after evaluating the second derivative.

Question 8:	Exam report
$y = 3x^2 - x^3 \quad A(-1,0)\ B(2,0)$ $a)\ y(2) = 3\times 2^2 - 2^3 = 12 - 8 = 4 \quad C(2,4)\ D(-1,4)$ The area of the rectangle is $3\times 4 = 12$ $b)i) \int (3x^2 - x^3)\,dx = x^3 - \frac{1}{4}x^4 + c$ $ii)\ Area = 12 - \int_{-1}^{2}(3x^2 - x^3)dx = 12 - \left[x^3 - \frac{1}{4}x^4\right]_{-1}^{2}$ $Area = 12 - (8-4) + \left(-1 - \frac{1}{4}\right) = 6\frac{3}{4}$ $c)i) \frac{dy}{dx} = 6x - 3x^2$ $ii) \frac{dy}{dx}(x=1) = 6 - 3 = 3$ $y(1) = 3 - 2 = 2$ Equation of the tangent : $y - 2 = 3(x-1)$ $y = 3x - 1$ $iii)$ y is decreasing when $\frac{dy}{dx} < 0$ $6x - 3x^2 < 0$ $(\div -3) \quad x^2 - 2x > 0$ $d)\ x^2 - 2x = x(x-2) > 0 \quad$ for $x < 0$ or $x > 2$	(a) Not everyone recognised that the height of the rectangle was the value of *y* when *x* = -1 or *x* = 2. Some who did made numerical errors. Even having found the height 4, some obtained the wrong area by taking *AB* as 4 or 2 (sometimes using Pythagoras' .Theorem) or by finding the perimeter instead. (b)(i) The integral was generally correct, though sometimes incorrect simplification occurred subsequently. A few integrated x^3 to $\frac{1}{3}x^4$ and a surprising number misread the integrand as $3x^2 - x^2$. There were also candidates who confused integration with differentiation or whose process was a hybrid of the two. (b)(ii) Almost everyone recognised that they should firstly evaluate the integral from -1 to 2 , but most stopped there, instead of going on to subtract the value of the integral from the area of the rectangle. There were a lot of sign errors in the work with some adding instead of subtracting or putting the two parts the wrong way round. A few wrongly substituted in the original function. Those who chose to work with the differences of two integrals seldom completed it correctly. (c)(i) Differentiation was done well on the whole. (c)(ii) Many substituted *x* = 1 into the derivative to find the gradient of the tangent and went no further. Most did not find the *y* coordinate of the point and so made no attempt at the equation of the tangent. A few non-linear equations were seen with a .gradient. of $6x - 3x^2$. (c)(iii) Very few candidates completed this part. Many made no attempt, and those who did tended to test a few values of *x* or to find the second derivative, which was of no value. Only the strongest candidates realised that $\frac{dy}{dx} < 0$ was the condition for *y* to be decreasing and that, after a couple of lines of algebra, the given inequality could be obtained. (d) Although most made an attempt at the quadratic inequality, few obtained both parts of the solution. It was imperative that candidates wrote $x > 2$, $x < 0$ and not $0 > x > 2$ as many incorrectly stated. It was disappointing to see how many candidates at this level could not solve the equation $x^2 - 2x = 0$, obtaining values such as -2, √2, 1+√2. Using the formula or completing the square sometimes led to 1±√1, which many candidates failed to simplify.

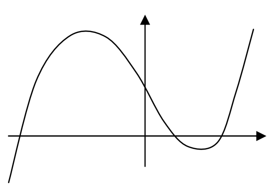
GRADE BOUNDARIES						
Component title	Max mark	A	B	C	D	E
Core 1 – Unit PC1	75	61	53	45	38	31

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

1(a)	$\left(\sqrt{5}\right)^2 + 2\sqrt{5} - 2\sqrt{5} - 4 = 1$	M1 A1	 2	Multiplying out or difference of two squares attempted Full marks for correct answer /no working
(b)	$\sqrt{8} = 2\sqrt{2}$; $\sqrt{18} = 3\sqrt{2}$ Answer $= 5\sqrt{2}$	M1 A1	 2	Either correct Full marks for correct answer /no working
	Total		**4**	
2(a)(i)	$15 + 4k = 7 \quad \Rightarrow 4k = -8 \quad \Rightarrow k = -2$	B1	1	**AG** (condone verification or $y = -2$)
(ii)	$\frac{1}{2}(x_1 + x_2)$ or $\frac{1}{2}(y_1 + y_2)$ Midpoint coordinates $\left(3, -\frac{1}{2}\right)$	M1 A1	 2	 One coordinate correct implies M1
(b)	Attempt at $\Delta y / \Delta x$ or $y = -\frac{3}{4}x + \frac{7}{4}$ Gradient $AB = -\frac{3}{4}$	M1 A1	 2	(Not x over y)(may use M instead of A/B) –0.75 etc any correct equivalent
(c)(i)	$m_1 m_2 = -1$ used or stated Hence gradient $AC = \frac{4}{3}$	1 A1√	 2	 Follow through their gradient of AB from part (b)
(ii)	$y - 1 = \frac{4}{3}(x - 1)$ or $3y = 4x - 1$ etc	B1√	1	Follow through their gradient of AC from part (c) (i) must be **normal** & (1,1) used
(iii)	$y = 0 \qquad \Rightarrow x - 1 = -\frac{3}{4}$ $x = \frac{1}{4}$	M1 A1	 2	Putting $y = 0$ in their AC equation and attempting to find x **CSO**. C has coordinates $\left(\frac{1}{4}, 0\right)$
	Total		**10**	
3(a)(i)	$(x - 2)^2$ $+ 5$	B1 B1	 2	$p = 2$ $q = 5$
(ii)	Minimum point (2, 5) or $x = 2$, $y = 5$	B2√	2	B1 for each coordinate correct or ft **Alt method** M1, A1 sketch, differentiation
(b)(i)	$12 - 2x = x^2 - 4x + 9$ $\Rightarrow x^2 - 2x - 3 = 0$	 B1	 1	Or $x^2 - 4x + 9 + 2x = 12$ **AG** (be convinced) (must have = 0)
(ii)	$(x - 3)(x + 1) = 0$ $x = 3, \quad -1$ Substitute one value of x to find y Points are (3, 6) and (–1, 14)	M1 A1 M1 A1	 4	Attempt at factors or quadratic formula or one value spotted Both values correct & simplified May substitute into equation for L or C y-coordinates correct linked to x values
	Total		**9**	

4(a)	$(m+4)^2 = m^2+8m+16$ $b^2-4ac=(m+4)^2-4(4m+1)=0$ $m^2+8m+16-16m-4=0$ $\Rightarrow m^2-8m+12=0$	B1 M1 A1	 3	Condone $4m+4m$ b^2-4ac (attempted and involving m's and no x's) or $b^2-4ac=0$ stated **AG** (be convinced – all working correct- $=0$ appearing more than right at the end)
(b)	$(m-2)(m-6)=0$ $m=2$, $m=6$	M1 A1	 2	Attempt at factors or quadratic formula **SC B1** for 2 or 6 only without working
	Total		**5**	
5(a)	$(x-4)^2+(y+3)^2$ $(11+16+9=36)$ RHS $=6^2$	B2 B1	 3	B1 for one term correct Condone 36
(b)(i) **(ii)**	Centre (4 , –3) Radius = 6	B1√ B1√	1 1	Ft their *a* and *b* from part (a) Ft their *r* from part (a)
(c)(i)	$CO^2=(-4)^2+3^2$ $CO=5$	M1 A1√	 2	Accept + or – with numbers but must add Full marks for answer only
(ii)	Considering CO and radius $CO<r \Rightarrow O$ is **inside** the circle	M1 A1√	 2	Ft outside circle when 'their CO' $>r$ or on the circle when 'their CO' $=r$ **SC** B1√ *if no explanation given*
	Total		**9**	
6(a)(i)	$p(2)=8+4-20+8$ $=0$, $\Rightarrow$ $x-2$ is a factor	M1 A1	 2	Finding p(2) M0 *long division* Shown = 0 **AND** conclusion/ statement about *x* –2 being a factor
(ii)	Attempt at quadratic factor x^2+3x-4 $p(x)=(x-2)(x+4)(x-1)$	M1 A1 A1	 3	or factor theorem again for 2nd factor or (*x*+4) or (*x*–1) proved to be a factor
(b)		B1 B1√ M1 A1	 4	Graph through (0,8) 8 marked Ft "their factors" 3 roots marked on *x*-axis Cubic curve through their 3 points Correct including *x*- intercepts correct Condone max on *y*-axis etc or slightly wrong concavity at ends of graph
	Total		**9**	

MPC1 (cont)

	Solution	Marks	Total	Comments
7(a)(i)	$\frac{dV}{dt}=2t^5-8t^3+6t$	M1 A1 A1	 3	One term correct unsimplified Further term correct unsimplified All correct unsimplified (no + c etc)
(ii)	$\frac{d^2V}{dt^2}=10t^4-24t^2+6$	M1 A1	 2	One term FT correct unsimplified **CSO**. All correct simplified
(b)	Substitute $t=2$ into their $\frac{dV}{dt}$ $(=64-64+12)=12$	M1 A1	 2	 **CSO**. Rate of change of volume is $12\text{m}^3\ \text{s}^{-1}$
(c)(i)	$t=1 \Rightarrow \frac{dV}{dt}=2-8+6$ $=0 \Rightarrow$ Stationary value	M1 A1	 2	Or putting their $\frac{dV}{dt}=0$ **CSO**. Shown to = 0 **AND** statement (If solving equation must obtain $t=1$)
(ii)	$t=1 \Rightarrow \frac{d^2V}{dt^2}=-8$ Maximum value	M1 A1√	 2	Sub $t=1$ into their second derivative or equivalent full test. Ft if their test implies minimum
	Total		**11**	
8(a)	$y_D=3+1=4$ or $y_C=12-8=4$ Area $ABCD$ $=3\times4=12$	M1 A1	 2	Attempt at either *y* coordinate
(b)(i)	$x^3-\frac{x^4}{4}$ $(+C)$	M1 A1 A1	 3	Increase one power by 1 One term correct unsimplified All correct unsimplified (condone no +C)
(ii)	Sub limits –1 and 2 into their (b) (i) ans $[8-4]-\left[-1-\frac{1}{4}\right] = 5\frac{1}{4}$ Shaded area = "their" (rectangle– integral) $=12-5\frac{1}{4}=6\frac{3}{4}$	M1 A1 M1 A1	 4	May use both –1, 0 and 0, 2 instead Alt method: difference of two integrals **CSO**. Attempted M2, A2
(c)(i)	$\frac{dy}{dx}=6x-3x^2$	M1 A1	 2	One term correct All correct (no +C etc)
(ii)	When $x=1$, $y=2$ when $x=1$, $\frac{dy}{dx}=3$ as 'their' grad of tgt Tangent is $y-2=3(x-1)$	B1 M1√ A1	 3	May be implied by correct tgt equation Ft their derivative when $x=1$ Any correct form $y=3x-1$ etc
(iii)	Decreasing when $\frac{dy}{dx}=6x-3x^2<0$ $3(2x-x^2)<0 \Rightarrow x^2-2x>0$	M1 A1	 2	Watch no fudging here!! May work backwards in proof. **AG** (be convinced no step incorrect)
(d)	Two critical points 0 and 2 $x>2$, $x<0$ ONLY	M1 A1	 2	Marked on diagram or in solution or M1 A0 for $0<x<2$ or $0>x>2$ **SC** B1 for $x>2$ (or $x<0$)
	Total		**18**	
	TOTAL		**75**	